Operational Amplifiers (Op Amps)

E80 – 2/5/13 - Professor Katherine Candler
Notes courtesy of Professor Sarah Harris

Agenda
- **Ideal Op Amps**
- **Real Op Amps**

Ideal Op Amps

Operational amplifiers (op amps) amplify an input signal and produce an output signal. The symbol for an op amp is shown below. Figure 1(a) shows the symbol with the power supplies, V_{cc}^+ and V_{cc}^- (also sometimes called V^+ and V^-) drawn explicitly. The power supplies are typically $\pm 15V$. Figure 1(b) shows a simplified symbol where the power supplies are implicit. **The power supplies must always be connected to operate the circuit**. v^+ and v^- indicate the voltages of the two input ports. Note: some references refer to these two input voltages as In^+ and In^-.
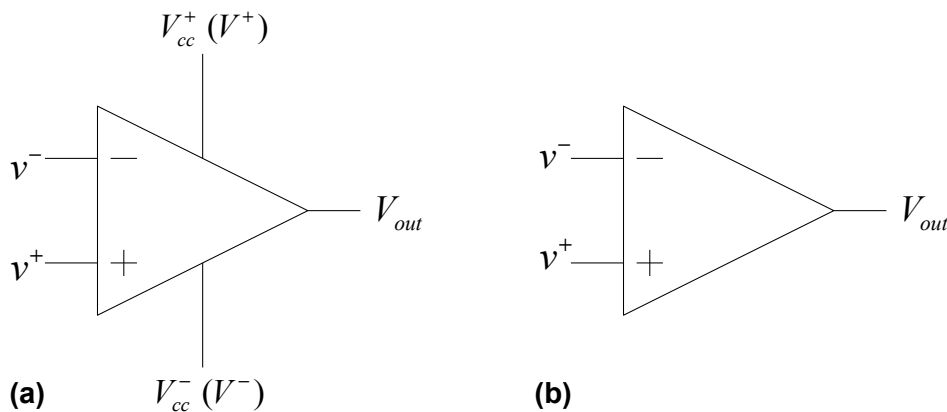

Figure 1. Op amp symbols: (a) with explicit power supplies, (b) simplified

Characteristics
Characteristics that are useful in analyzing an op amp circuit are:
1. Current into v^+ and v^- is 0.
2. Voltage at the two inputs is equal: $v^- = v^+$ when circuit is configured in negative feedback (i.e., the output is connected to the negative input terminal)
3. Input impedance looking into the input terminals is infinite: $R_{in} = \infty$
4. Output impedance is small: $R_{out} \approx 0$

Let's use these characteristics, particularly the first two, in analyzing the following example circuits.

Example 1: Inverter

Consider the circuit in Figure 2. Because no current flows into either input of the op amp (characteristic 1), $i_1 = i_2$.
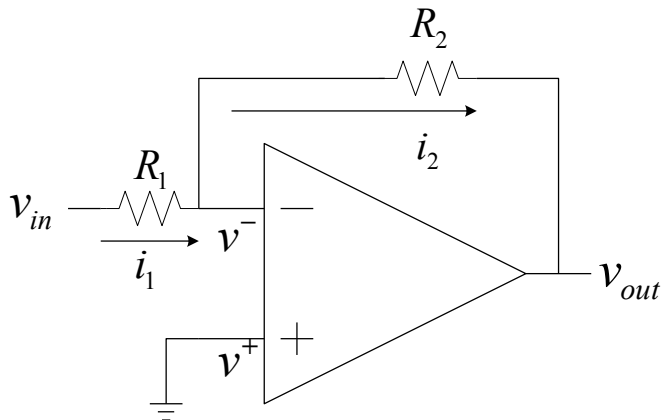

Figure 2. Inverter

Writing the currents in terms of voltages, we get:

$$i_1 = i_2$$

$$\frac{v_{in} - v^-}{R_1} = \frac{v^- - v_{out}}{R_2}$$

Since the voltages at v^+ and v^- are the same (characteristic 2) and $v^+ = 0$ V, $v^- = 0$ V. Substituting into the previous equation, we get:

$$\frac{v_{in}}{R_1} = \frac{-v_{out}}{R_2}$$

$$\frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1}$$

So, for example, if $R_2 = R_1$, v_{out} has the same magnitude but opposite phase of v_{in}. If $R_2 > R_1$, the output is both amplified and inverted. Note that the voltage of the amplified output cannot exceed the voltage of the external supplies, $\pm 15V$.

We can also analyze the circuit in the frequency domain. Suppose R_1 and R_2 were replaced by generic impedances Z_1 and Z_2 as shown in Figure 3.
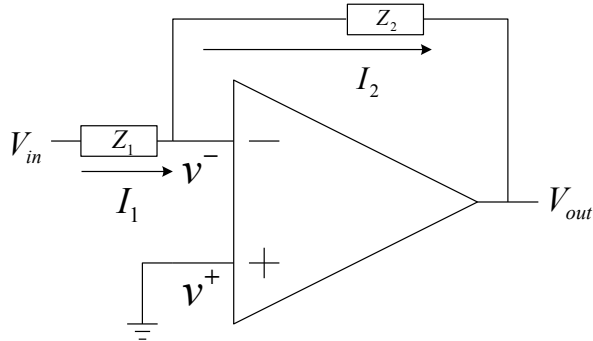
Figure 3. Circuit with impedances

$$I_1 = I_2$$

$$\frac{V_{in} - v^-}{Z_1} = \frac{v^- - V_{out}}{Z_2}$$

As before, since the voltages at v^+ and v^- are the same (characteristic 2) and $v^+ = 0$ V, $v^- = 0$ V. Substituting into the previous equation, we get:

$$\frac{V_{in}}{R_1} = \frac{-V_{out}}{R_2}$$

$$\frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1}$$

Example 2: Voltage follower / buffer

Consider the circuit in Figure 4. Because the voltages at v^+ and v^- are the same (characteristic 2) and $v^+ = v_{in}$, $v_{out} = v^- = v_{in}$.
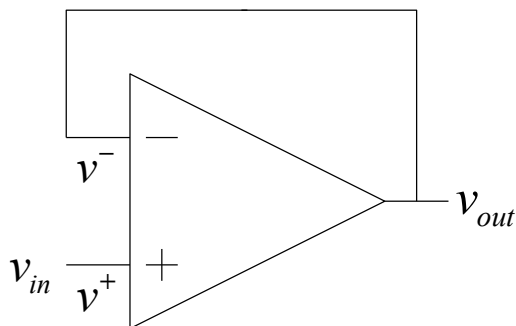

Figure 4. Voltage follower / buffer

Thus, $\dfrac{v_{out}}{v_{in}} = 1$. The output voltage *follows* the input voltage. This device, where the output is equal to the input, is also called a buffer. A buffer is useful for isolating the output from a noisy input, or vice versa. It also can convert a high output impedance circuit into a low output impedance circuit (characteristic 4).

Example 3: Circuit comparison with and without buffer

Consider the circuits in Figure 5. Compare the output voltages of the two circuits measured by an oscilloscope with a 1 MΩ input impedance.
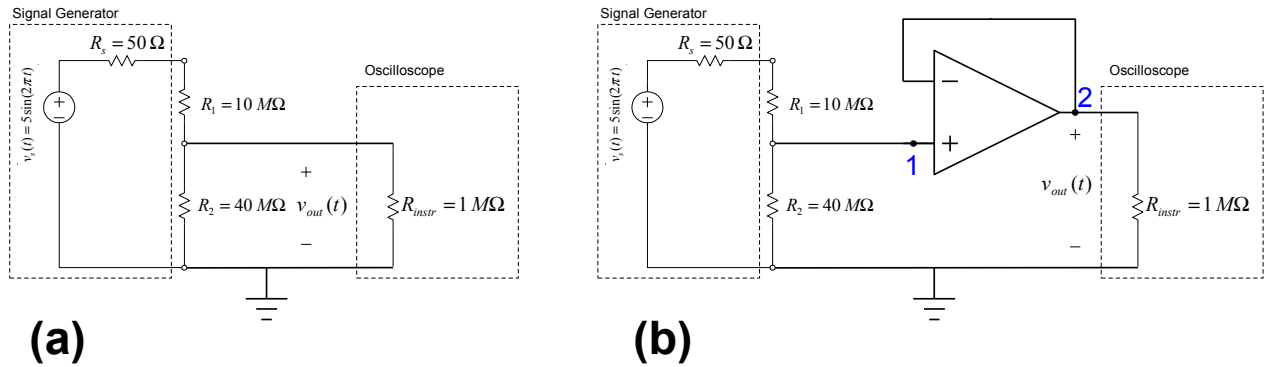

Figure 5. Comparison of circuit with and without buffer

As we've analyzed before, the measured output voltage for the circuit in Figure 5(a) is:

$$v_{out} = \frac{R_2 \parallel R_{instr}}{(R_2 \parallel R_{instr}) + R_1 + R_s} v_s \approx \frac{R_{instr}}{R_{instr} + R_1} v_s = \frac{1}{11} v_s$$

For the circuit in Figure 5(b), the impedance seen looking into node 1 is infinite (characteristic 4), so the voltage measured at node 1 is:

$$v_1 = \frac{R_2}{R_2 + R_1 + R_s} v_s \approx \frac{R_2}{R_2 + R_1} v_s = \frac{4}{5} v_s$$

Since the output impedance of node 2 is approximately 0Ω (characteristic 3), all of the voltage delivered to node 2 is dropped across the 1 M Ω internal impedance of the oscilloscope, and the measured output voltage is:

$$v_{out} = v_1 = \frac{4}{5} v_s$$

Thus, inserting the buffer eliminated instrument loading effects.

Example 4: Differentiator

Consider the circuit in Figure 6. We can analyze the circuit in the frequency domain using impedances and the previous derived results.
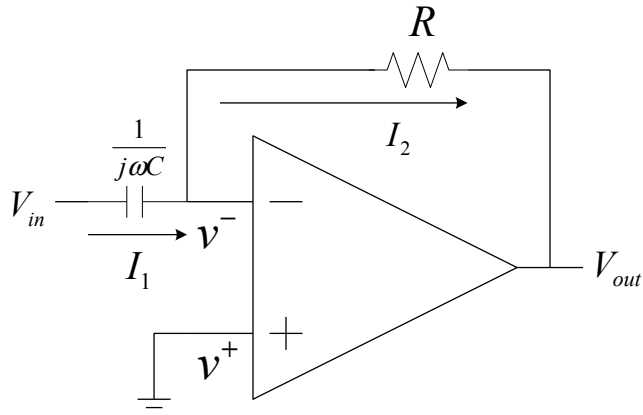

Figure 6. Differentiator

$$Z_1 = \frac{1}{j\omega C}$$

$$Z_2 = R$$

$$\frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1} = -\frac{R}{1/j\omega C} = -j\omega RC$$

The output is the inverted derivative of the input.

Example 5: Integrator

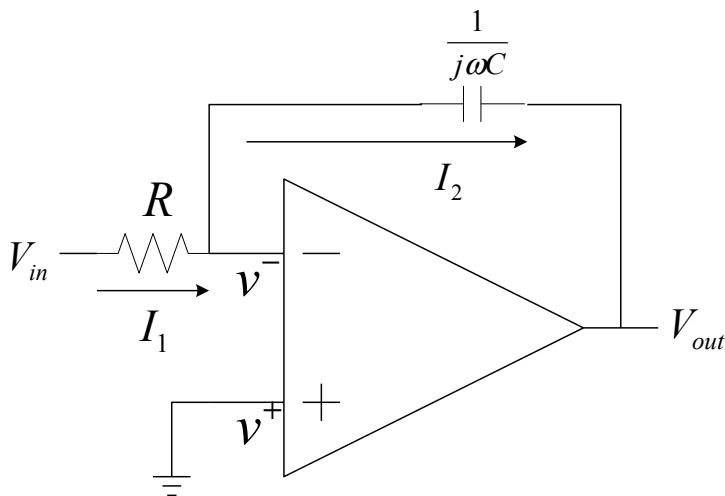Consider the circuit in Figure 7. We can analyze the circuit using impedances.

Figure 7. Integrator

$$Z_1 = R$$

$$Z_2 = \frac{1}{j\omega C}$$

$$\frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1} = -\frac{1/j\omega C}{R} = -\frac{1}{j\omega RC}$$

The output is the inverted integral of the input.

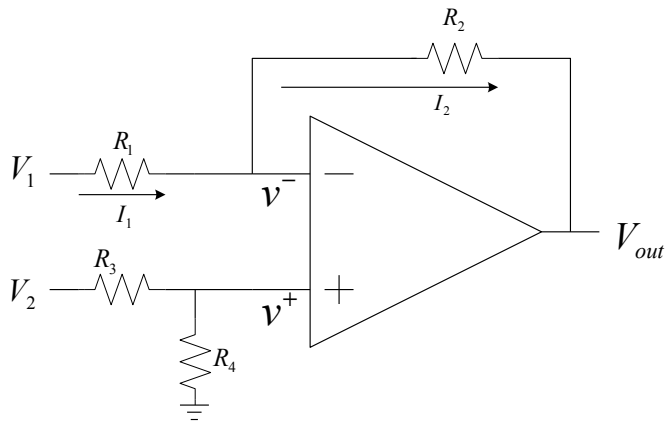## Example 6: Differential amplifier

Consider the circuit in Figure 8.

Figure 8. Differential amplifier

$$I_1 = I_2$$

$$\frac{V_1 - v^-}{R_1} = \frac{v^- - V_{out}}{R_2}$$

$$v^+ = (\frac{R_4}{R_3 + R_4})V_2$$

Substituting and after some algebra, we get:

$$V_{out} = (\frac{R_4}{R_3 + R_4})(\frac{R_1 + R_2}{R_1})V_2 - (\frac{R_2}{R_1})V_1$$

$$when \ R_1 = R_3 \ and \ R_2 = R_4, \quad V_{out} = (\frac{R_2}{R_1})[V_2 - V_1]$$

A differential amplifier amplifies the difference between two voltages. If all the resistance values are equal, the output is just the difference between the two input voltages.

A differential amplifier is particularly useful in eliminating common noise between the two input voltages. For example, if $v_1(t) = u_1(t) + n(t)$, and $v_2(t) = u_2(t) + n(t)$ where $u_1(t)$ and $u_2(t)$ are signals of interest and $n(t)$ is the common noise, then

$v_{out}(t) = u_2(t) - u_1(t)$ (when all resistors are equal) and the common noise is eliminated from the output signal.

A differential amplifier can also be used to remove or add a DC offset to a signal. This is called "level shifting". For example, again suppose all resistors are equal, $v_1(t) = -1V$, and $v_2(t) = \sin(\omega t)$, then the output is the sine wave shifted up by 1 V DC.

Example 7: Inverting summer
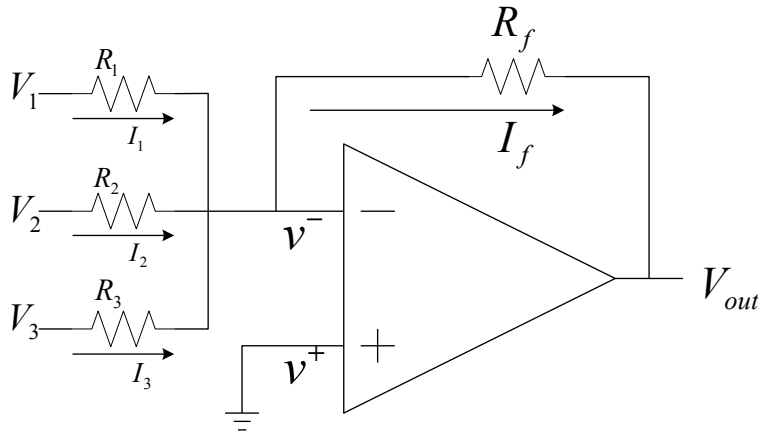Consider the circuit in Figure 9.

Figure 9. Inverting summer

We analyze the circuit, again using characteristics 1 and 2.

$$I_1 + I_2 + I_3 = I_f$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = -\frac{V_{out}}{R_f}$$

$$V_{out} = -R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right)$$

If $R_1 = R_2 = R_3 = R_f$, then v_{out} is just the inverted sum of the inputs. A block diagram symbol for the inverting summer circuit is shown in Figure 10. A non-inverting summer could be created by following the inverting summer with an inverting op amp (Figure 3 with $R_1 = R_2$).
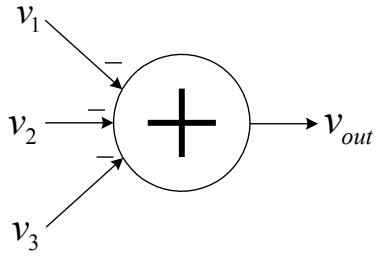
Figure 10. Inverting summer

Example 8: Another Summer
Consider the circuit in Figure 11. Recall that all voltages are referenced to ground.
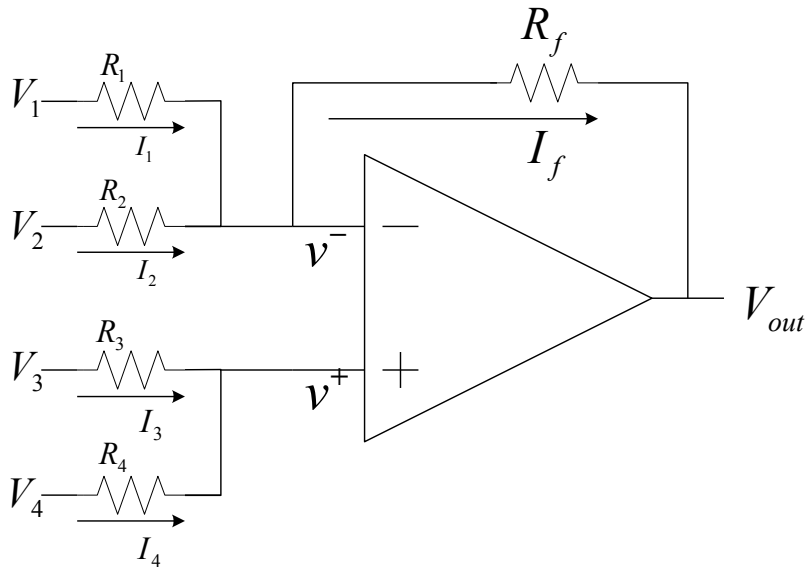

Figure 11. Another Summer

$$I_1 + I_2 = I_f$$

$$\frac{V_1 - v^-}{R_1} + \frac{V_2 - v^-}{R_2} = \frac{v^- - V_{out}}{R_f}$$

$$I_3 = -I_4$$

$$\frac{V_3 - v^+}{R_3} = -\frac{(V_4 - v^+)}{R_4}$$

$$v^+ = (\frac{R_4}{R_3 + R_4})V_3 + (\frac{R_3}{R_3 + R_4})V_4$$

$$v^+ = AV_3 + BV_4,$$

$$where \ A = (\frac{R_4}{R_3 + R_4}), B = (\frac{R_3}{R_3 + R_4})$$

Substituting, we get:

$$\frac{V_1 - (AV_3 + BV_4)}{R_1} + \frac{V_2 - (AV_3 + BV_4)}{R_2} = \frac{(AV_3 + BV_4) - V_{out}}{R_f}$$

$$V_{out} = -CV_1 - DV_2 + EV_3 + FV_4,$$

$$where\ C = (\frac{R_f}{R_1}), D = (\frac{R_f}{R_2}), E = (\frac{R_4}{R_3 + R_4})(\frac{R_f}{R_1} + \frac{R_f}{R_2} + 1), F = (\frac{R_3}{R_3 + R_4})(\frac{R_f}{R_1} + \frac{R_f}{R_2} + 1)$$

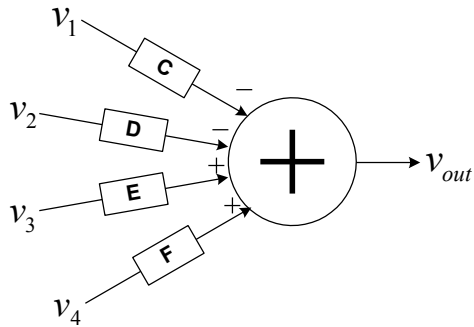A block diagram of this summer is shown in Figure 12.

Figure 12. Block diagram of summer

For example, if $R_1 = R_2 = R_3 = R_4 = R_f$, then the weights on the inputs are: C=D=1 and E=F=1.5.

Example 9: Low-pass filter
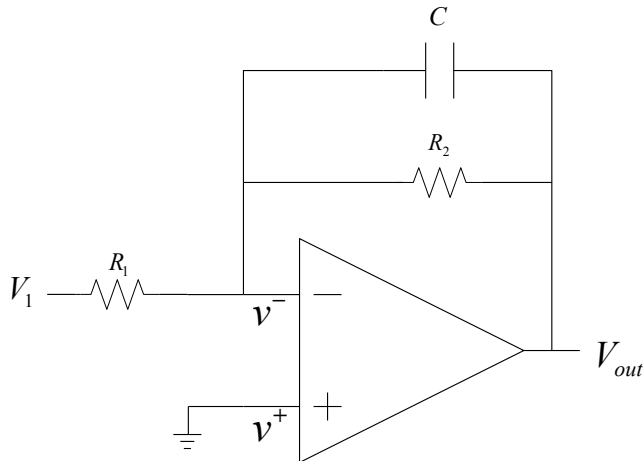Consider the circuits in Figure 13.

Figure 13. Low-pass filter

$$Z_1 = R_1$$

$$Z_2 = \left[j\omega C + \frac{1}{R_2} \right]^{-1} = \frac{R_2}{1 + j\omega R_2 C}$$

$$\frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1} = -\frac{R_2 / R_1}{1 + j\omega R_2 C} = -\frac{A}{1 + j\omega\tau}, \ where \ A = \frac{R_2}{R_1}, \ \tau = R_2 C$$

For example, suppose $R_2 = 1k\Omega$, $R_1 = 100\Omega$, $C = 1\mu F$, then $A = 10$, $\tau = 1ms$ and the Bode plot of the system is shown in Figure 14 below.
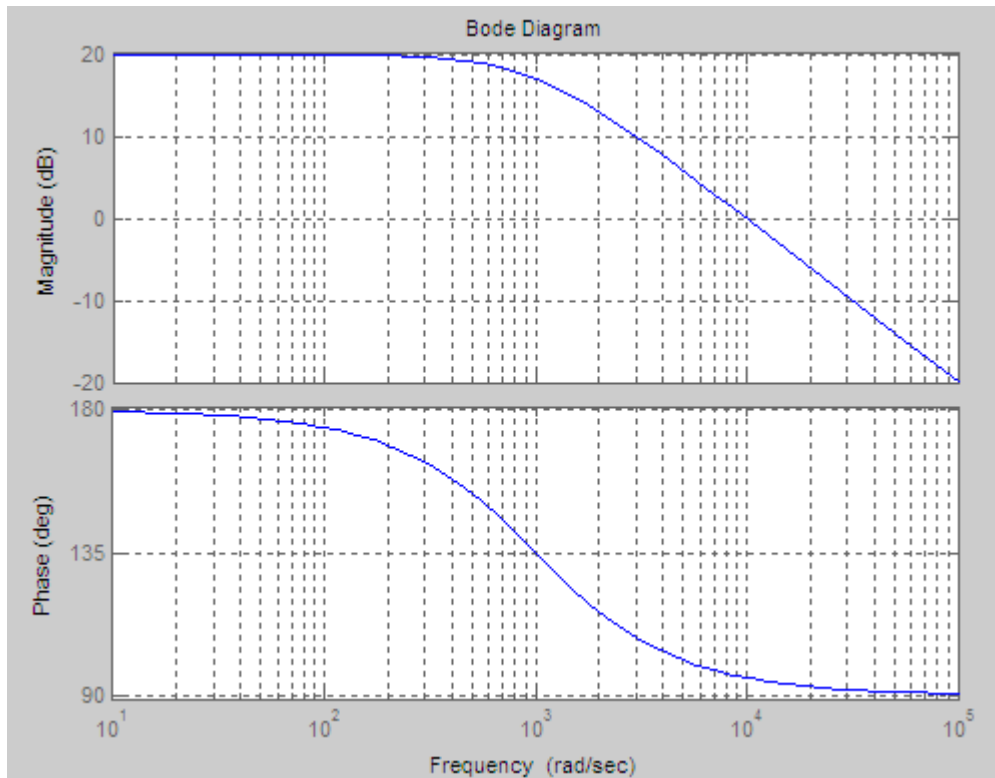

Figure 14. Bode plot of system in Figure 13. Bandwidth = 10^3 rad/s (160 Hz)

Example 10: High-pass filter
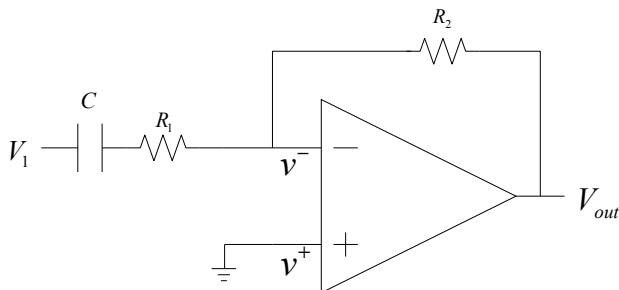Consider the circuits in Figure 15.

Figure 15. High-pass filter

$$Z_1 = \frac{1}{j\omega C} + R_1$$

$$Z_2 = R_2$$

$$\frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1} = -\frac{R_2}{\frac{1}{j\omega C} + R_1} = -\frac{j\omega R_2 C}{1 + j\omega R_1 C} = -\frac{j\omega \tau_2}{1 + j\omega \tau_1}, \text{ where } \tau_1 =, R_1 C \ \tau_2 = R_2 C$$

For example, suppose $\tau_1 = 10^{-6}\,s$, $\tau_2 = 10^{-3}\,s$, then a Bode plot of the system is shown in Figure 16 below.
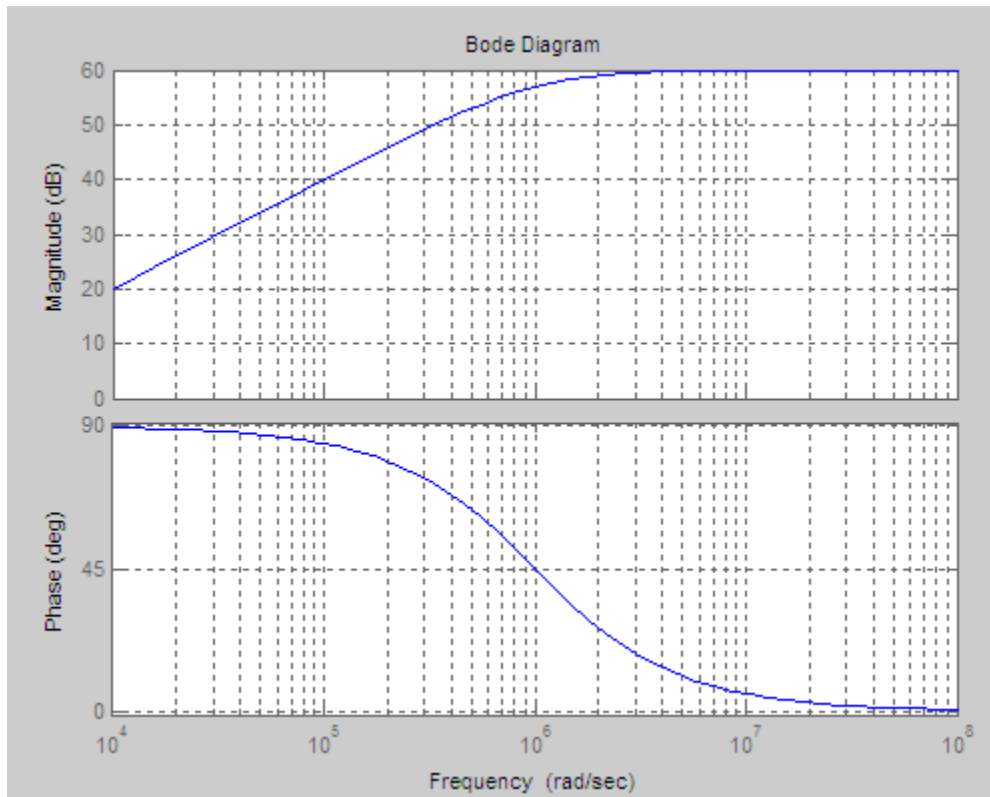

Figure 16. Bode plot of system in Figure 15. Cutoff frequency = 10^6 rad/s (160 kHz)

Example 11: Band-pass filter
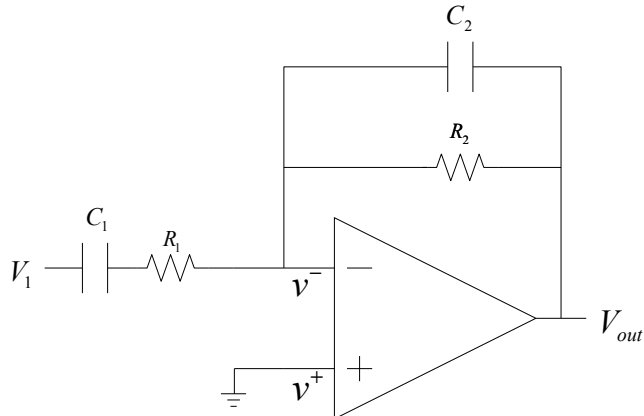Consider the circuits in Figure 17.

Figure 17. Band-pass filter

$$Z_1 = \frac{1}{j\omega C_1} + R_1$$

$$Z_2 = \left[j\omega C_2 + \frac{1}{R_2} \right]^{-1} = \frac{R_2}{1 + j\omega R_2 C_2}$$

$$\frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1} = -\frac{j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)} = -\frac{j\omega \tau_3}{(1 + j\omega \tau_1)(1 + j\omega \tau_2)},$$

$$where \ \tau_1 = R_1 C_1, \tau_2 = R_2 C_2, \tau_3 = R_2 C_1$$

For example, suppose $\tau_1 = 10^{-6}$, $\tau_2 = 10^{-8}$, $\tau_3 = 10^{-3}$, then the Bode plot for the system is as shown in Figure 18.
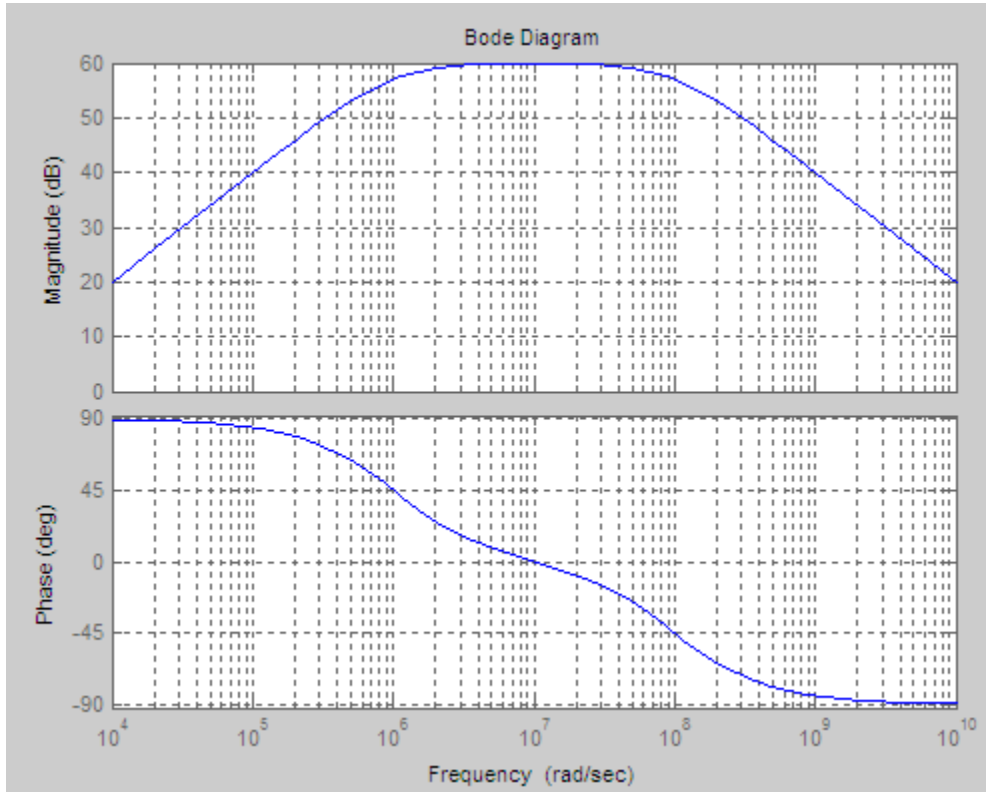
Figure 18. Bode plot of system in Figure 17. Bandwidth = 10^2 rad/s (16 Hz)

Real Op Amps

Real op amps have characteristics that are approximated by the ideal characteristics stated on page 1:

Characteristics

Characteristics that are useful in analyzing an op amp circuit are:

1. Current into v^+ and v^- is small ($\sim 10^{-9} - 10^{-12}$ A).
2. Voltage at the two inputs is non-zero but approximately equal when op-amp is configured with negative feedback (i.e., when the output is connected to the negative input terminal).
3. Input impedance looking into the input terminals is large: $R_{in} \approx 10^7 \Omega$ to $10^{12} \Omega$.
   The larger input impedances are found on op amps with FET input stages (like the TL081).
4. Output impedance is small: $R_{out} \approx 60 \Omega$

Figure 19 shows a model of an op amp. Internal circuitry of an op amp can be found in its data sheet. The open-loop gain (A) of an op amp is large ($10^5 - 10^6$). The open-loop gain is the gain when there is no connection between the output and the negative input terminal, as we have seen in the initial examples.

The input is: $v_{in} = v^+ - v^-$. The output is: $v_{out} = Av_{in} = A(v^+ - v^-)$.
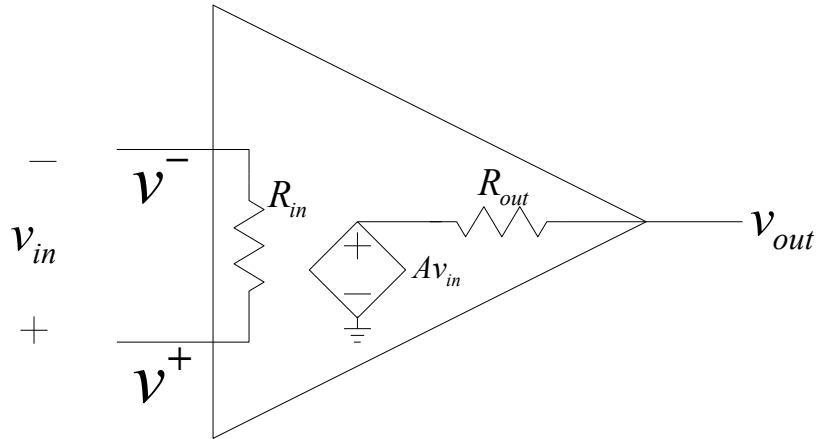
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Figure 19. Op amp model

A/D Converters

An A/D (analog to digital) converter converts an analog (continuous) signal to a digital (discrete) signal. Without feedback, an op amp amplifies the input signal until it saturates at either the positive or negative external supply voltage ($\pm 15V$). A group of op amps can be used to convert an analog signal to a "digital" signal, where the output is either ON (+15V) or OFF (-15V). This is shown in Figure 20 for converting a 0-3 V signal to a digital signal with 1 V accuracy.
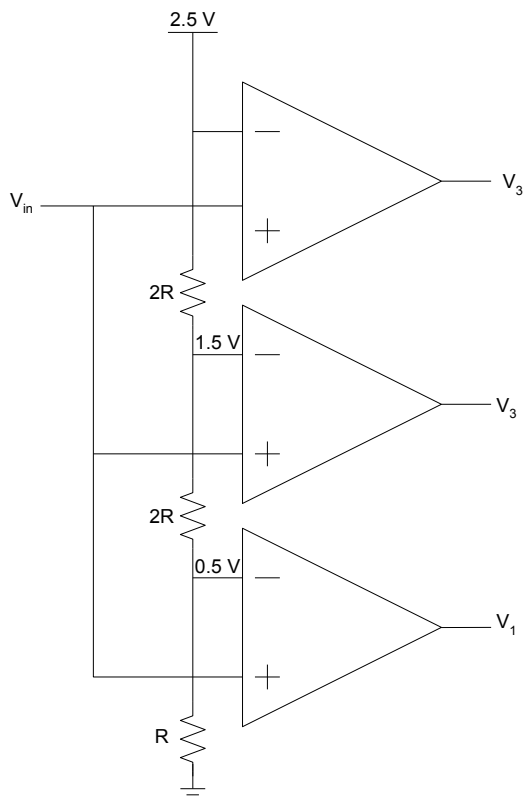

Figure 20. A/D Converter

We define a "1" output as 15V and a "0" output as -15V. Table 1 below shows the output voltages for the respective input voltages. Note that the A/D converter can't distinguish between a 2V input and 2.2V input. It only distinguishes between 1 V differences. A digital circuit could convert the 3-bit output to a binary representation. This is a 2-bit A/D converter.

V_{in} (V)	0	1	2	3
V_3 V_2 V_1	000	001	011	111
Binary	00	01	10	11

Table 1. A/D converter outputs

Lab Hints

- Don't forget to connect the power supplies ($\pm 15V$) to each of your op amps before you try to use them.
- Test each op amp chip using a simple circuit (like a voltage follower) to confirm that the chip operates correctly.