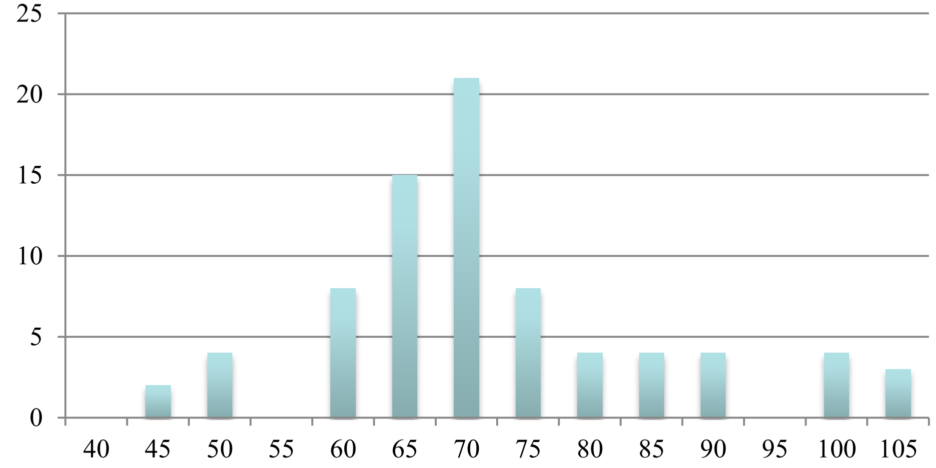
E80
Experimental Engineering

Thrust and Flight Modeling

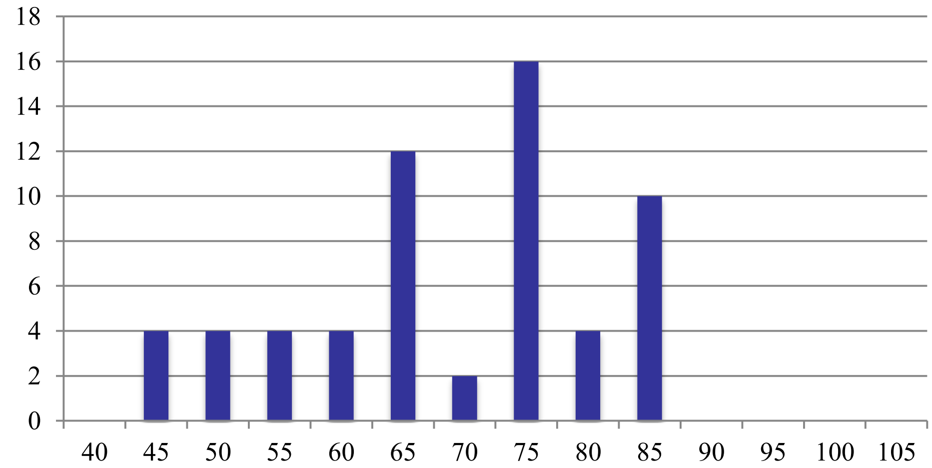
Static Motor Tests and Flight Modeling Lab

http://twistedsifter.com/2012/10/red-bull-stratos-space-jump-photos/

First Flight



BEM



Outline

- Static Motor Tests and Flight Modeling Lab Overview
- Flight Modeling:
  - 1 Degree of Freedom (DOF) Model
  - 3 DOF Model

Static Motor Rotation Lab Objectives

- Measure the thrust curves, mass flow rate of combustion gases and specific impulse for two rocket motors.
- Construct analytical and 1-D (1 DOF) and 2-D (3 DOF) numerical models of rocket flight.
- Compare the analytical and numerical models with the output of RockSim or OpenRocket.

Where will the rocket go?

Flight modeling

- What key forces dictate the flight trajectory?

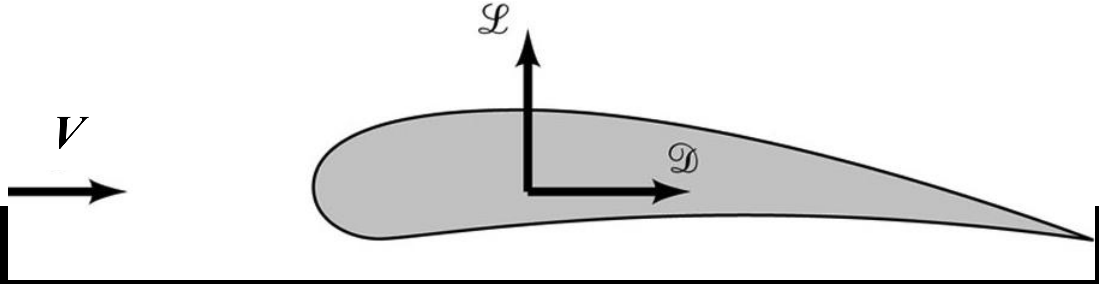
Thrust, Drag, Gravity, Lift

Reminder: Lift and Drag

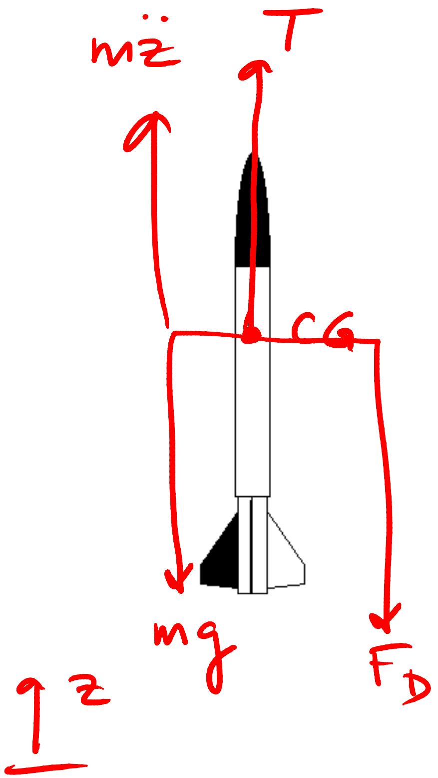
The sum of pressure and shear stress is the *resultant force*. It is split into two components:

1. **Lift:** The component of resultant force that is ***perpendicular*** to the *incoming net velocity vector* (effective flow direction).

2. **Drag:** The component of resultant force that is ***parallel*** to the *incoming net velocity vector* (effective flow direction).



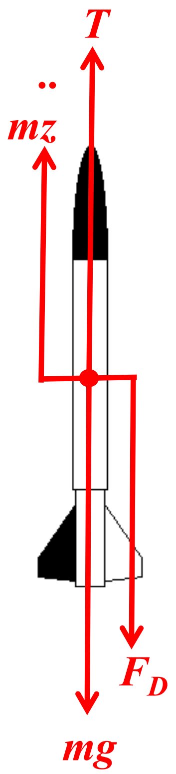
One DOF Model: Free Body Diagram



One DOF Model: Governing Equation

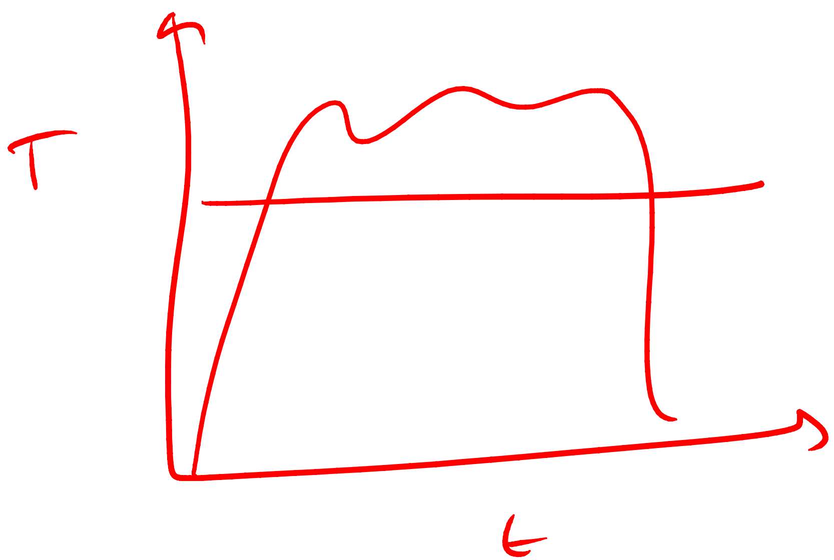
$$F = m\ddot{z} = T - mg - F_D$$

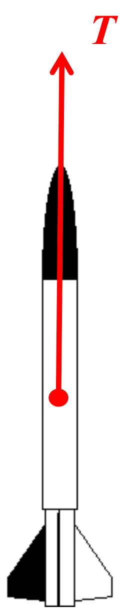
$$m = m(t)$$

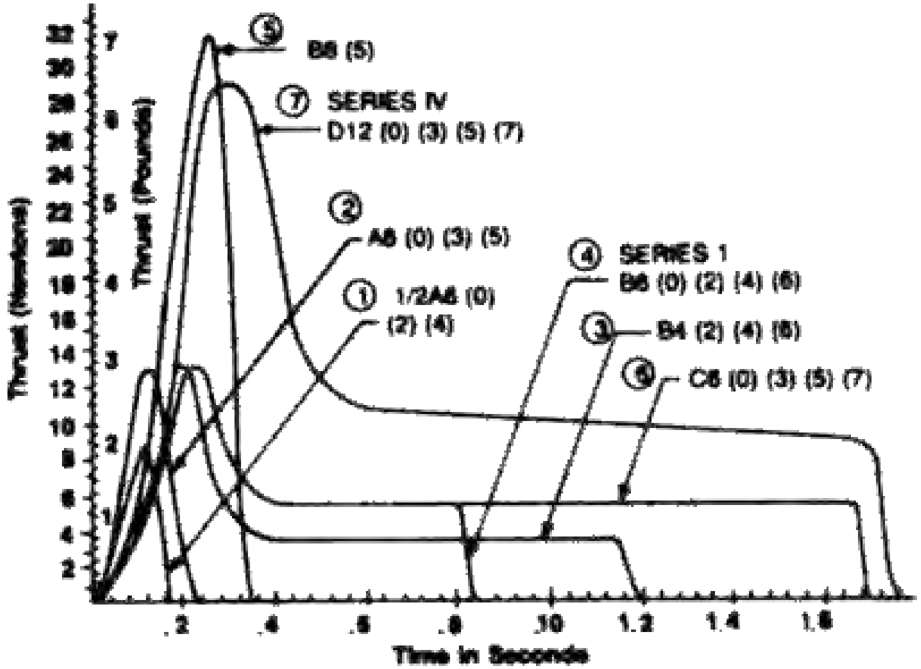


Modeling Thrust

- Is thrust constant during flight? No. T(t)



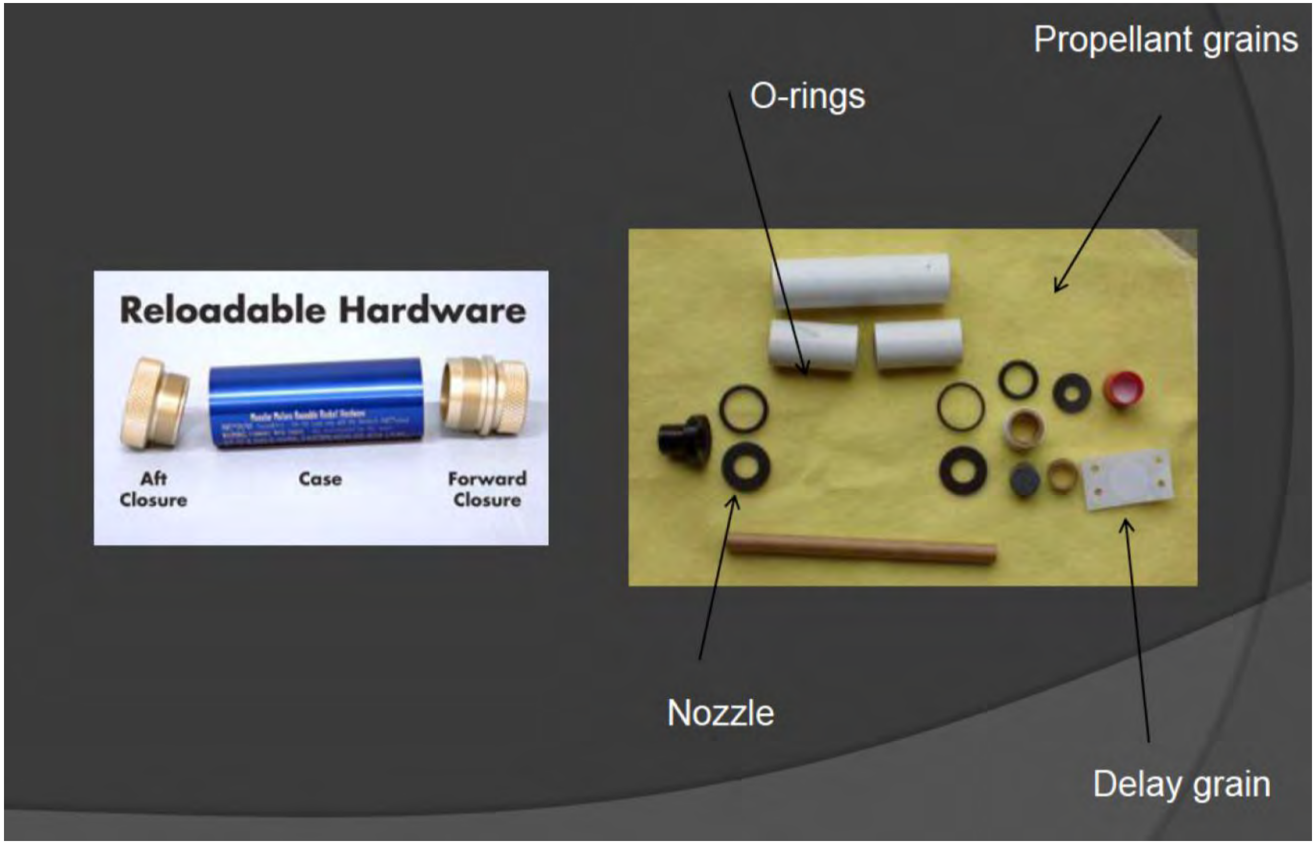




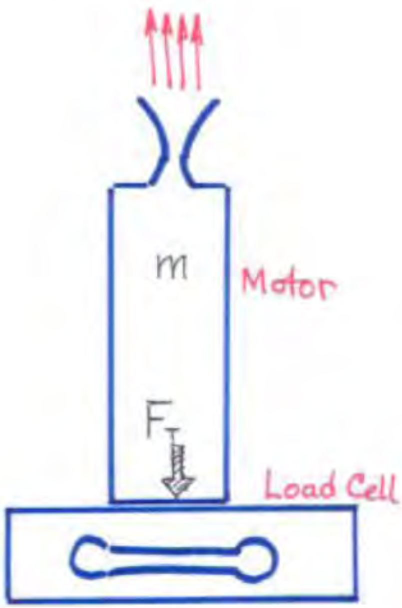
Engine Type	Total Impulse		Average Thrust		Propellant Weight	Thrust Curve Number
	Pound-seconds	Newton seconds	Pounds	Newtons		
1/2A6-	0.28	1.25	1.35	5.80	0.00344 lb.	1
A8-	0.56	2.50	1.80	7.70	0.00918 lb.	2
B4-	1.12	5.00	0.90	4.15	0.01836 lb.	3
B6-	1.12	5.00	1.35	5.80	0.01374 lb.	4
B8-	1.12	5.00	1.79	8.00	0.01374 lb.	5
C6-	2.25	10.00	1.35	5.80	0.02748 lb.	6
D12-	4.48	20.00	2.66	11.80	0.05496 lb.	7

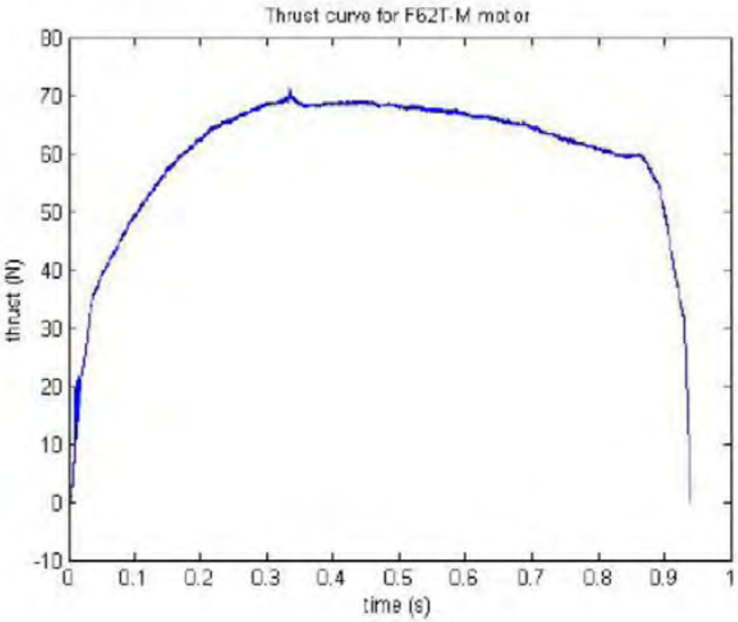
http://v-serv.com/usr/estesmotors.htm

Static Motor Rotation Lab



Static Motor Thrust Curve





http://www.eng.hmc.edu/NewE80/StaticTestVideos.html

Static Motor Lab, Section 2:

- Calculate the total impulse.
- Calculate the average thrust and average mass flow rate.
- Calculate the exit velocity of the combustion gases from the nozzle. What assumptions did you have to make?
- Calculate the specific impulse, I_{sp}.

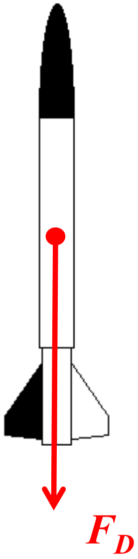
Drag Force

$$F_D = \frac{1}{2} \rho V^2 A C_D$$

A = reference area

$$C_D = f(Re, \alpha)$$

↑ angle of attack



Analytical One DOF Model

- GE:

$$m\ddot{z} = T - mg - F_D$$

- Assumptions:
  - Const m
  - Constant T (over a time interval)
  - Constant C_D

Numerical One DOF Model

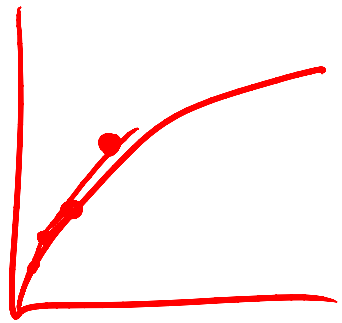
- GE:

$$m(t)\ddot{z} = T(t) - m(t)g - \frac{1}{2}\rho\dot{z}^2 A C_D$$

$$C_D(\dot{z})$$

- Many options for numerical solution methods, e.g.
  - OpenRocket uses **Runge-Kutta** (RK4)
  - One option is **Explicit Euler** ignoring high order terms…

Explicit Euler



$$\dot{y}(t) = f(y, t)$$

For each time step of size h,

$$y_{n+1} = y_n + h f(y_n, t_n)$$

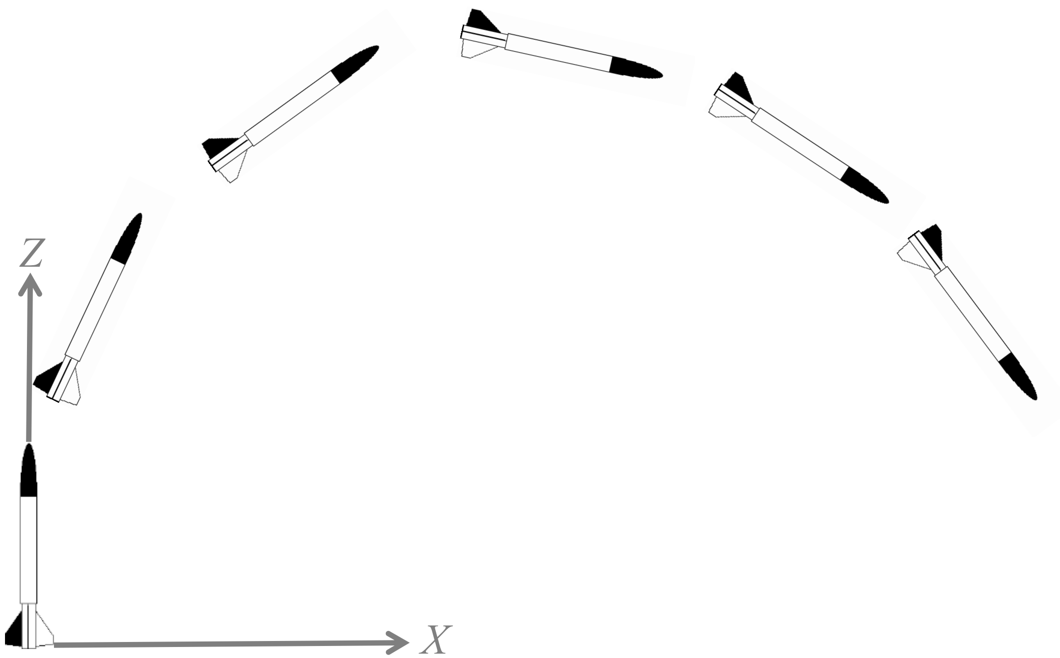
$$t_{n+1} = t_n + h$$

One DOF Model

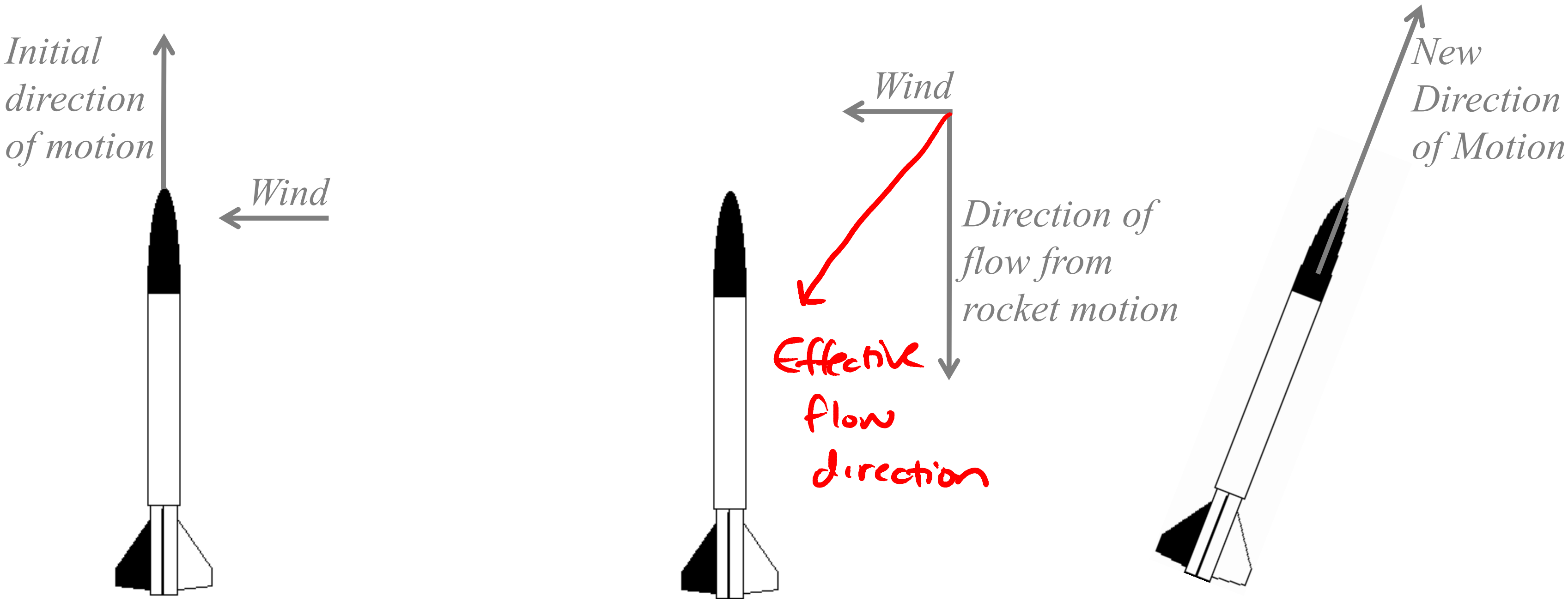
```
for t = 0 to maxTime
{
        T = …
        m = …
        Fd = …

        z_dd(t) = 1/m*(T-m*g-Fd);
        z_d(t) = z_d + z_dd*Δt
        z(t) = z + z_d*Δt
}
```

Three DOF Model

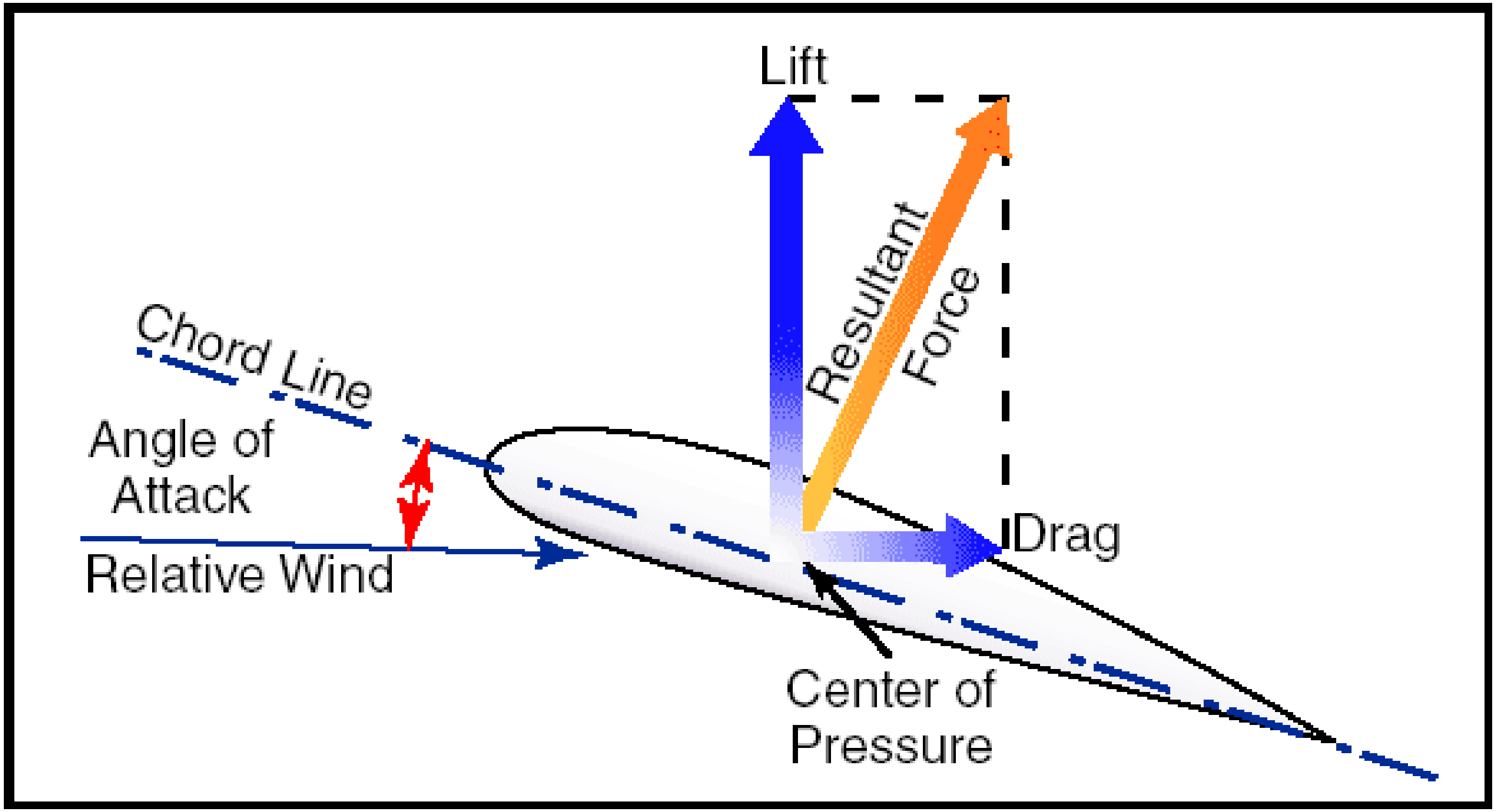
- What are the 3DOF?



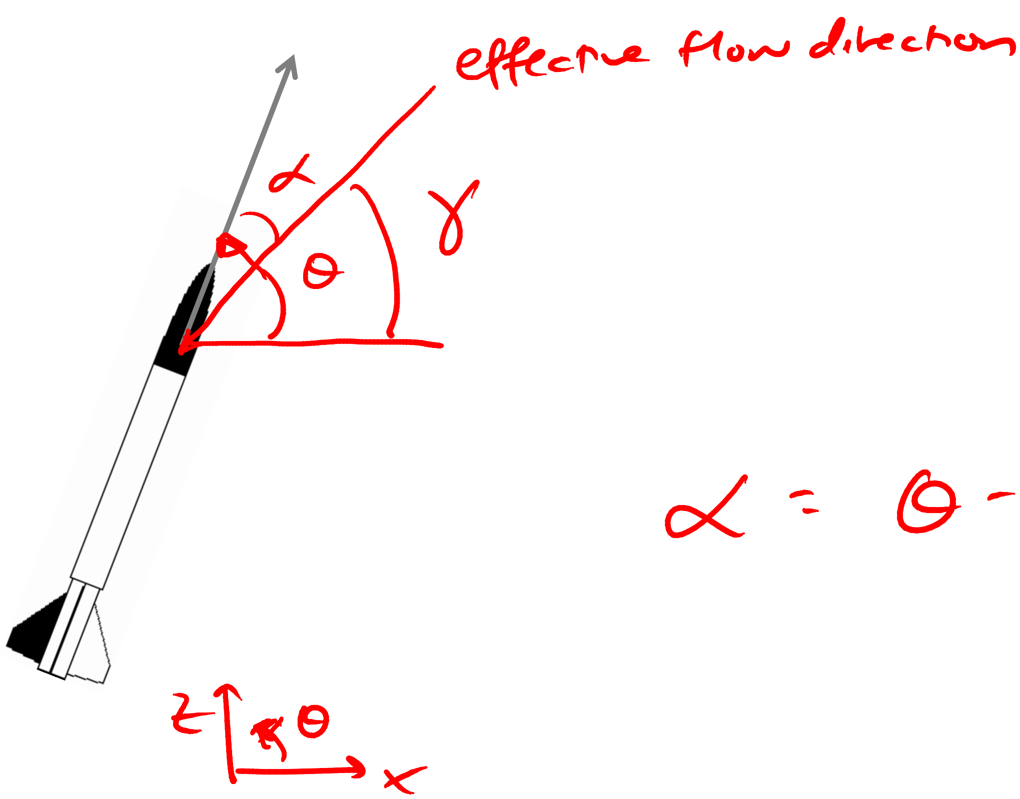
Why does the rocket rotate?

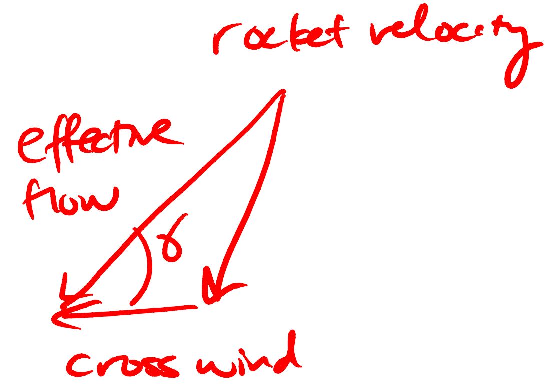


Reminder: Angle of Attack



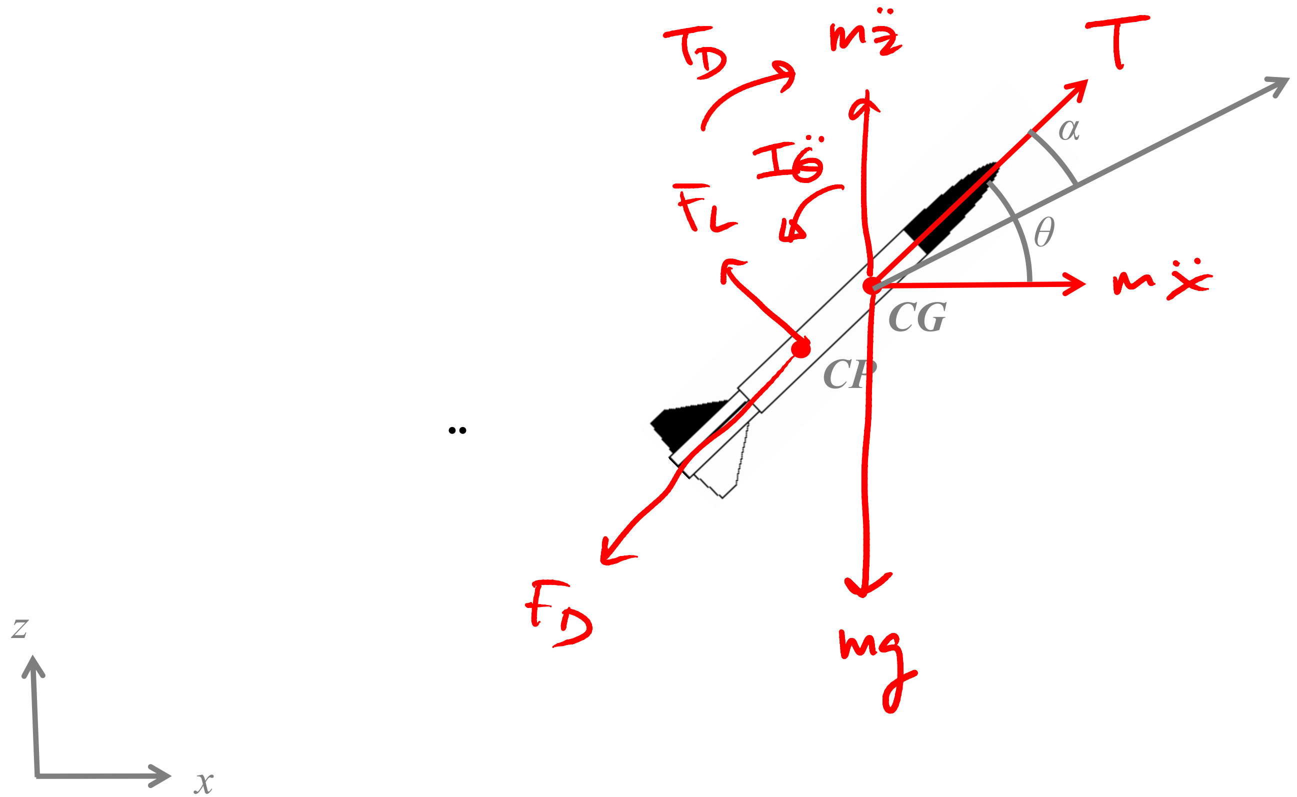
Angle of Attack





$$\alpha = \theta - \gamma$$

Three DOF Free Body Diagram



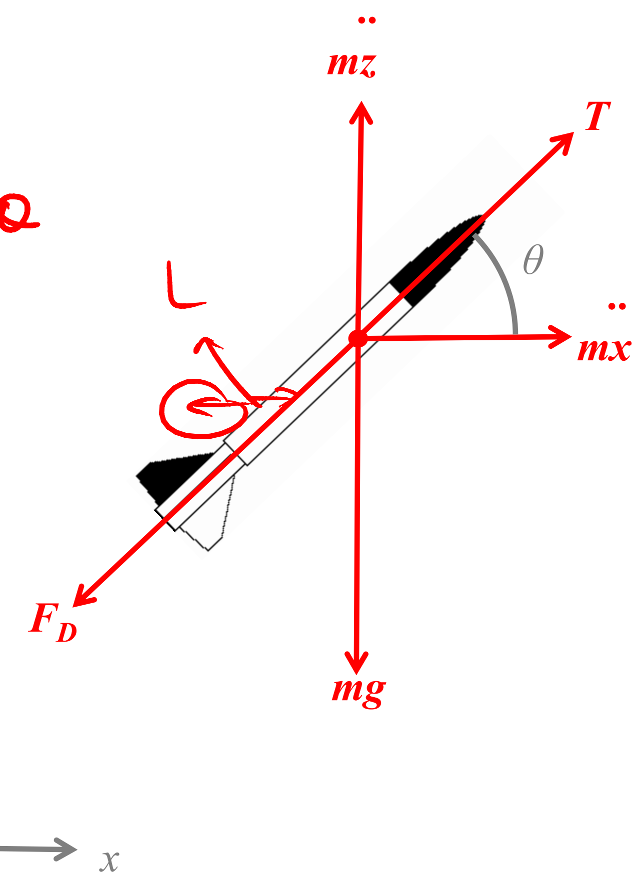
Non-Rotational Forces

- z-direction

$$m\ddot{z} = T\sin\theta - mg - F_D\sin\theta + F_L\cos\theta$$

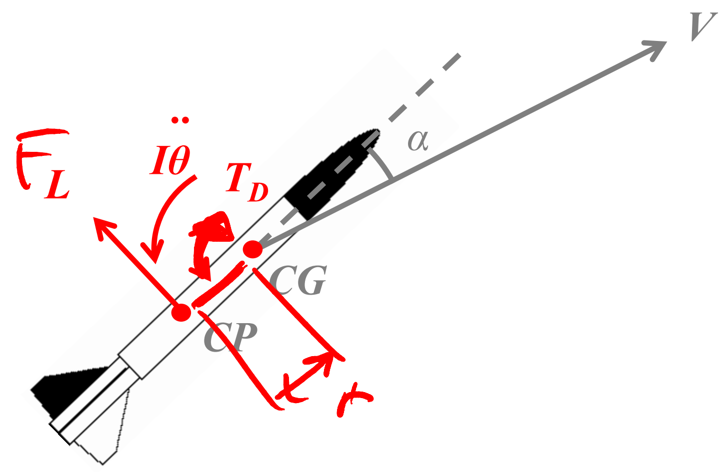
- x-direction

$$m\ddot{x} = T\cos\theta - F_D\cos\theta - F_L\sin\theta$$



Torque Balance

$$I\ddot{\theta} = -T_D - rF_L$$



Rotational Damping

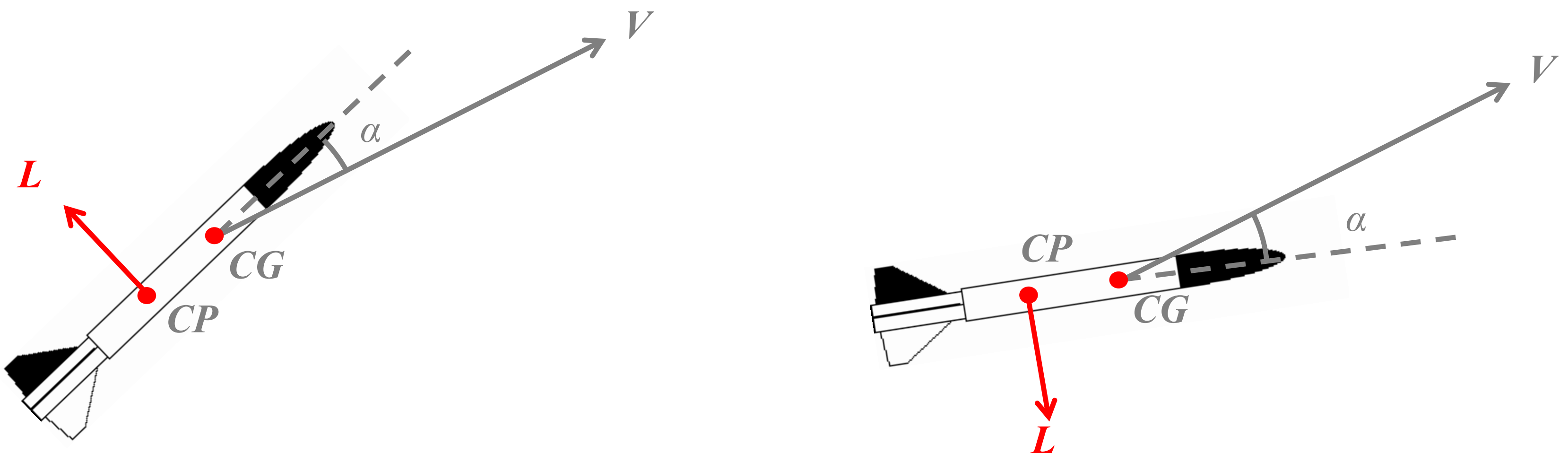
- The **rotational damping** can be modeled as

$$T_D = c\,\dot{\theta}$$

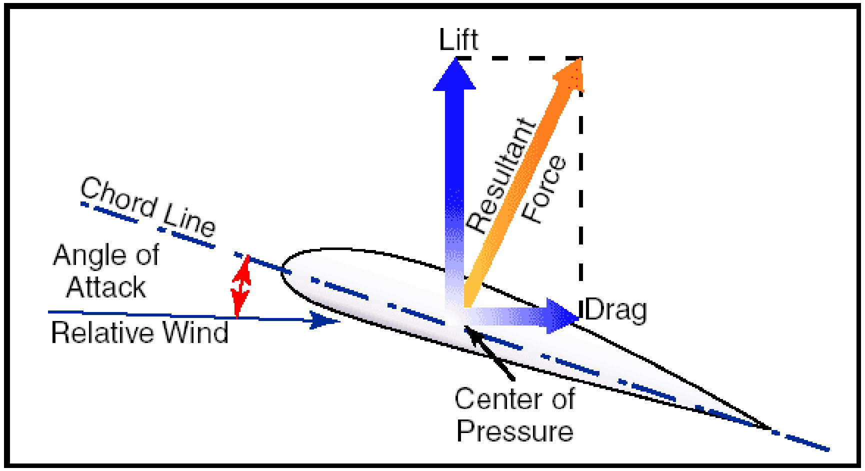
c = damping coefficient

Rocket Stability

- Is this stable?
  - Depends on location of C_P versus C_G



Reminder: Complication #3 Angle of Attack

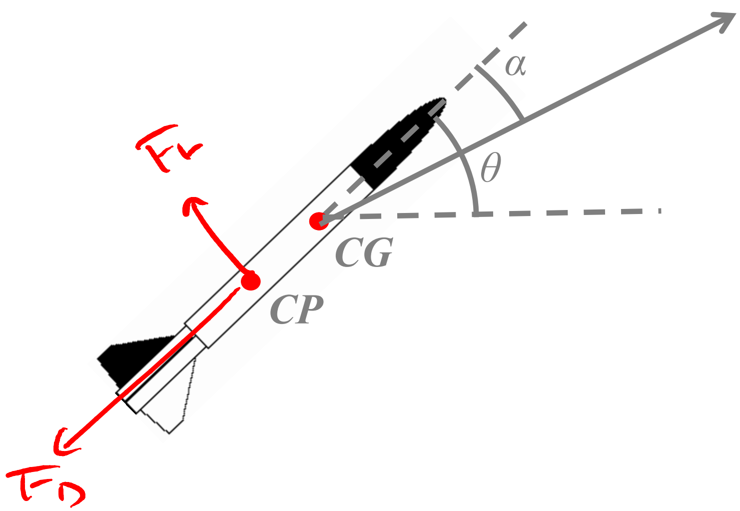


$$C_L, C_D = f(\alpha, Re)$$

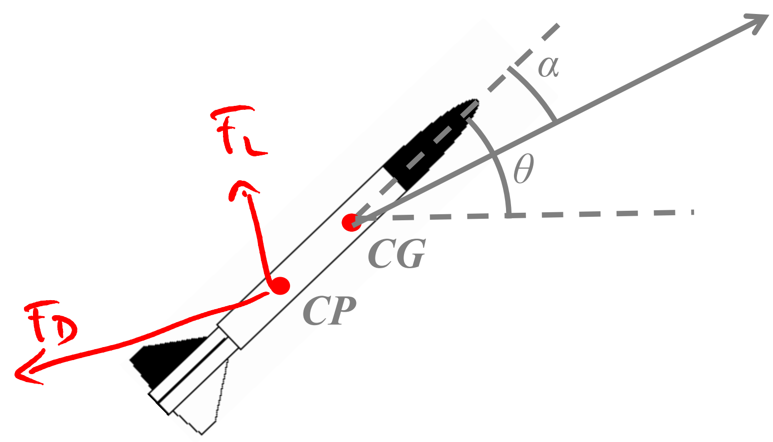
Drag and Lift direction

- Drag and lift can be defined w.r.t.

Rocket axis



Effective flow direction



Three DOF Model

```
for t = 0 to maxTime
{
        T = …
        m = …
        Fd = …
        L = …
        Td = …
        alpha = …

        z_dd(t) = …
        x_dd(t) = …
        θ_dd(t) = …
        …
}
```

To Linde Field

- Good luck!