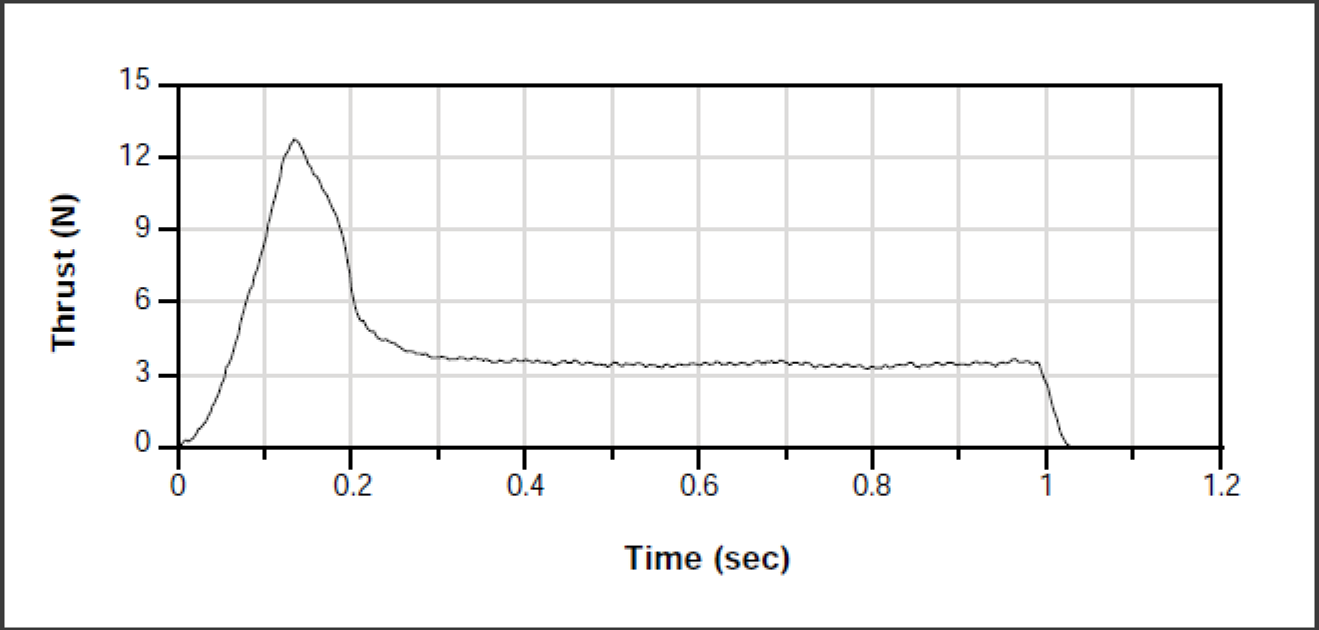
Thrust and Flight Modeling

- So… just how did OpenRocket or Rocksim predict the trajectory, velocity, and acceleration of our vehicle?
- What did we input into these numerical tools?

Rocket Motors

- Device for turning chemical (thermal) energy into kinetic energy
- Thrust = $\dot{m}V_e + (P_e - P_o)A_e$
  - The faster we can get the exit velocity out of the nozzle, the more thrust we get

Thrust Curve of an Estes B4 Motor

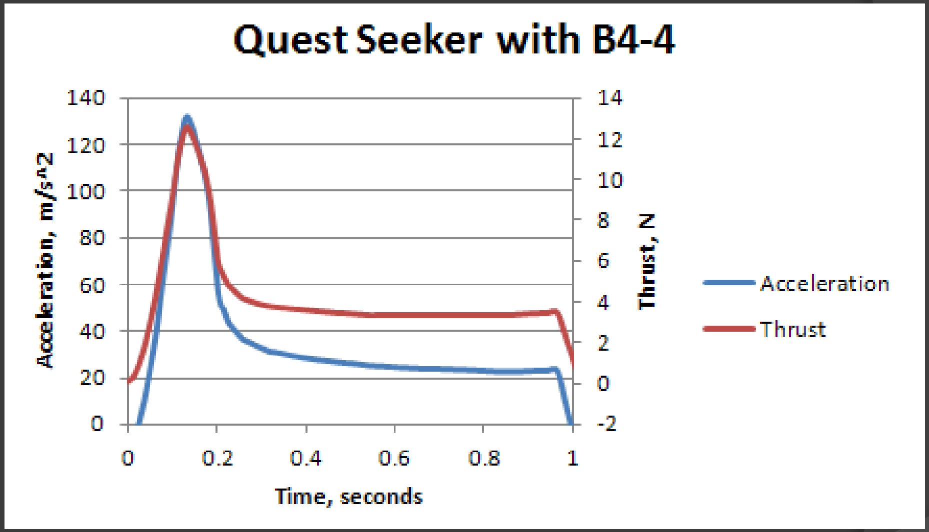


Thrust Curve Input

- We need the thrust over the time the rocket motor is firing
- OpenRocket and Rocksim have thrust curves for many motors built in
- See thrustcurve.org as well
- Also thrust curve data are available on E80 web site for previous static motor firings

What does thrust do to our vehicle?

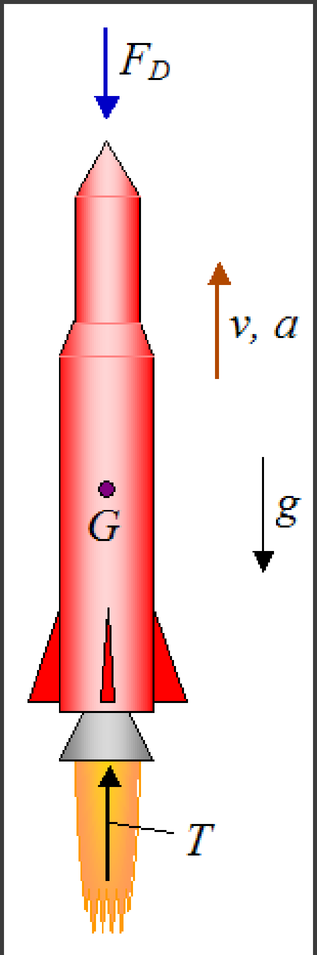
- OpenRocket prediction of acceleration vs. time for Quest Seeker with a B4-4 motor
- Secondary y-axis shows thrust of B4-4 motor



What will affect our rocket's flight?

- So we know thrust is important.
- What else?

Vertical Powered Flight



http://www.real-world-physics-problems.com/rocket-physics.html

Vertical Powered Flight

- Thrust
  - From thrust curve
- Weight
- Drag force
  - How do we get this?
    - Coefficient of drag
    - $F_D = \frac{1}{2}\rho V^2 \mathrm{A} C_D$
    - NOTE: Drag is a function of velocity

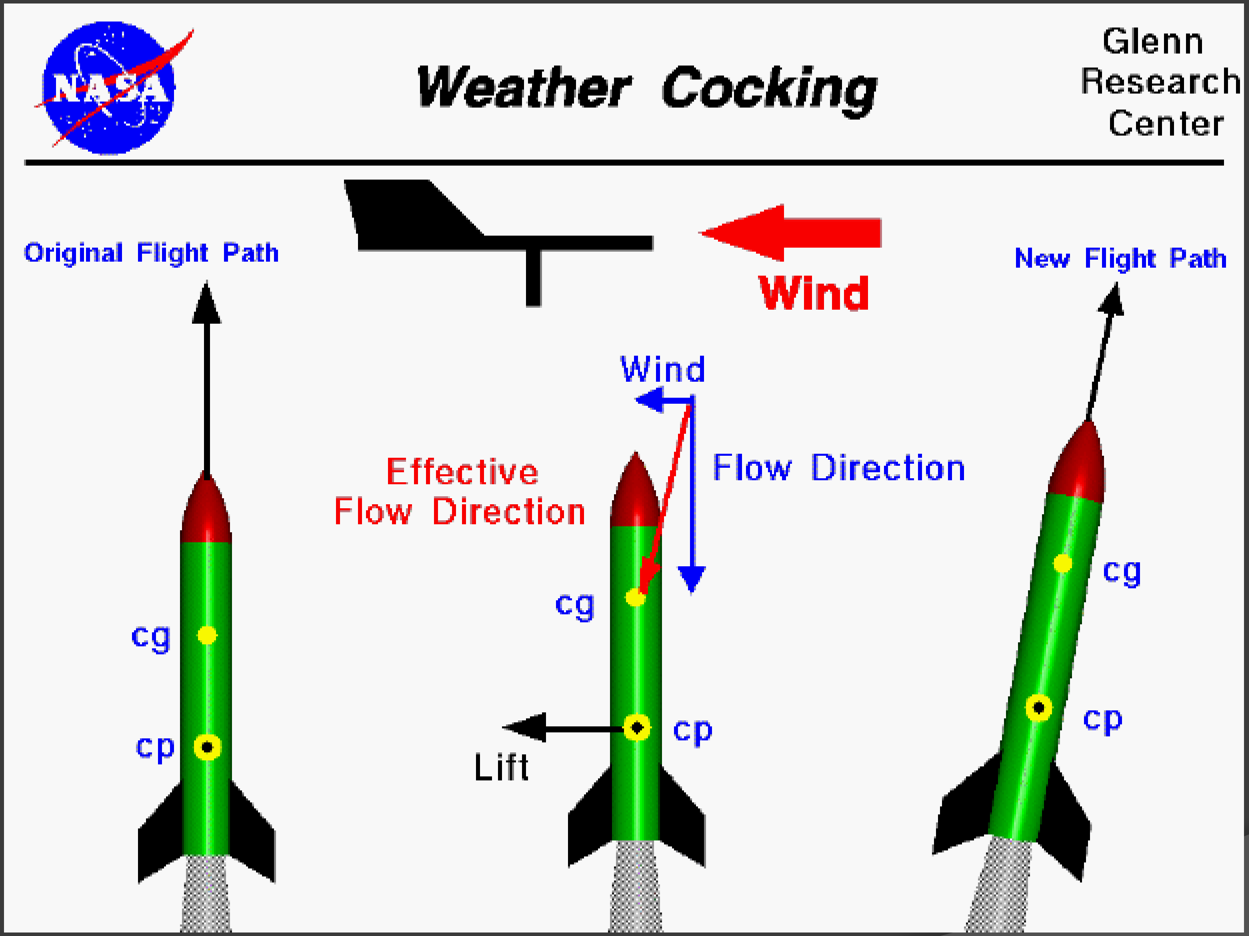
1DOF Equations of Motion

- Newton's 2nd Law: sum of forces = mass times acceleration (linear momentum)
- All is good when we are in vertical flight
  - Vehicle acceleration is along what we think of as "vertical"
  - Thrust, weight, drag all lined up
- $ma = \mathrm{m}\frac{dv}{dt}$=Thrust – Drag – Weight
- Does the rocket always fly vertically during flight?

Weather Cocking

http://g3nqf.co.uk/weather-vane-not-a-weather-cock

Rocket path is not always vertical



http://www.grc.nasa.gov/WWW/k-12/VirtualAero/BottleRocket/airplane/rktcock.html

Weather Cocking

- Lift is generated, since the rocket is at an angle of attack to the wind
- Where does the lift force act?
  - Center of Pressure (Cp)
  - Is this the same thing as the center of mass?
- Due to the lift force and location of the Cp, a moment is generated about the CG and the rocket **rotates**

Weather Cocking

- When does it stop rotating?
  - When the rocket is aligned in the direction of the relative wind (when the angle of attack is zero), the lift force goes to zero, and so does the moment due to that lift force
- Well, ideally it stops rotating…
- When designing our vehicle we need to take into account damping, as a well-designed rocket is probably not critically damped or overdamped

Damping coefficient (E59 stuff)

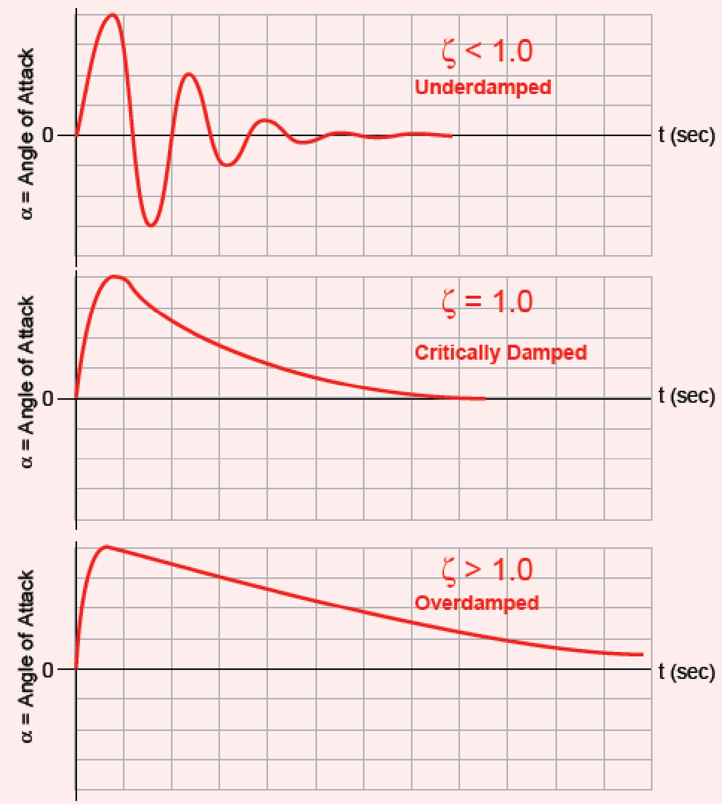


Figure 1: The damping ratio tells us how a rocket is going to respond to a disturbance in flight. For example, whether or not it will oscillate.

http://www.apogeerockets.com/downloads/Newsletter197.pdf

Dynamic characteristics

- Usually we have to trade-off between increase in drag (maybe bigger fins) and living with some oscillations
- Topics in Advanced Rocketry recommends a damping coefficient between 0.05 and 0.3
- Rocksim will calculate the damping ratio explicitly, and that you can probably do a hand-calc using information from OpenRocket (do a google search)

Back to Trajectory Modeling

- So, in order to predict our trajectory if we have a side wind, we'll need
  - Aerodynamic forces
  - Angle of attack
  - Rocket geometry
  - Windspeed
  - …

Trajectory Modeling

- We still write Newton's 2nd Law, but what do we also have to write now to describe the rotation?
- What else happened that we need to account for?

3DOF Equations of Motion

- Need to solve angular momentum equation until the rocket has rotated into the direction of the relative wind (until angle of attack is zero)
  - $J\frac{d\omega}{dt}$=M
- Need to solve two linear momentum equations
- But now we have local and global coordinate systems, and as the rocket rotates, it continually changes orientation

Coordinate Frames

- Thrust and Aerodynamic Forces are relative to the vehicle (local) frame
  - Note: we are now taking "Lift" as the force normal to the rocket long axis and not perpendicular to the relative wind
  - Why?
- Weight is easy in the global frame (though not at all difficult in the local frame)
- However, we are after a trajectory (position) in the global frame
  - Euler Rotation Theorem
  - Quaternions

How to solve eqns of motion numerically?

- Could do
  - Explicit Euler
  - Implicit Euler
  - Runge-Kutta (OpenRocket does RK4)
- Recall your differencing equations from the First Flight Lab
- $v(t + \Delta t) = v(t) + \frac{dv}{dt}\Delta t + \frac{d^2v}{dt^2}\frac{\Delta t^2}{2} + \cdots$

Explicit Euler

- $\frac{dv}{dt} = \frac{v(t+\Delta t)-v(t)}{\Delta t}$ (ignoring H.O.T.)
- $m(\frac{v(t+\Delta t)-v(t)}{\Delta t})$=Thrust(t)-Drag(t)-Weight(t)
- $v(t+\Delta t)=\frac{\Delta t}{m}$ [Thrust(t)-Drag(t)-Weight(t)] +$v(t)$
- We know everything on the RHS at time t
- We calculate our new velocity at time $t+\Delta t$
- Then march forward in time
  - Consider the size of your time step if you choose Explicit Euler (Prof. Spjut's video lecture from last year does a demo)
  - Also see the VIs and an excel spreadsheet linked to the lab for examples

1DOF vs. 3DOF

- 1DOF = one degree of freedom
  - Rocket goes up vertically, no side wind, therefore no rotation
  - Solve one linear momentum equation
    - Can predict maximum altitude
- 3DOF = three degrees of freedom
  - Side wind causes rocket to rotate into wind
  - Need to solve one angular momentum equation as well as two linear momentum equations
    - End up predicting trajectory in two coordinates
- 6DOF = six degrees of freedom
  - When you absolutely positively got to nail down every $^*#& variable in the room[1] (3 linear momentum equations; 3 angular momentum equations)

1.) Apologies to Quentin Tarantino ("Jackie Brown")

http://www.rocketdynearchives.com/areai/bowlarea.html

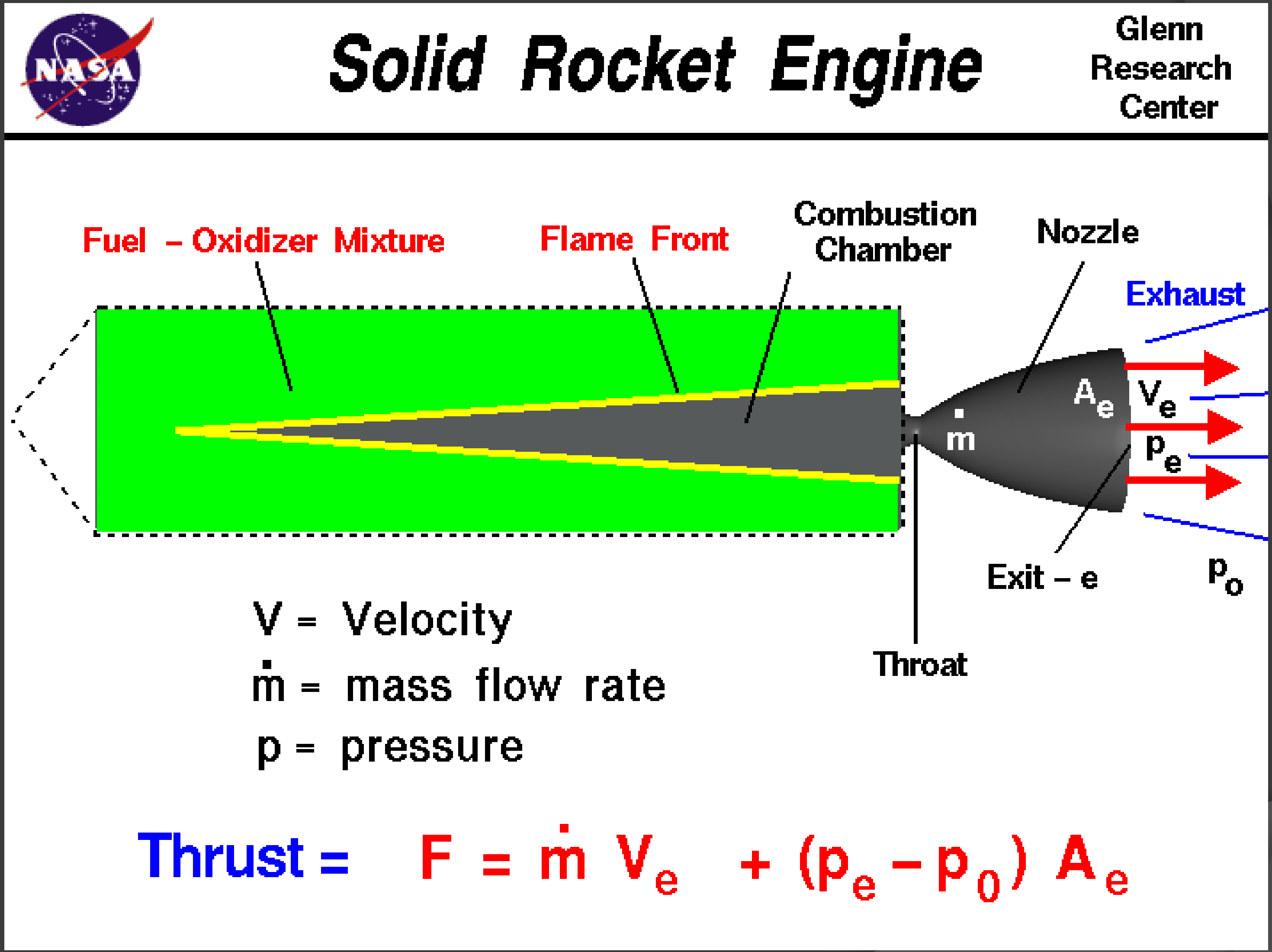
http://crgis.ndc.nasa.gov/historic/File:01-086-30.jpg

Test Stand Videos

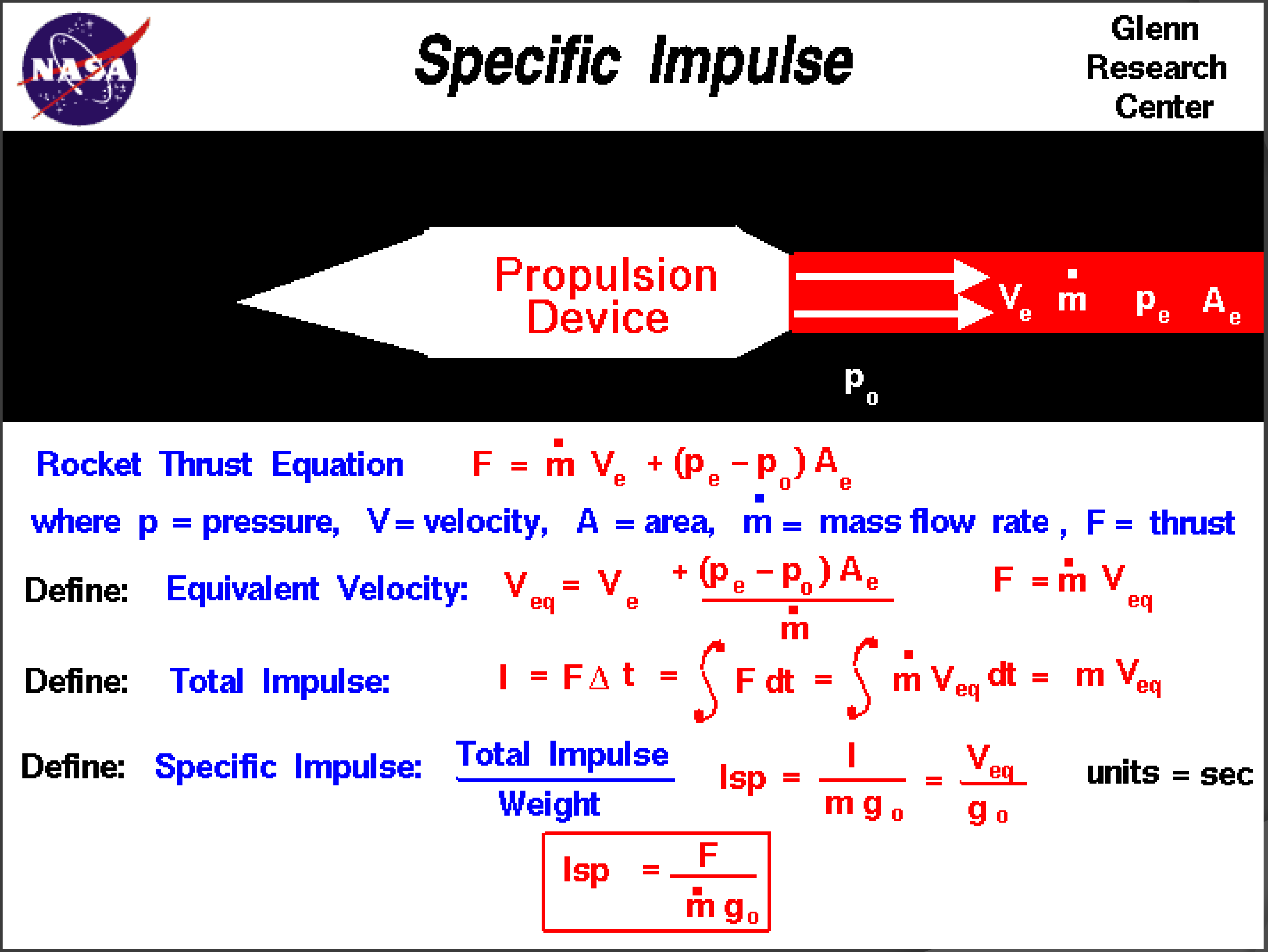
- https://www.youtube.com/watch?v=TaJhrFQoxh8
- https://www.youtube.com/watch?v=rsPnkJ5vLfc

Rocketdyne Santa Susana Field Laboratory

- Aside: Horribly contaminated site, Simi Valley residents and Rocketdyne workers justifiably upset about this
- Nuclear reactor test site—uranium fuel rods ruptured and partially melted in 1959
  - No containment
- Runoff of various contaminants
  - Groundwater contamination (perchlorate)



http://www.grc.nasa.gov/WWW/k-12/airplane/Images/srockth.gif



http://www.grc.nasa.gov/WWW/k-12/airplane/Images/specimp.gif

Solid Motors

Propellant grains

O-rings

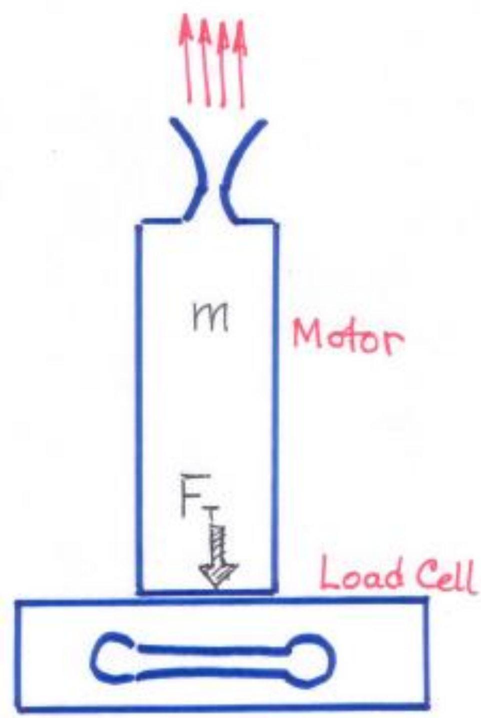
Nozzle

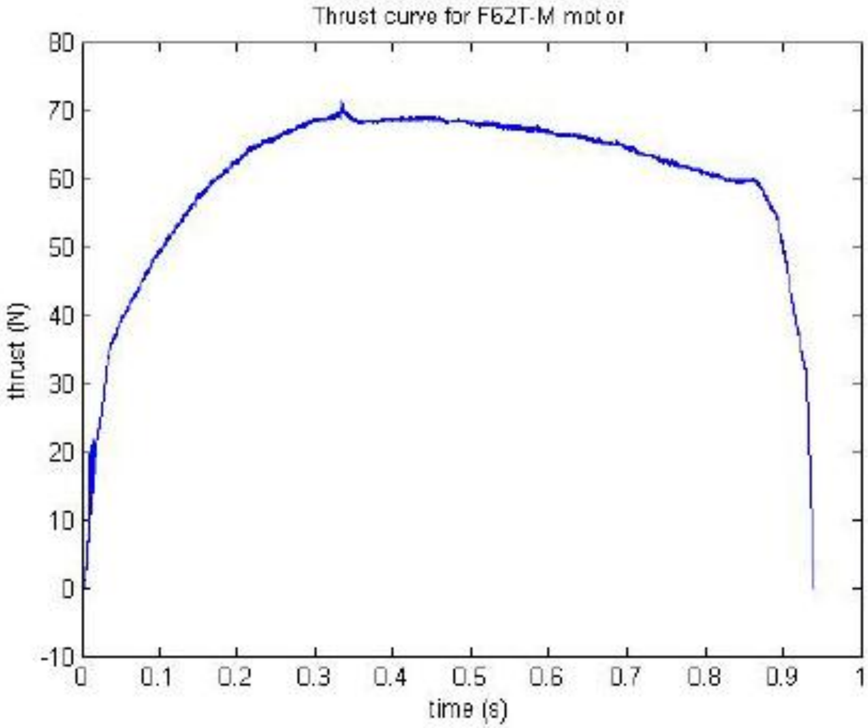
Delay grain

Solid rocket motor

- https://www.youtube.com/watch?v=YcDwAJNAjnU

Static Motor Test





http://www.eng.hmc.edu/NewE80/PDFs/E80StaticMotorTestMotion2012.pdf

Static Motor Test

$$F_T = \dot{m}V_e + \left(P_e - P_0\right)A_e = \dot{m}V_{eq}$$

$$\dot{m} = \frac{m_{final} - m_{initial}}{t_{burn}} = \frac{\Delta m}{t_{burn}}$$

$$V_{eq} \cong \frac{F_{T,average}}{\dot{m}}$$

http://www.eng.hmc.edu/NewE80/PDFs/E80StaticMotorTestMotion2012.pdf

Propellant

- Estes motors: black powder--charcoal, sulfur, and saltpeter (potassium nitrate)
- Aerotech motors: composite propellant--ammonium perchlorate, synthetic rubber and a metal fuel like aluminum

Area-Velocity Ratio (Nozzles)

- $\frac{dA}{A} = (M^2 - 1)\frac{dV}{V}$
- 1-D, steady, isentropic, no viscous forces
- What does this say for subsonic flow?
  - $0 \leq M < 1$

Subsonic Flow

- $\frac{dA}{A} = (M^2 - 1)\frac{dV}{V}$
- $0 \leq M < 1$
- M^2-1 is a negative number
- So an increase in velocity (dV > 0) means we have a decrease in area (dA < 0) and vice versa
- To speed up a subsonic flow, we decrease area
  - This probably matches your intuition from physical experience
- What about supersonic flow? M > 1

Supersonic Flow

- $\frac{dA}{A} = (M^2 - 1)\frac{dV}{V}$
- M > 1
- M^2-1 is a positive number
- Here, an increase in velocity (dV >0) is accomplished by dA > 0
  - Increase in area leads to a faster flow
- Why?

Compressible vs. Incompressible Flow

- Recall we assume incomp flow for M < 0.3
  - Density is not constant in supersonic flow
- $\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \text{constant}$
  - For incomp flow, density cancels out and we get $A_1 V_1 = A_2 V_2$ = constant
    - Decrease in area means we have an increase in velocity to keep mass flow rate constant

Compressible Flow

- $\dot{m}=\rho_1 A_1 V_1 = \rho_2 A_2 V_2$=constant
- What must be happening if we have to increase area to get a higher velocity in supersonic flow?

Compressible Flow

- $\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$=constant
- The density decreases faster than the velocity increases, so the area must increase to maintain mass continuity
  - Probably not intuitive since we mostly do not live in a supersonic world

Sonic

- M=1
- Area-velocity equation says dA=0
  - This will be a local minimum in area
  - Termed “throat” area
- Why minimum?

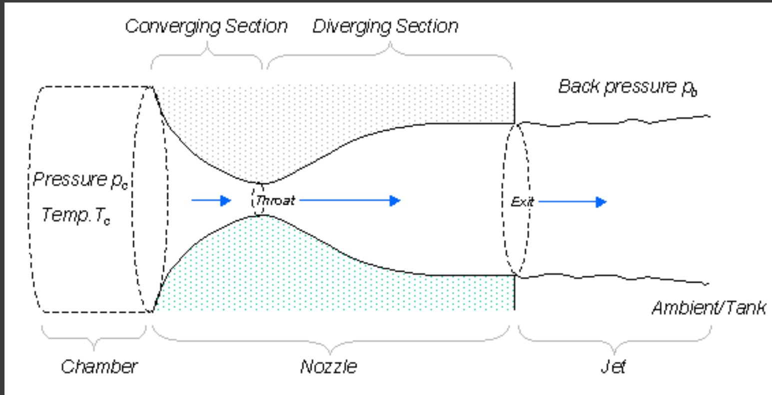


Figure 1. Converging Diverging Nozzle Configuration

http://www.engapplets.vt.edu/fluids/CDnozzle/fig1.gif

E80 Static Motor Test Lab

- Calibrate Load Cell
- Measure Thrust Curve of Rocket Motor
- Measure average mass flow rate
- Model 1DOF and 3DOF flight trajectory
- Compare with OpenRocket or Rocksim

http://www.eng.hmc.edu/NewE80/PDFs/E80StaticMotorTestMotion2012.pdf