E80 S'08 Thrust & Kinetics Lecture

What is Newton II?

Usually written as

$$\sum \vec{F} = m\vec{a}$$

Oversimplification

$$\sum \vec{F} = \frac{d}{dt}(m\vec{v}) = \dot{m}\vec{v} + m\vec{a}$$

For constant m, we get usual.

What makes rockets fly?

High velocity substances exiting out the back.

At steady state $\dot{m}$ and $\vec{v}$ of the gases are constant so

$$\sum \vec{F} = \dot{m}\vec{v} \quad \text{at steady state}$$

$$F = \dot{m}v \quad \text{1-D}$$

For the transient

$$F = \frac{d}{dt}(mv) \quad \text{1-D}$$

Note: The mass of the rocket is changing.

Where do you get the hot gases?

Usually from combustion or oxidation reactions.

Reaction kinetics

For solid propellant - heterogeneous R.K.

Before we go there, we need some fluid mechanics. and some thermo.

It can be shown for an ideal gas that

$$c_v dT = T d\hat{s} - P d\hat{V}$$

c_v - heat capacity at constant volume

T - temperature (abs)

$\hat{s}$ - specific entropy

P - absolute pressure

$\hat{V}$ - specific volume

and.

$$d\hat{s} = \frac{C_V}{T}dT + R\frac{d\hat{V}}{\hat{V}}$$

or

$$d\hat{s} = \frac{C_P}{T}dT - R\frac{dP}{P}$$

C_p - heat capacity at const P

R - universal gas constant = $N_A k$

If we design our nozzle carefully

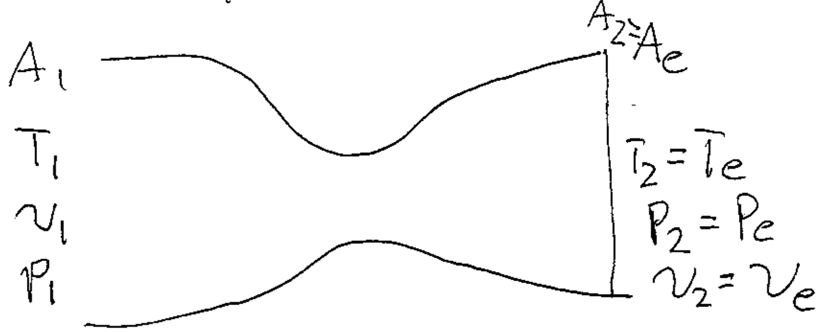
$d\hat{s} = 0$ or $\Delta\hat{s} = 0$ Adiabatic Reversible

$$\frac{C_P}{T}dT = R\frac{dP}{P}$$

If $C_p \neq C_p(T)$ Never true, usually acceptable

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{R}{C_P}} = \left(\frac{P_2}{P_1}\right)^{\left[1-\frac{1}{\gamma}\right]} \qquad \gamma \equiv \frac{C_P}{C_V}$$

If we analyze the nozzle with 1st Law



$$\Delta \dot{H} + \Delta \dot{E}_k = \cancelto{0}{\dot{Q}} - \cancelto{0}{\dot{W}_s}$$

$$\dot{m}\left(\hat{H}_2 - \hat{H}_1\right) + \frac{1}{2}\dot{m}\left(v_2^2 - v_1^2\right) = 0$$

$$\hat{H}_2 - \hat{H}_1 = c_p\left(T_2 - T_1\right)$$

$$v_2^2 - v_1^2 = 2c_p\left(T_2 - T_1\right) = 2c_p T_1\left[1 - \left(\frac{P_2}{P_1}\right)^{1-\frac{1}{\gamma}}\right]$$

There are equivalent expressions. Be very careful with units.

$$v_e = \left\{\frac{RT_1}{m_w}\,\frac{2\gamma}{\gamma-1}\left[1 - \left(\frac{P_e}{P_1}\right)^{1-\frac{1}{\gamma}}\right]\right\}^{1/2}$$

m_w - Gas molecular weight $\frac{kg}{kmol}$ for SI

The de Laval nozzle is sonic in the throat and supersonic beyond. Take compressible flow to see why.

A plot of F_{thrust} vs t is a plot of $\frac{d}{dt}(mv)$ vs t. $\dot{m}$, v_e

* if nozzle is not ideal and exit pressure $P_e = P_{amb}$ there is a $P_e A_e$ additional term on the thrust

Chemical Kinetics

For homogeneous reactions

$$r\left[\frac{mol}{m^3 s}\right] \equiv \frac{1}{\nu_i}\frac{1}{V}\frac{dn_i}{dt}$$

n_i - moles of i
ν_i - stoic coeff.

For heterogeneous reaction

$$r_s \equiv \frac{1}{\nu_i}\frac{1}{S}\frac{dn_i}{dt}$$

S - surface area

Possible steps

1) diffuse oxidizer to surface
2) adsorb on surface
3) react on surface
4) desorb from surface
5) diffuse product from surface.

Additional complications
Diffuse through bulk
Surface may evaporate and react in gas phase.

In our case

$$2\,NH_4ClO_4(s) \rightarrow 2O_2(g) + N_2(g) + 4H_2O(g) + Cl_2(g) \quad (1)$$

$$4\,Al(s) + 3O_2(g) \rightarrow 2Al_2O_3(s) \quad (2)$$

$$2\,Al(s) + 3Cl_2(g) \rightarrow 2AlCl_3(g) \quad (3)$$

Holding everything together is HTPB
Hydroxyl-terminated polybutadiene
It also burns

$$HO(C_4H_4)_nOH(s) + O_2(g) \rightarrow CO_2(g) + H_2O(g)$$

(Not balanced)

A complete analysis of the kinetics is incredibly involved.

We're going to ignore lots of details

and assume

$$r = kP^n$$

The reasons are subtle but involve assuming that the surface rate is so fast that the surface and the gas just above it are in equilibrium at a pressure, P, and that as P_1 (in the motor) changes T_{rxn} changes so

kP^n is adequate.

We can't measure P_1, T_1, or $S(t)$

All we can measure is S_0 and

F_{thrust}.

We don't know the exact mixture of NH_4ClO_4, Al, and HTPB but we can guess at the ratios.
We don't know $\dot{m}$ or v_e independently.
What can we do? YMMV

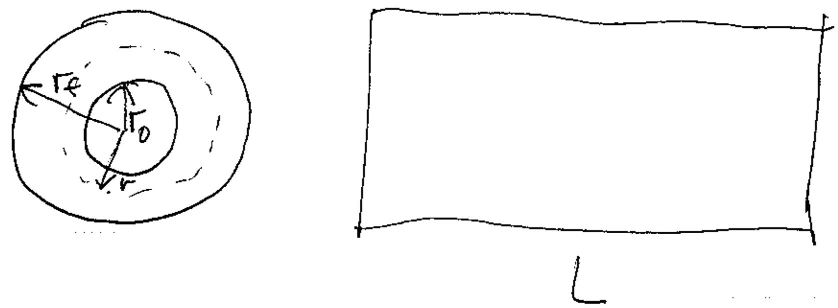
1) Weigh before and after. Then we know the integral of $\dot{m}$.

2) Assume some relationship between $\dot{m}$ & v_e
e.g. $\dot{m} \propto v_e$, ~~$\dot{m}$ = const~~, .

3) Assume an initial and burn geometry.
e.g. cylinder

Regress the data

Example: Assume propellant is hollow cylinder of total constant length



At any instant $A = 2\pi r L$

$$dV = 2\pi r L\, dr$$

$$dm = \rho\, dV$$

$$\frac{dm}{dt} = \text{rate} \cdot A$$

$$\rho \frac{dV}{dt} = (k p^n)(2\pi r L)$$

$$\rho \cancel{(2\pi r L)} \frac{dr}{dt} = k p^n \cancel{(2\pi r L)}$$

$$dr = \frac{k p^n}{\rho} dt$$

$$r - r_0 = \frac{k p^n}{\rho} t$$

$$\dot{m} = 2\pi L k p^n \left(\frac{k p^n}{\rho} t - r_0 \right)$$

$$\dot{m} = \frac{2\pi L}{\rho} k^2 p^{2n} t - 2\pi L k p^n r_0$$

Fitting a straight line to $\dot{m}$ vs t should give kP^n from the slope and intercept.

Alternately

$$kP^n = \frac{\pi L r_0 \rho + \sqrt{\dot{m}\, 2\pi L \rho t + (\pi r_0 \rho L)^2}}{2\pi L t}$$

(double check math)

Can plot kP^n vs t and see if it makes sense.

Goals for experiment

1) Get thrust curves

2) See if you can regress some kinetic parameters

3) Compare kinetic parameters for different motors