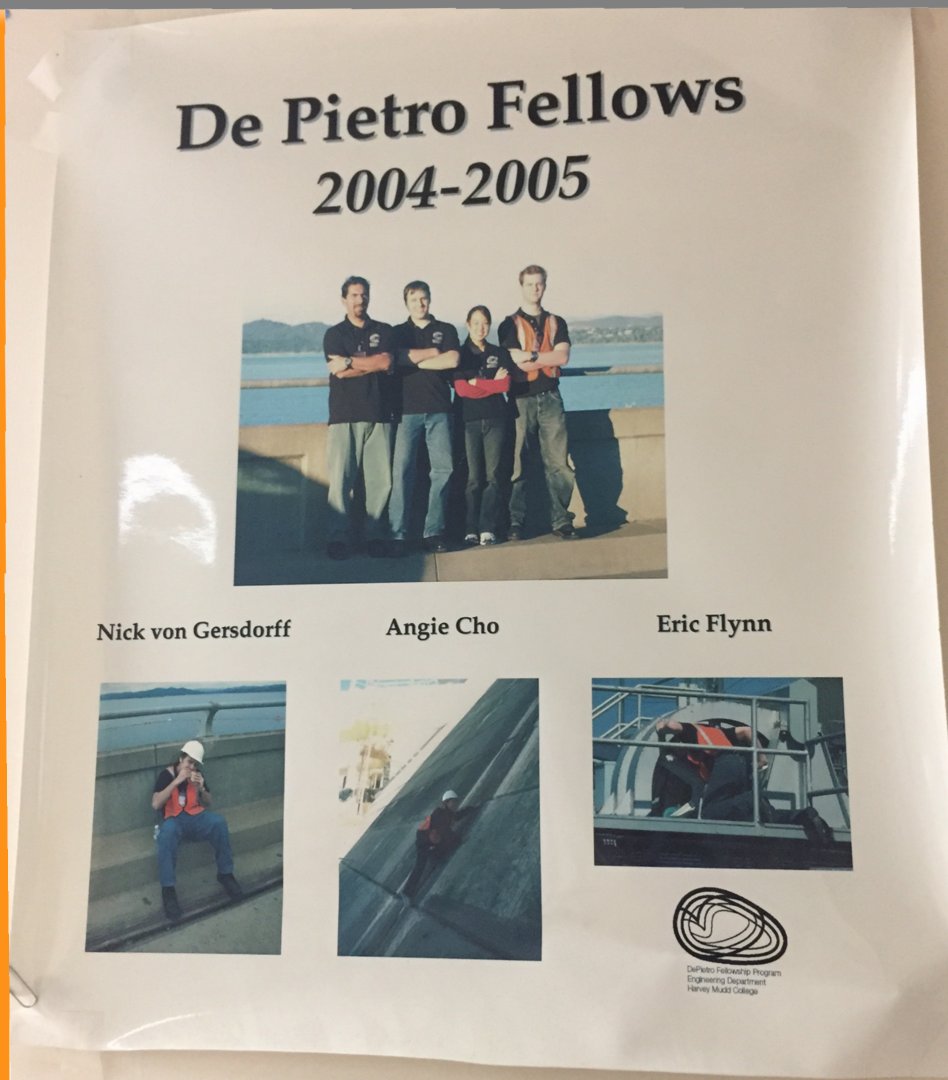
E80 Spring 2015

Vibration and System Identification

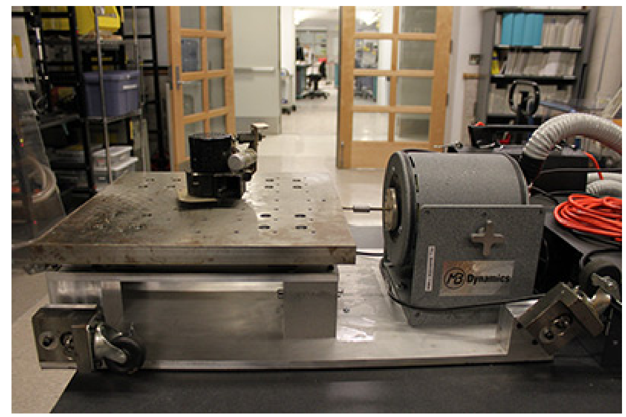
(adapted from 2013 and 2014 lectures)

Prof. Angie Lee

Folsam Dam: vibration testing



Shaker: sinusoidal input



http://itll.colorado.edu/test_measurement_equipment/vibration_testing/

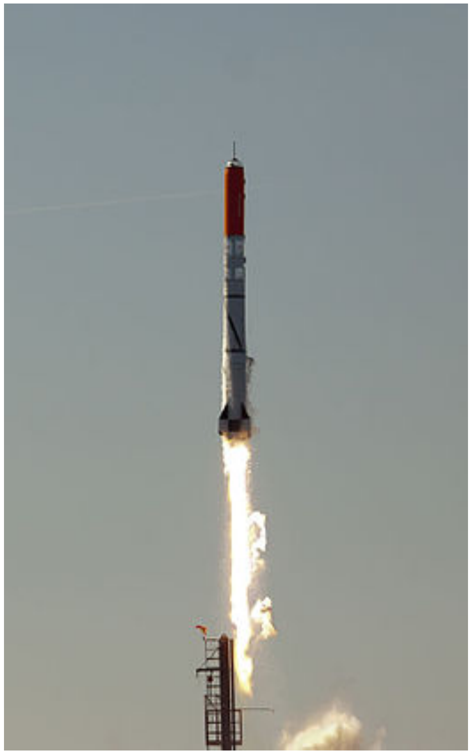
Cold gas thruster: impulse

Lecture outline

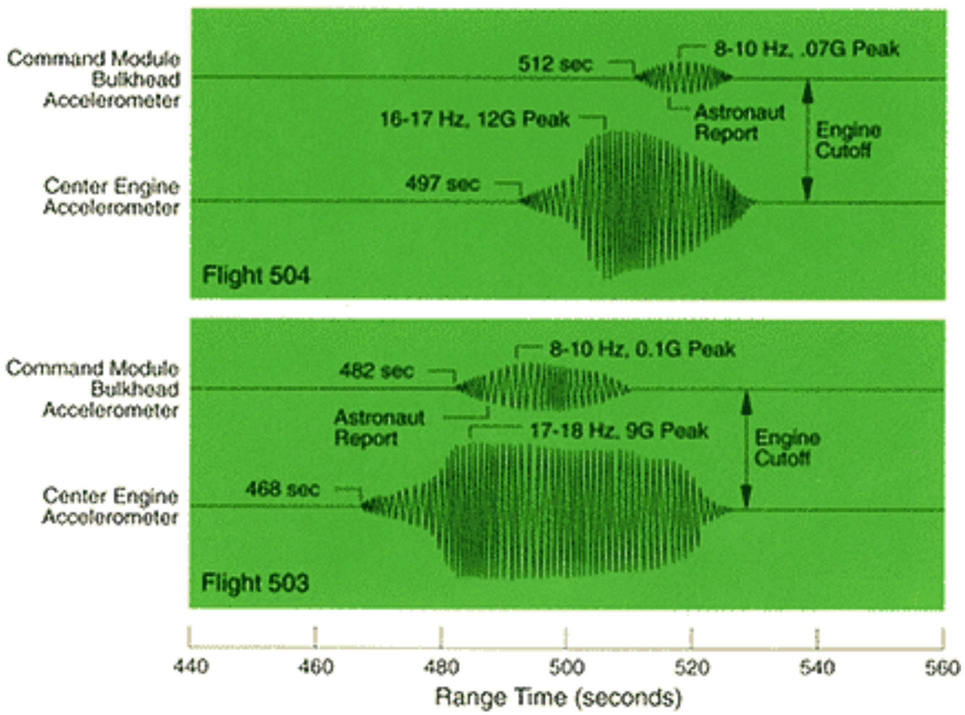
- Motivation
- Vibration testing (experiment)
  - Shaker tests, impact hammer tests
  - Lab test --> field test --> flight test
- Vibration analysis (modeling)
  - Spring-mass-damper model (mathematical modeling)
  - Continuum model (numerical modeling)
  - Validation

Rocket vibration

- **HEAT1X-Tycho Brahe inaugural flight**
- **Pilot's POV – 9 Hz oscillation**
- http://www.youtube.com/watch?v=-rASHRBo9Rg&feature=player_embedded



Saturn rocket vibration



"Pain was directly associated with motion of the eyeballs and testicles, as well as from internal heating that resulted from sloshing of the brain and viscera. The vibration frequency was also in the range of normal brain waves, adding confusion to decision making, hand and arm movement, and even speech."

- Jim Fenwich on Pogo oscillations

http://www.pwrengineering.com/articles/pogo.htm

Space shuttle main engine turbopumps

"The high-pressure pumps rotated at speeds reaching 36,000 rpm on the fuel side and 24,000 rpm on the oxidizer side. At these speeds, minor faults were exacerbated and could rapidly propagate to catastrophic engine failure."

"…the vibration spectral data contained potential failure indicators in the form of discrete rotordynamic spectral signatures. These signatures were prime indicators of turbomachinery health…"

"Wings in Orbit" edited by Wayne Hale and Helen Lane

Rocket failure, March 2012

“While the lower stages of the North Korean rocket continued to function for several minutes, resonance at the top of the launch vehicle resulted in ‘**catastrophic disassembly** of the third stage at Max Q,’ said Charles Vick, senior technical and space policy analyst at GlobalSecurity.org. ‘The vibrations just tore it apart.’”

http://www.nytimes.com/2012/04/13/world/asia/north-korea-launches-rocket-defying-world-warnings.html?pagewanted=all&_r=0

http://www.eetimes.com/electronics-news/4370955/Severe-vibrations-likely-brought-down-N--Korean-rocket

Cantilever vibration modes

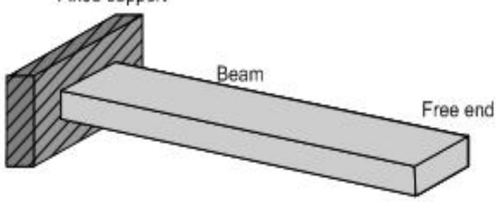


Fig. 4.1 (a): A cantilever beam

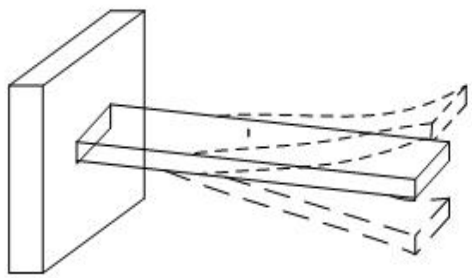


Fig. 4.1 (b): The beam under free vibration

http://iitg.vlab.co.in/?sub=62&brch=175&sim=1080&cnt=1

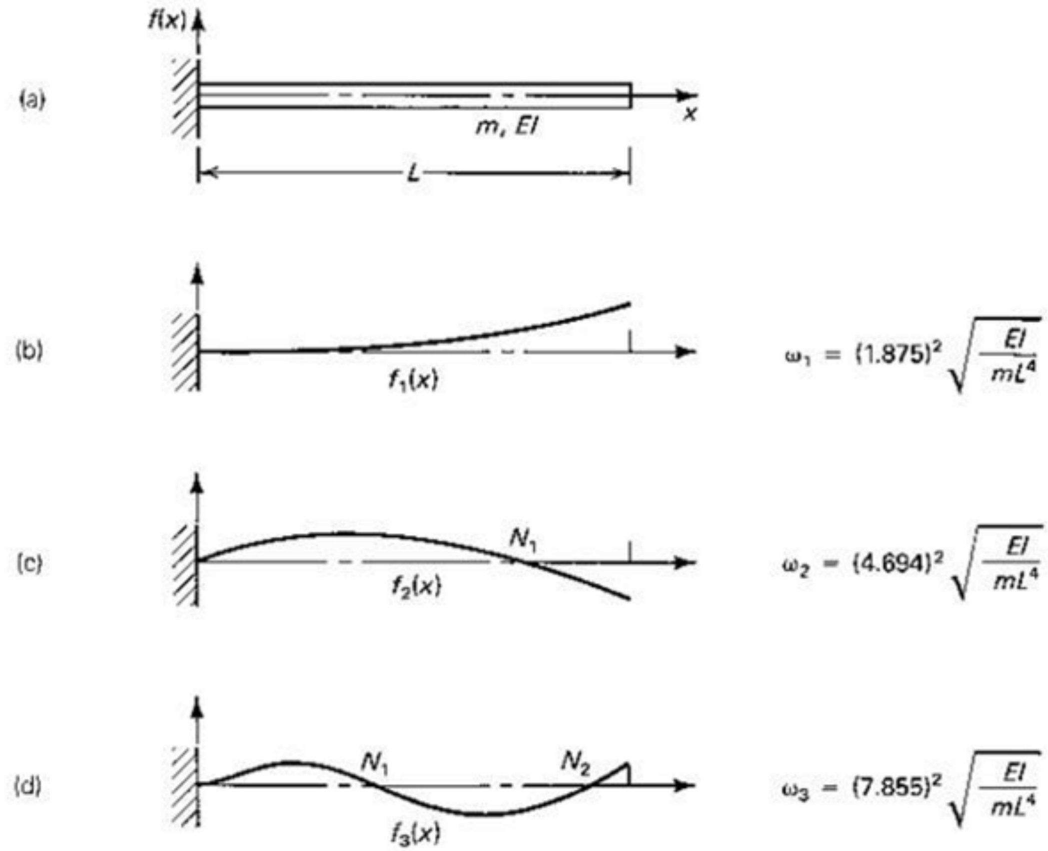


Fig. 4.3: The first three undamped natural frequencies and mode shape of cantilever beam

https://www.youtube.com/watch?v=kun62B7VUg8

Vibration testing

- Lab tests
  - Shaker tests
  - https://www.youtube.com/watch?v=o8H_NT7Ziao
  - https://www.youtube.com/watch?v=pCXTZDfTdG0
  - https://www.youtube.com/watch?v=XkmgMkDKAyU
  - Impact hammer tests
  - https://www.youtube.com/watch?v=tBRjPN8m6zE

Vibration analysis

- Need to determine loading (what is causing the vibration?)
- Modeling
  - Mathematical model
    - Lumped element model (spring-mass-damper)
    - Continuum model
  - Numerical/computational model
    - SolidWorks simulation
- Verify model with experimental data

Spring-mass-damper model

- Around a resonance frequency, you can model as

$$m_e \ddot{y} = f - ky - c\dot{y}$$

$$m\ \ddot{y} + c\dot{y} + ky = f$$

$$\ddot{y} + \frac{c}{m_e}\dot{y} + \frac{k}{m_e}y = \frac{f}{m_e}$$

$$\ddot{y} + 2\zeta\omega_n \dot{y} + \omega_n^2 y = f / m_e$$

$$\omega_n = \sqrt{\frac{k}{m_e}} \qquad \zeta = \frac{c}{2\sqrt{m_e k}}$$

Frequency response function (FRF)

- Position

$$\frac{Y}{F} = \frac{\frac{1}{m_e}\left(\frac{1}{\omega_n}\right)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2 + 2\zeta\frac{\omega}{\omega_n}j}$$

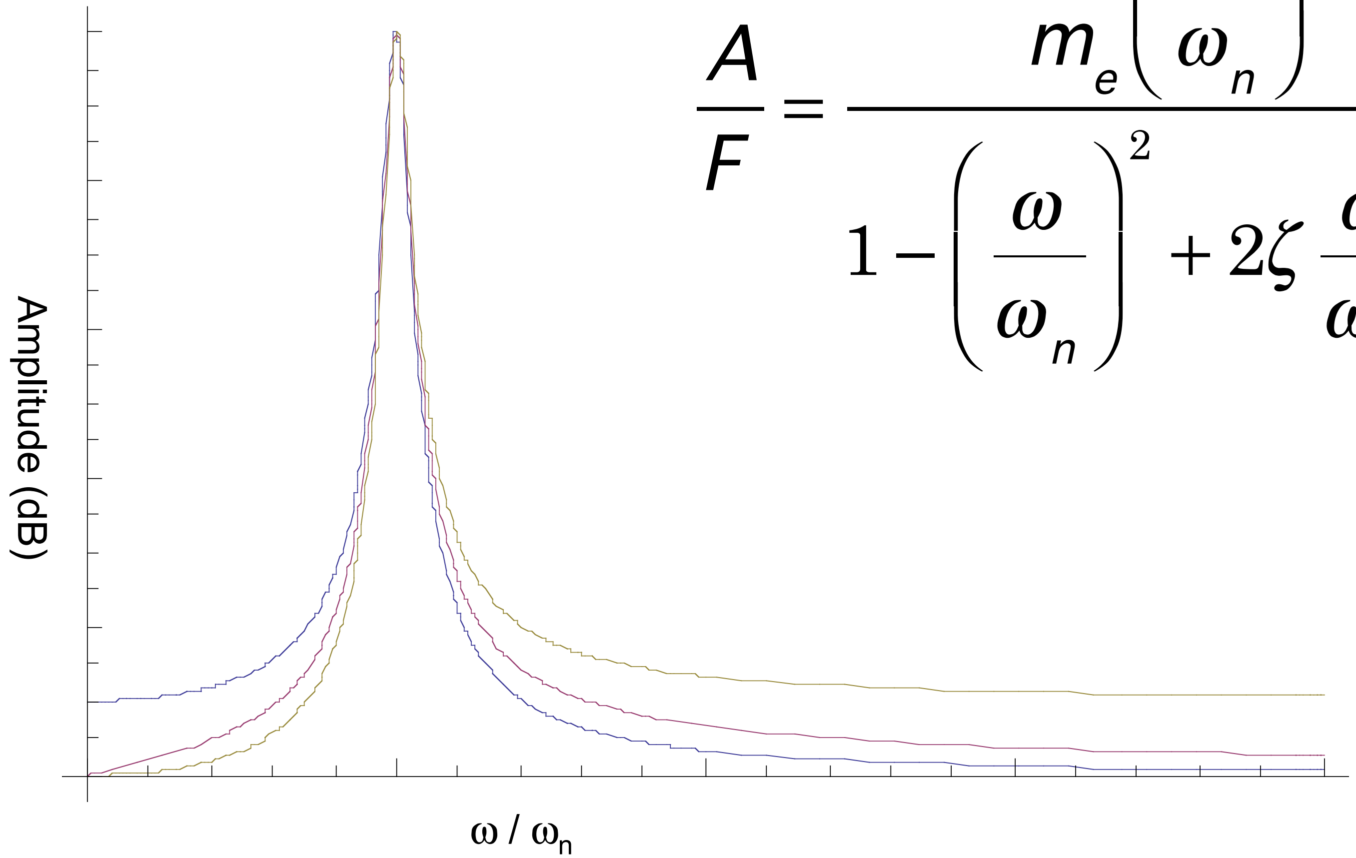
- Velocity

$$\frac{V}{F} = \frac{j\omega\frac{1}{m_e}\left(\frac{1}{\omega_n}\right)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2 + 2\zeta\frac{\omega}{\omega_n}j}$$

Frequency response function (FRF)

- Acceleration

$$\frac{A}{F} = \frac{-\frac{1}{m_e}\left(\frac{\omega}{\omega_n}\right)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2 + 2\zeta\frac{\omega}{\omega_n}j}$$

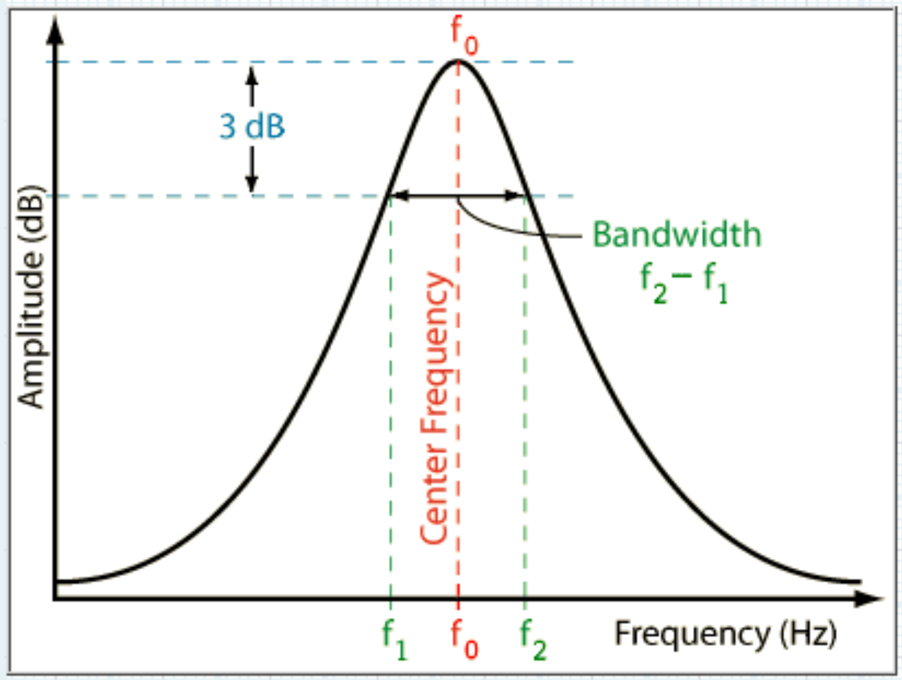


Damping coefficient

- From the peak $\omega_r = \omega_n \sqrt{1-\zeta^2}$
- From the half-power bandwidth $\Delta\omega = \omega_{+hp} - \omega_{-hp}$

$$Q = \frac{\omega_r}{\Delta\omega}$$

$$\zeta = \frac{1}{2Q}$$

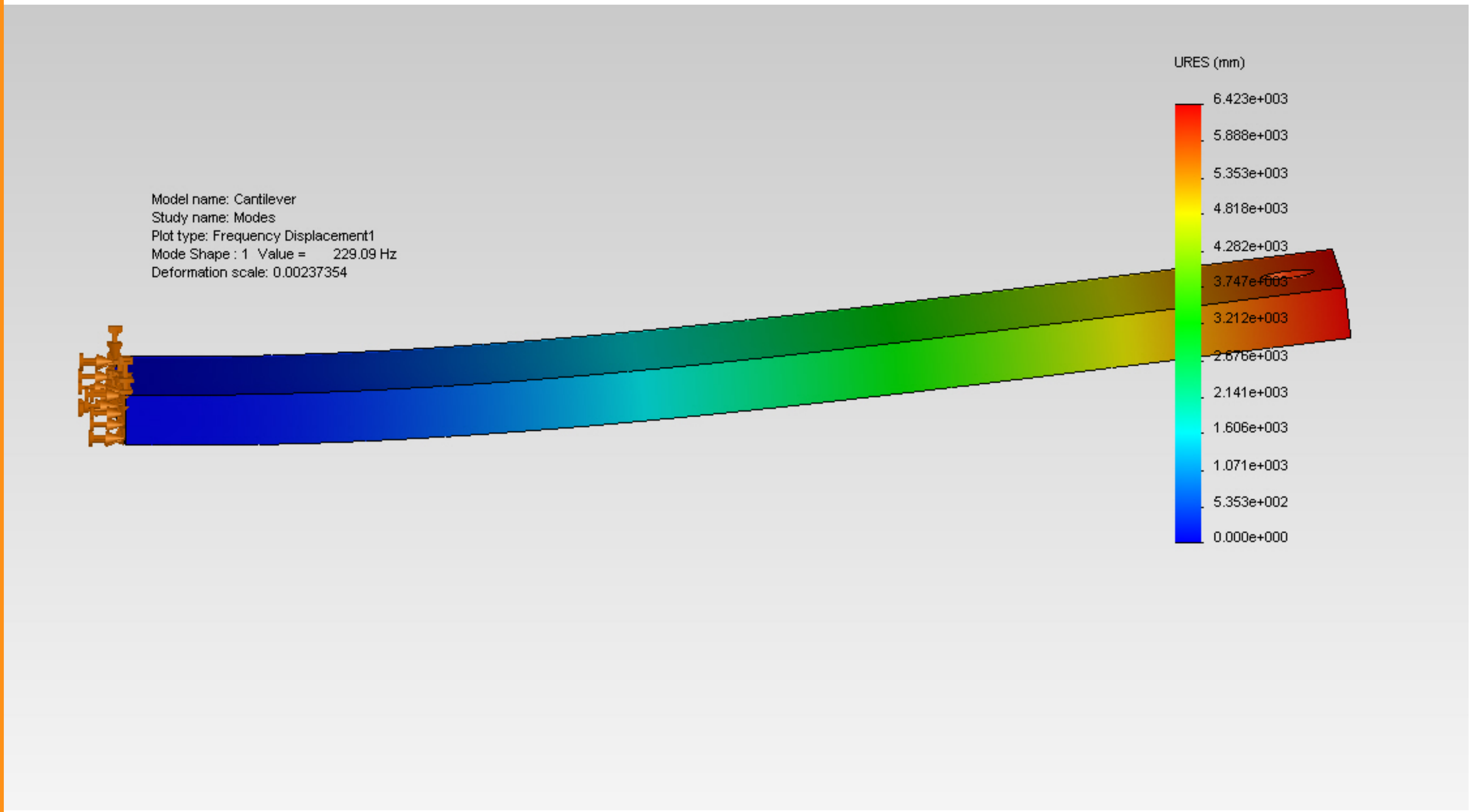


http://www.sengpielaudio.com/calculator-cutoffFrequencies.htm

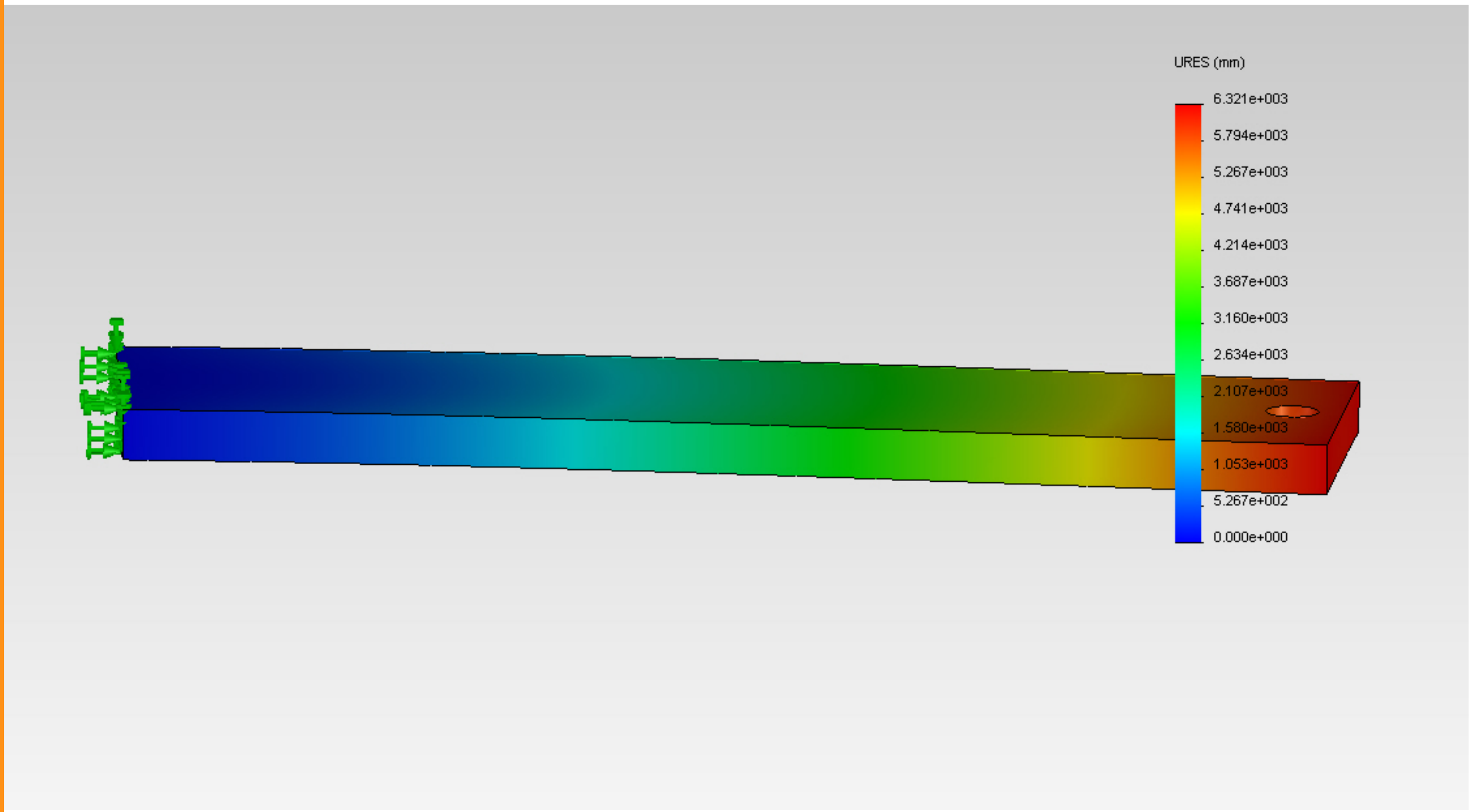
SolidWorks simulations

- Cantilever beam
- Rocket

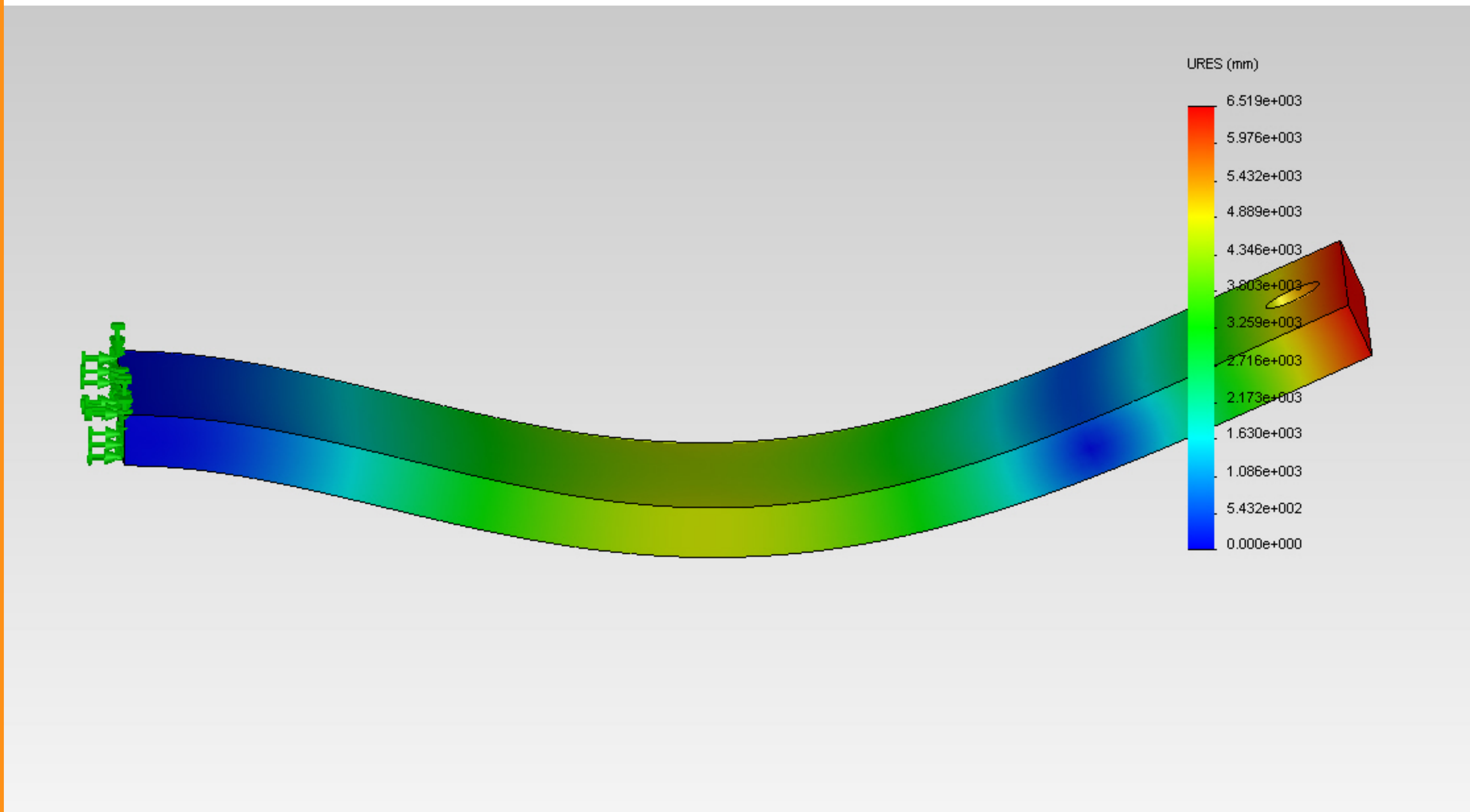
Mode 1: 299.09 Hz



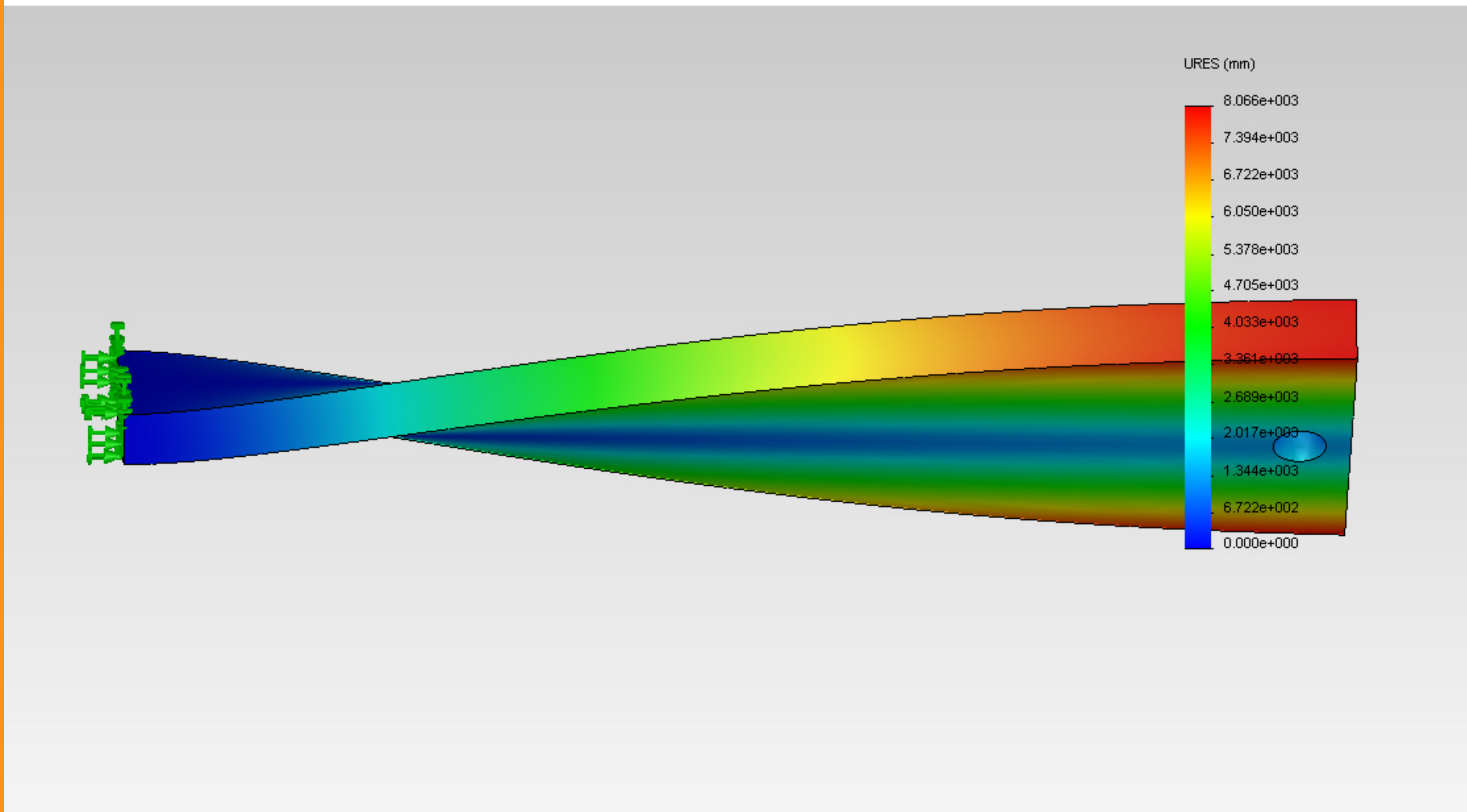
Mode 2: 1297.9 Hz



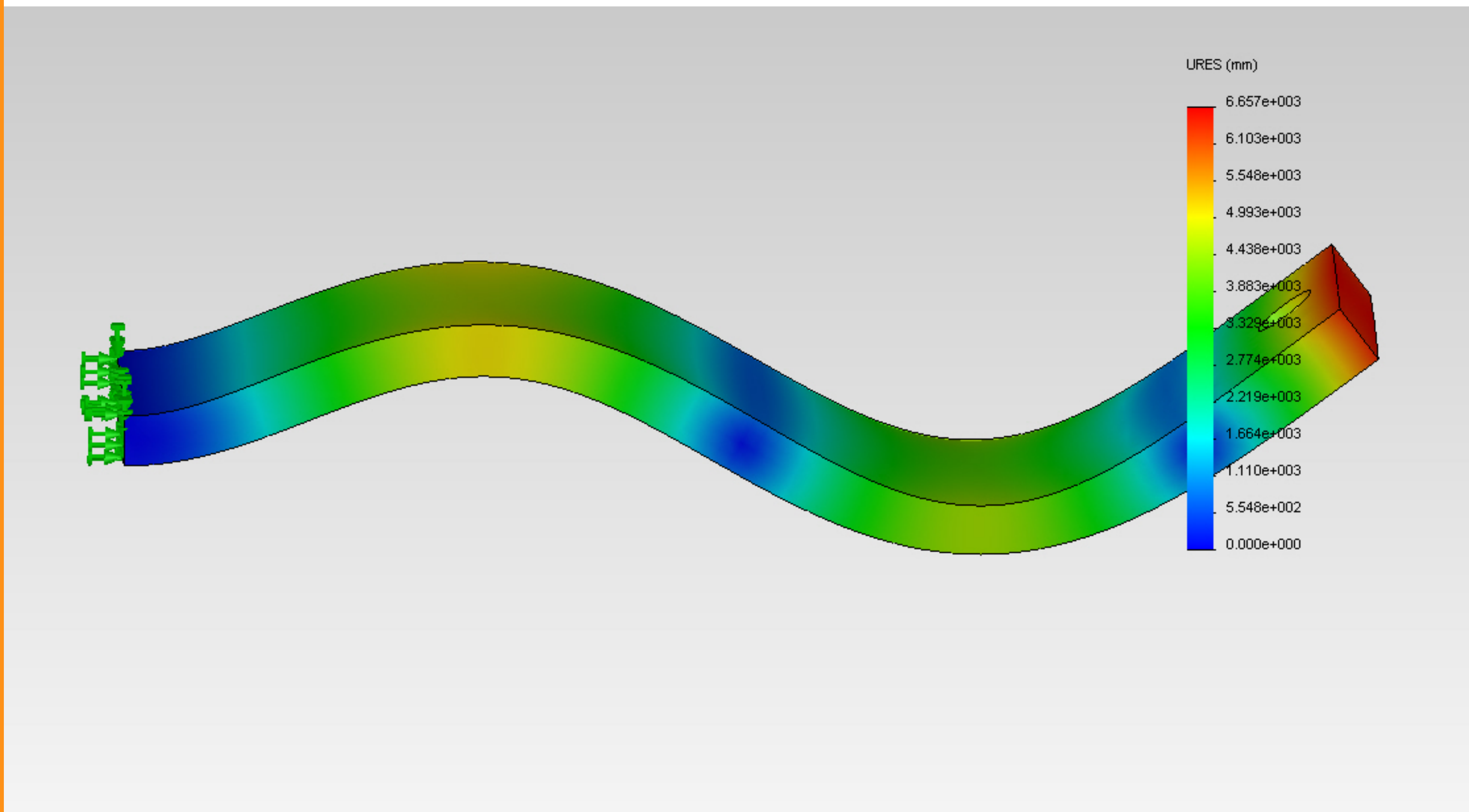
Mode 3: 1417.6 Hz



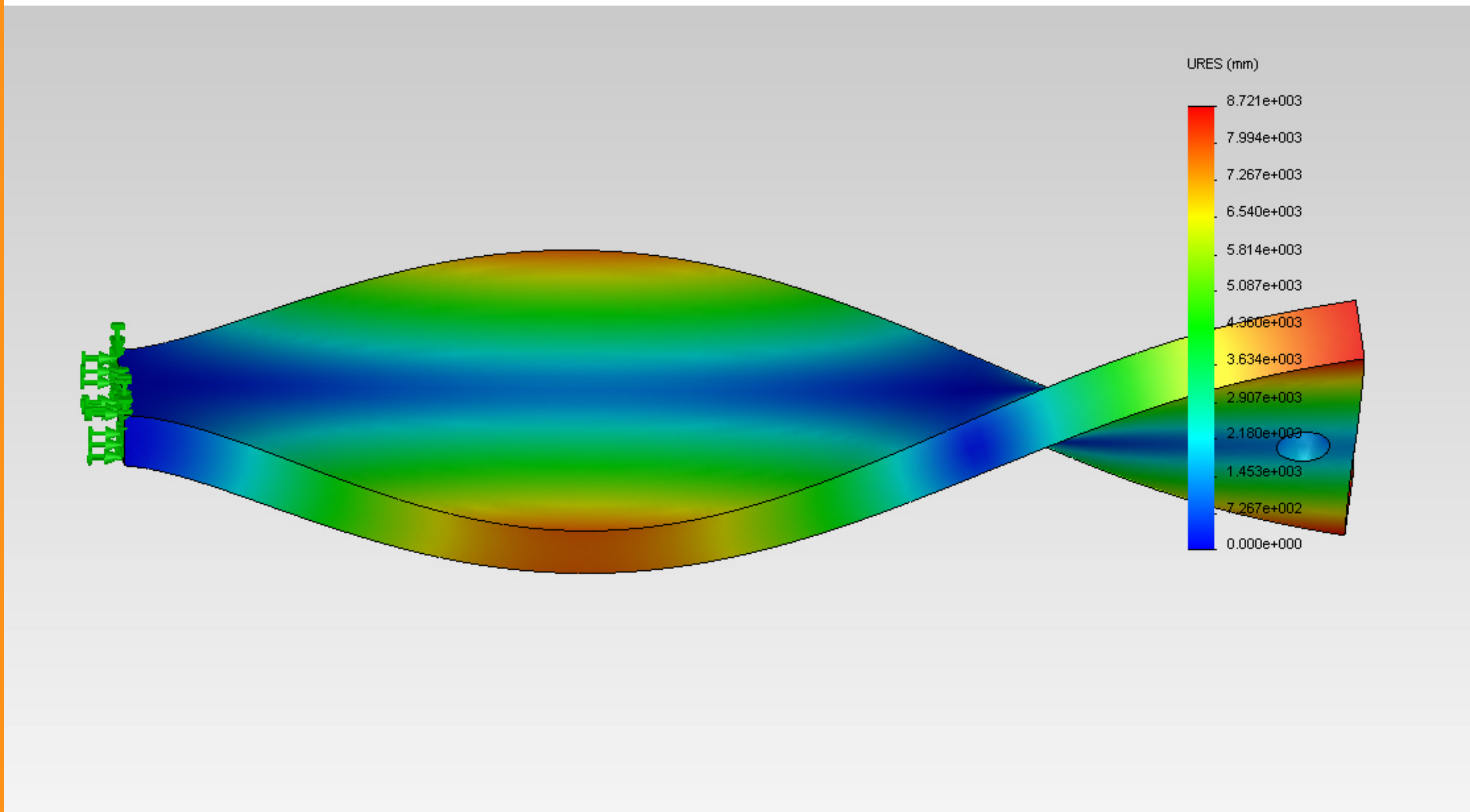
Mode 4: 1679.3 Hz



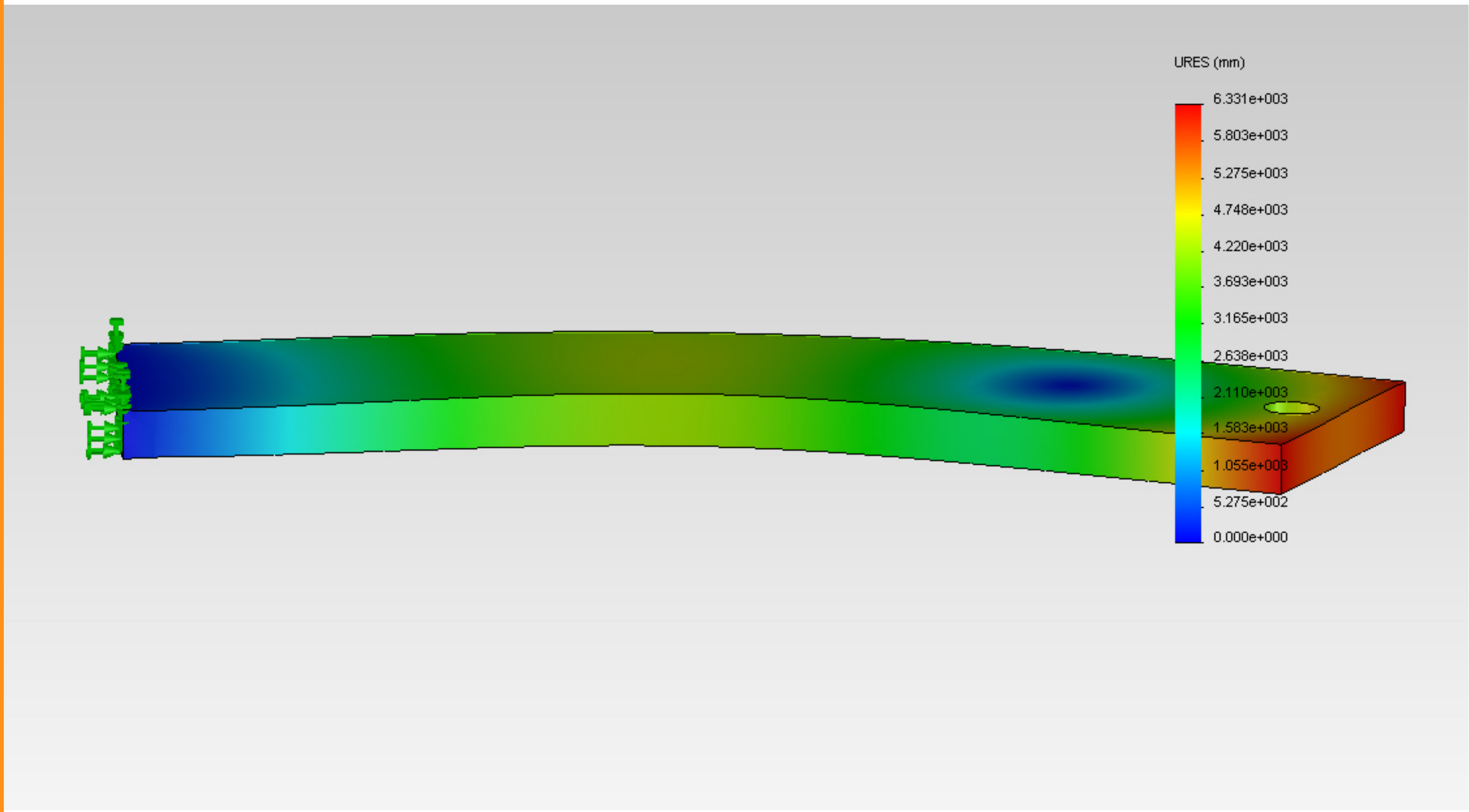
Mode 5: 3917.6 Hz



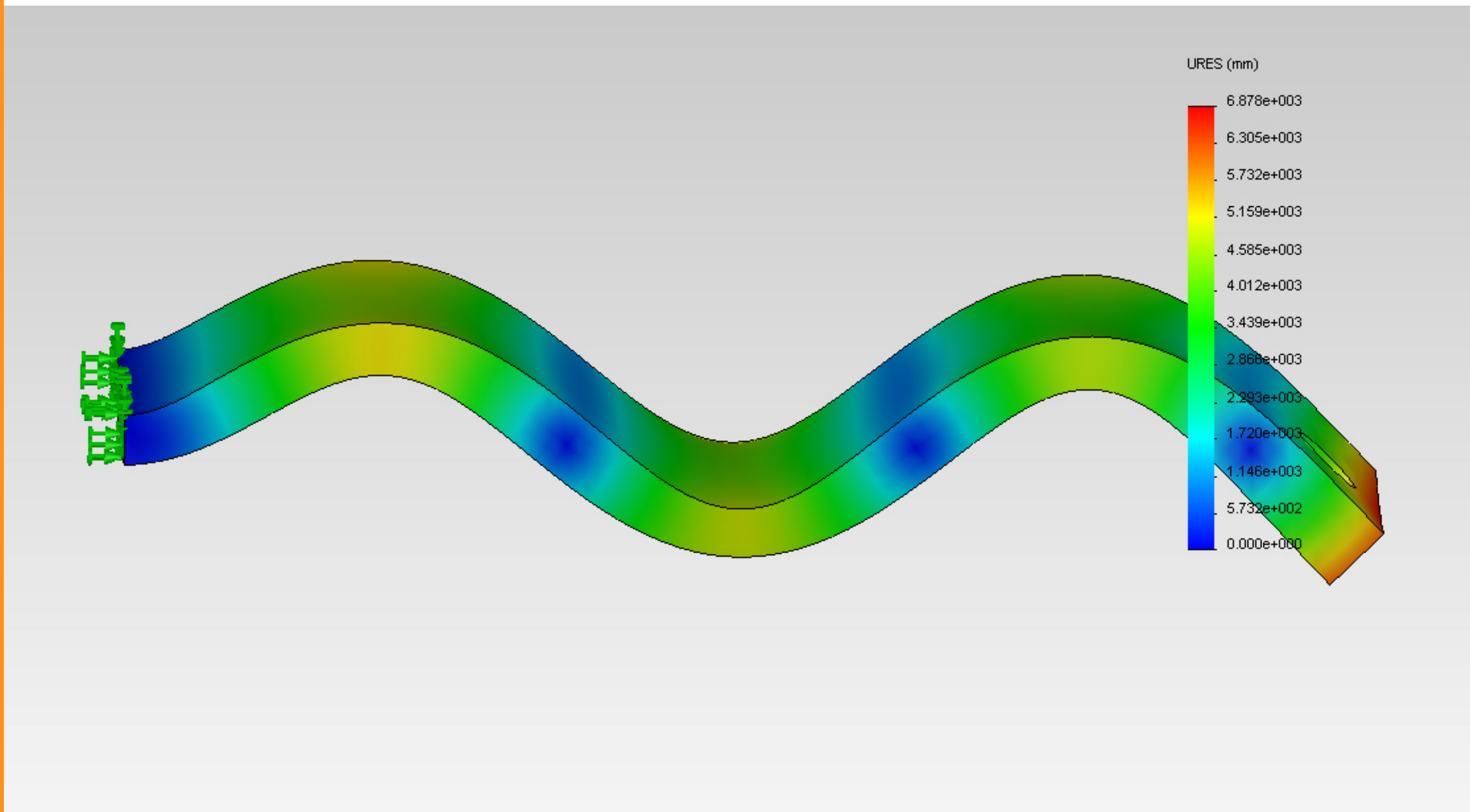
Mode 6: 5149.6 Hz



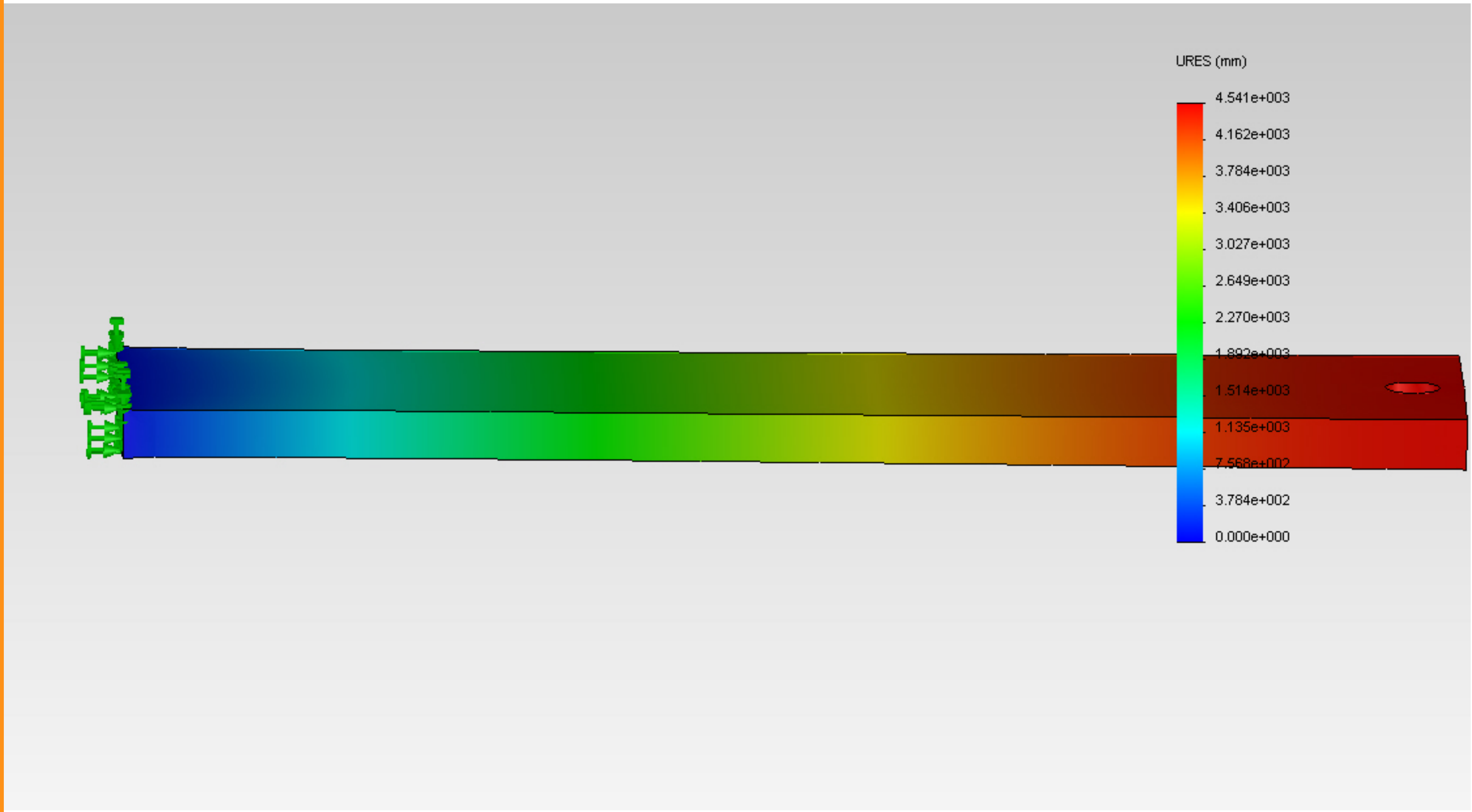
Mode 7: 6538.1 Hz



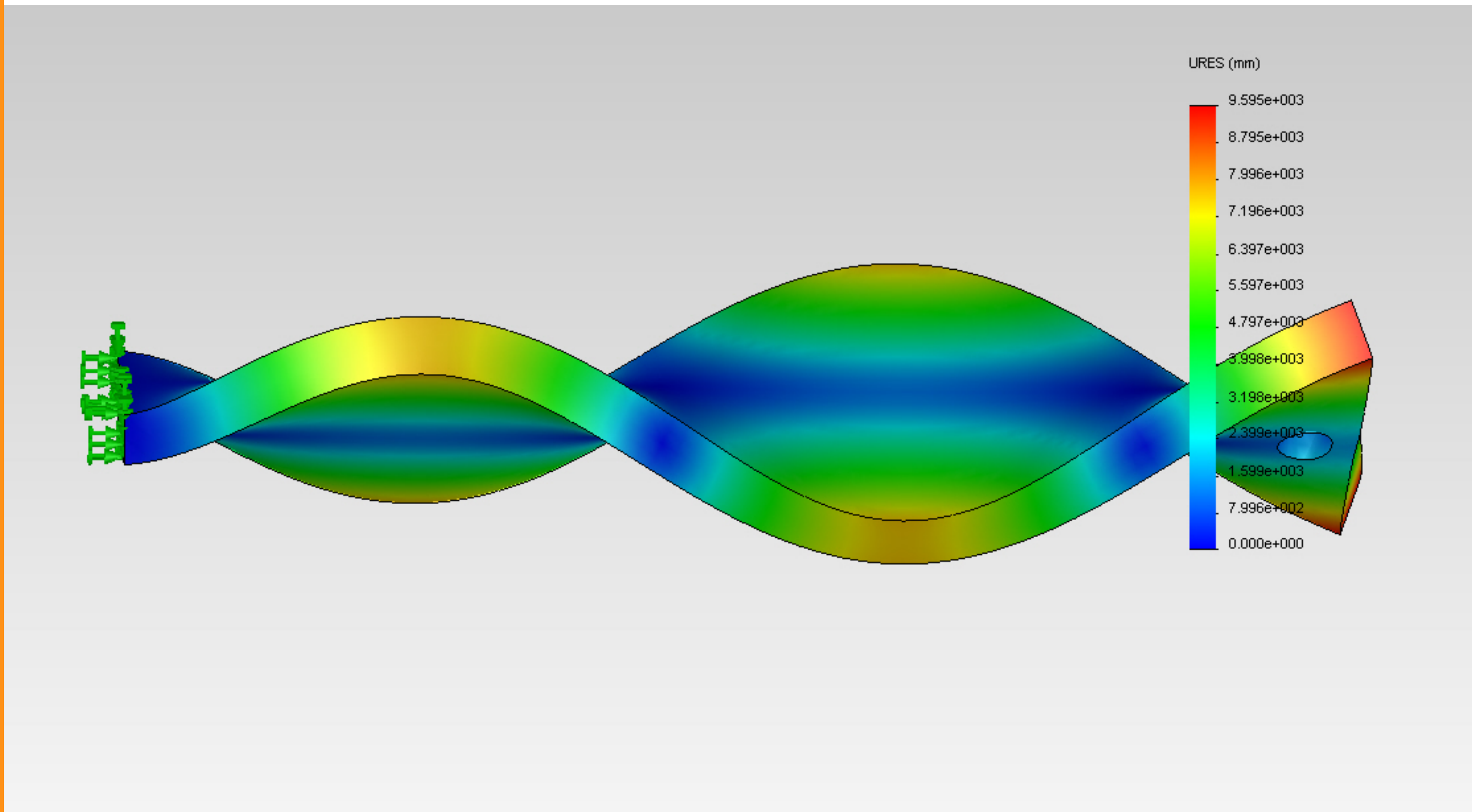
Mode 8: 7545.1 Hz



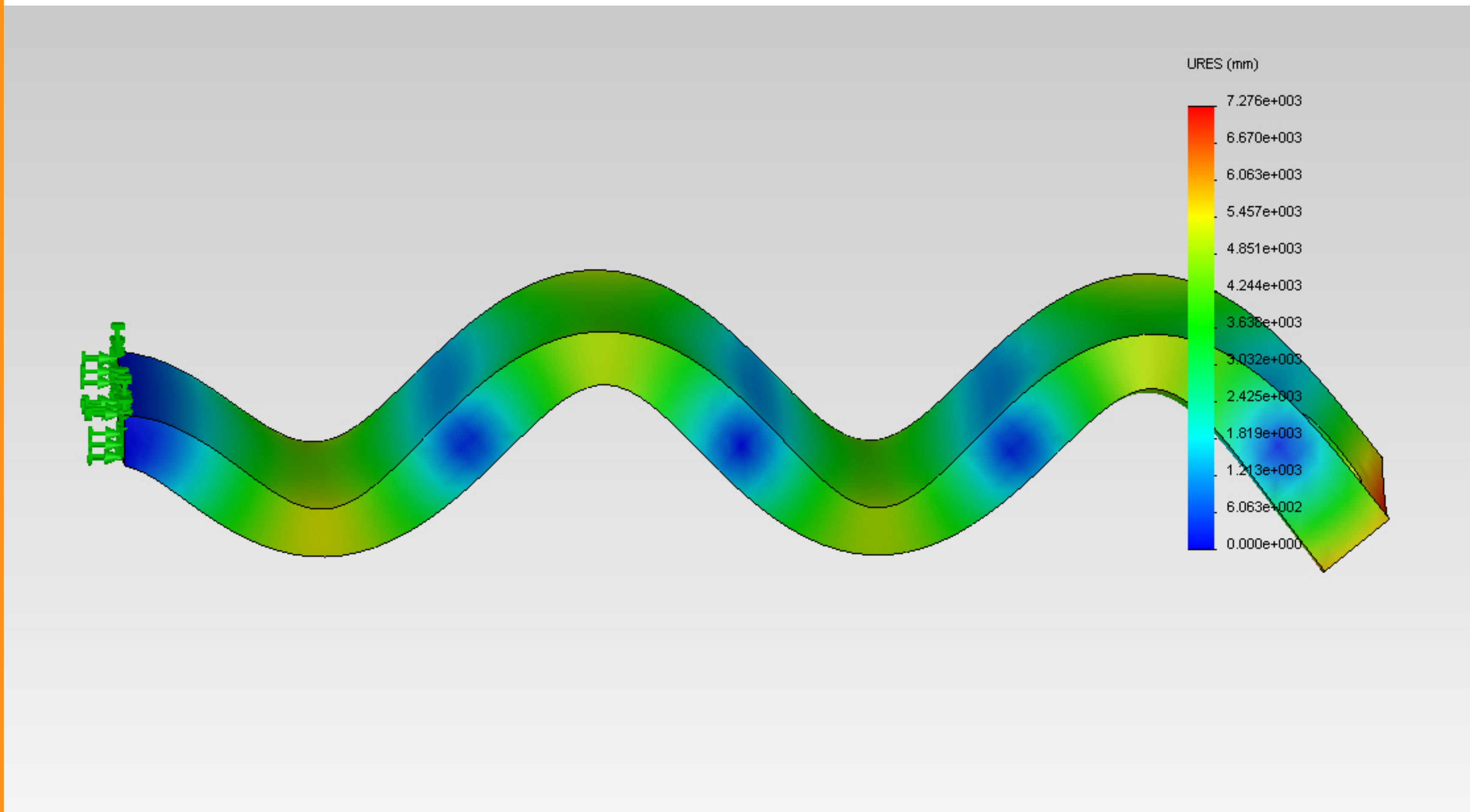
Mode 9: 8377.9 Hz



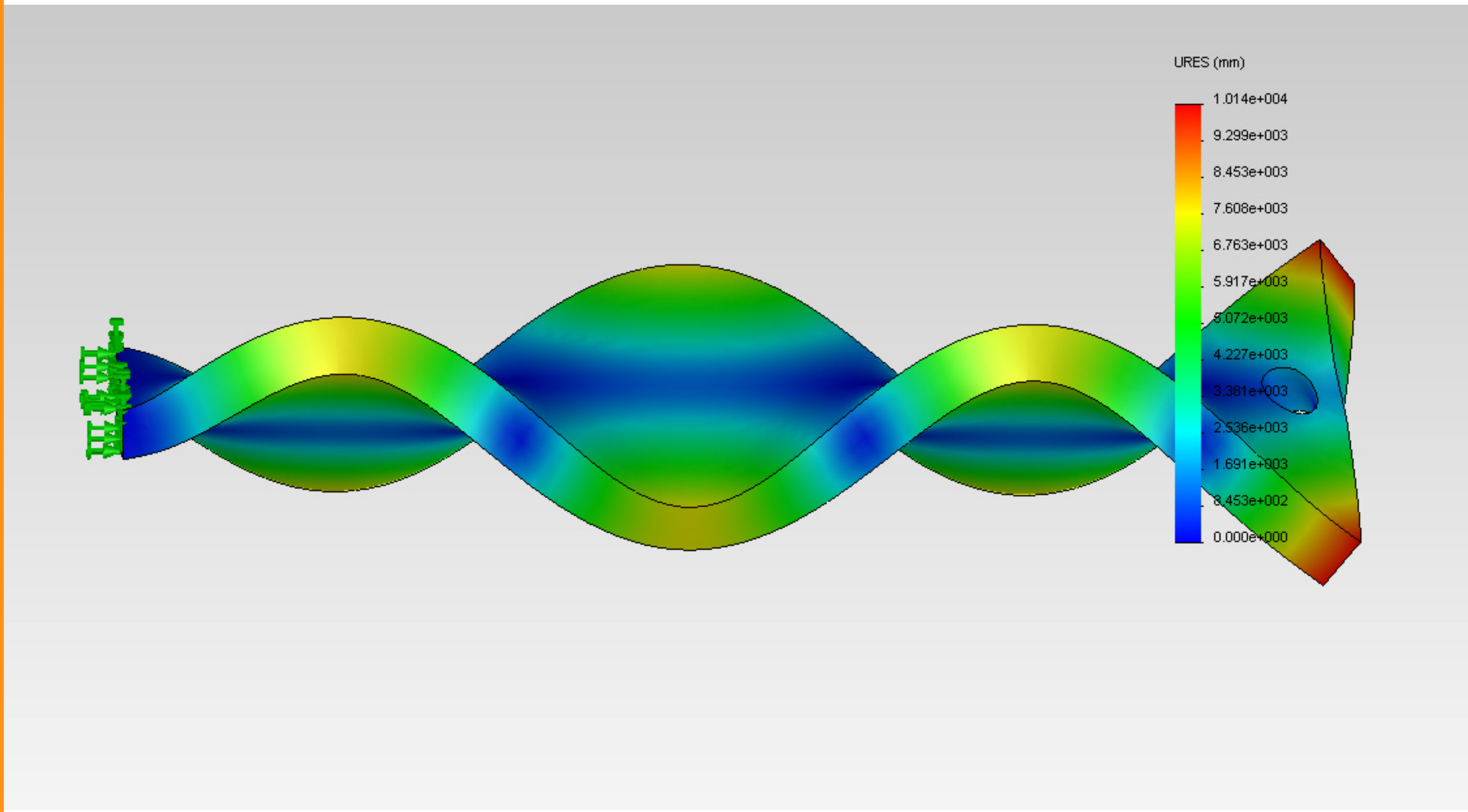
Mode 10: 8933.4 Hz



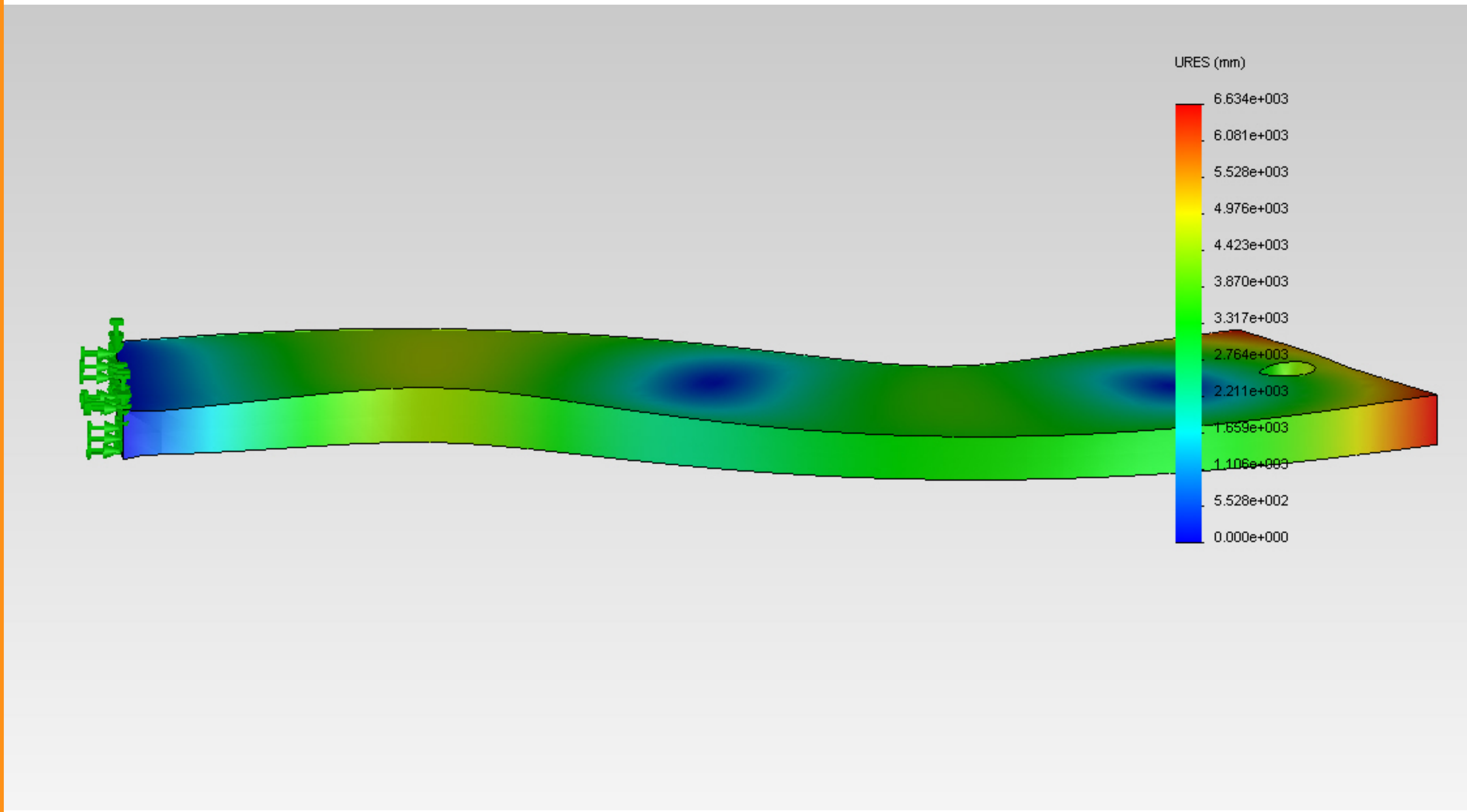
Mode 11: 12199 Hz



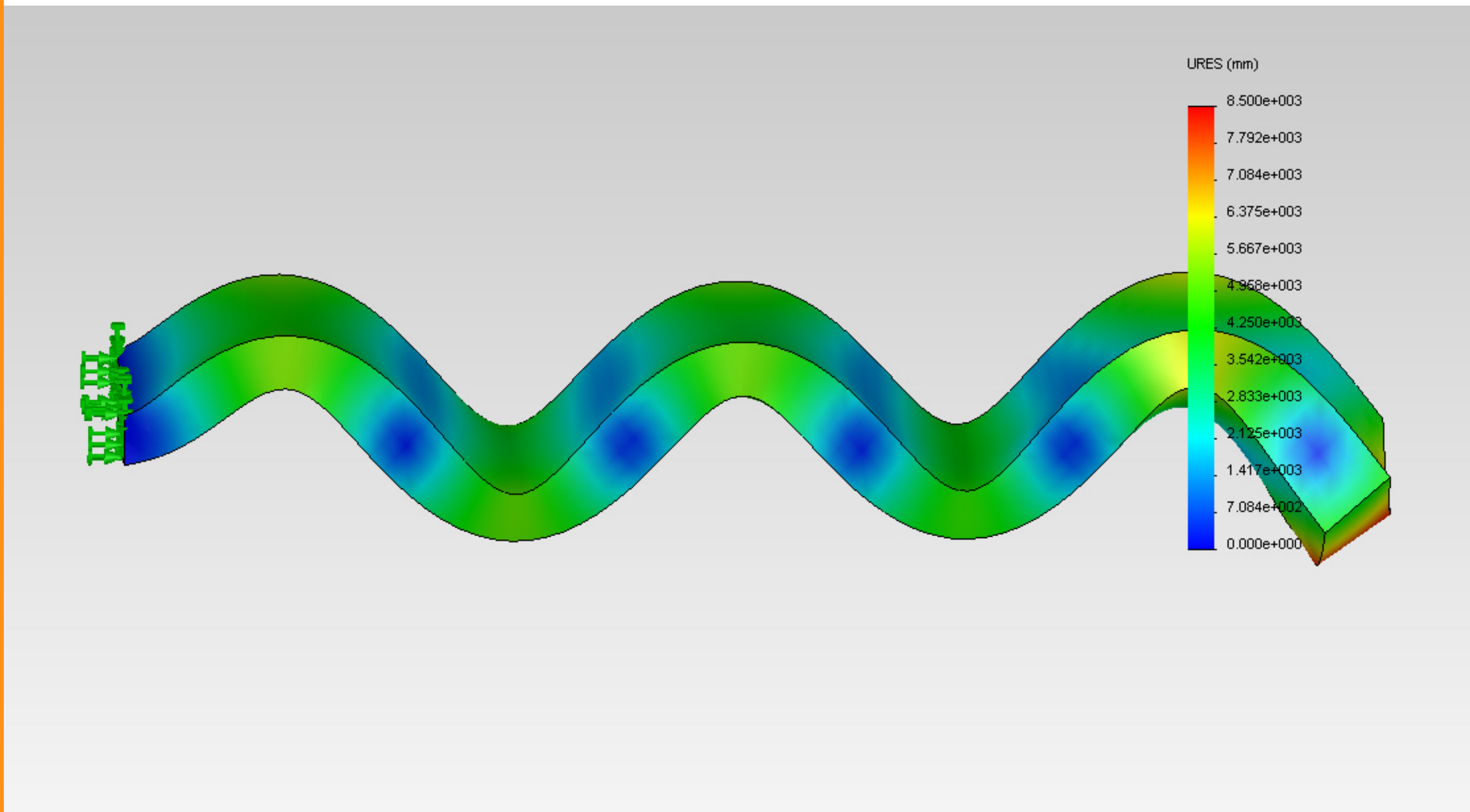
Mode 12: 13198 Hz



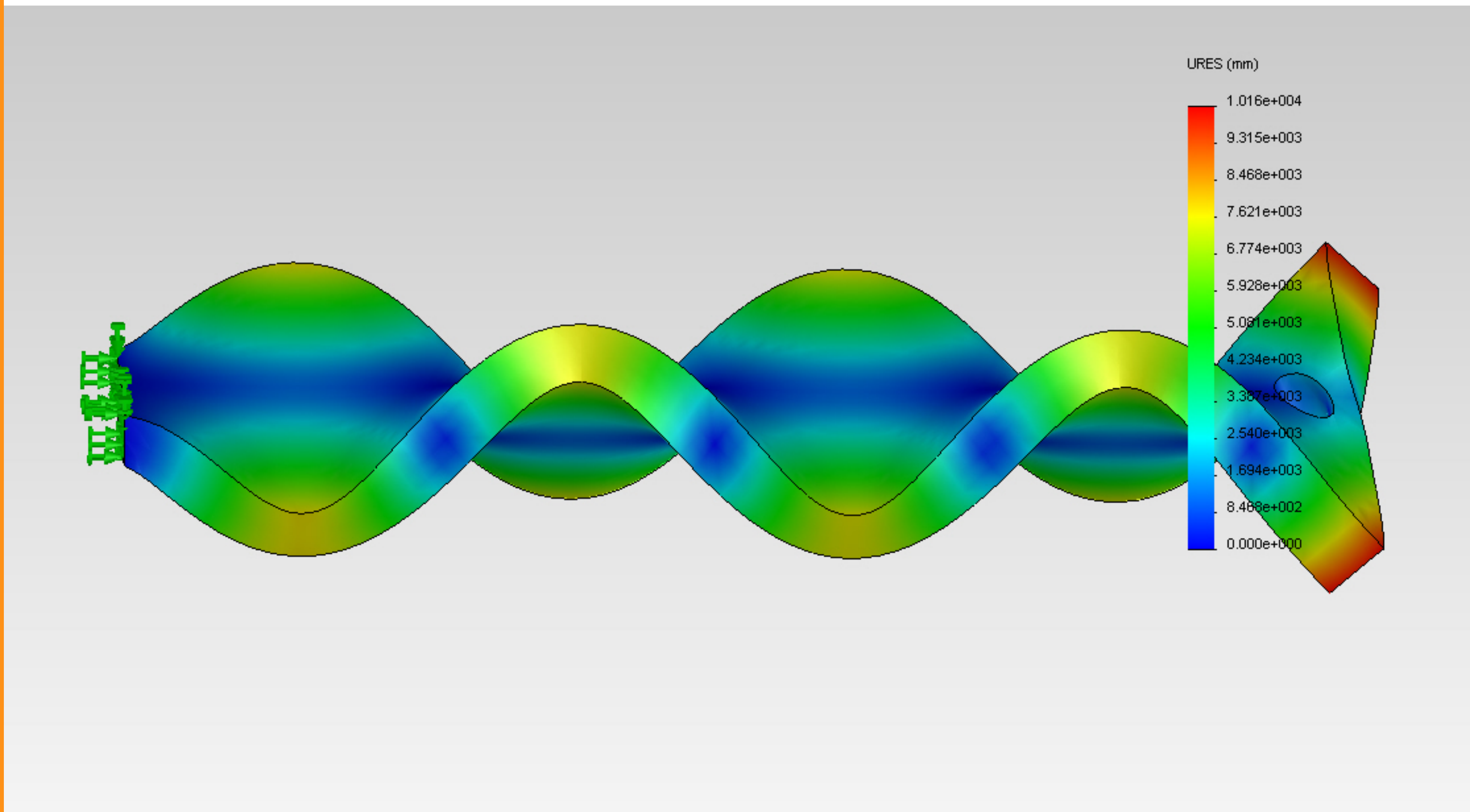
Mode 13: 14941 Hz



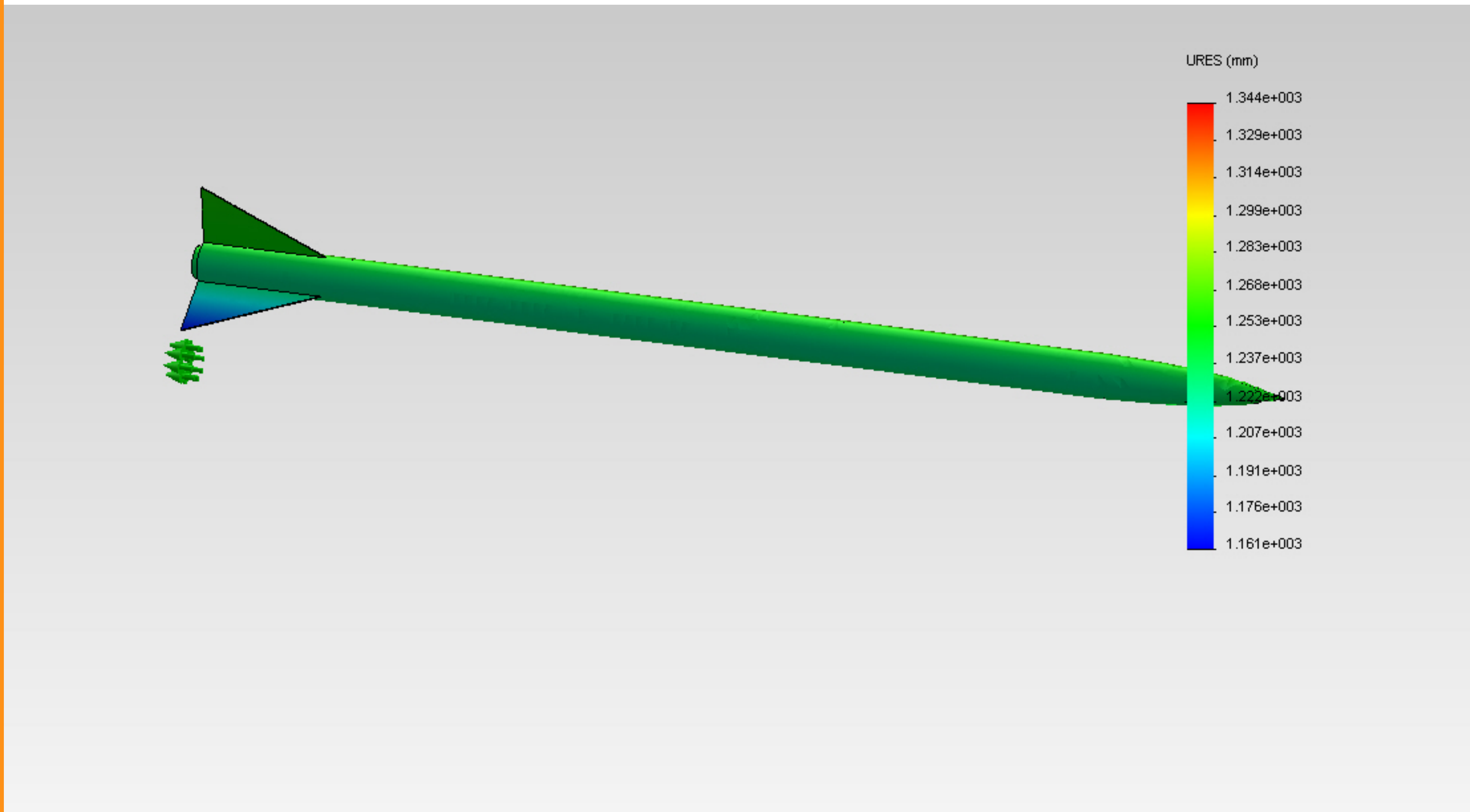
Mode 14: 17714 Hz



Mode 15: 18072 Hz

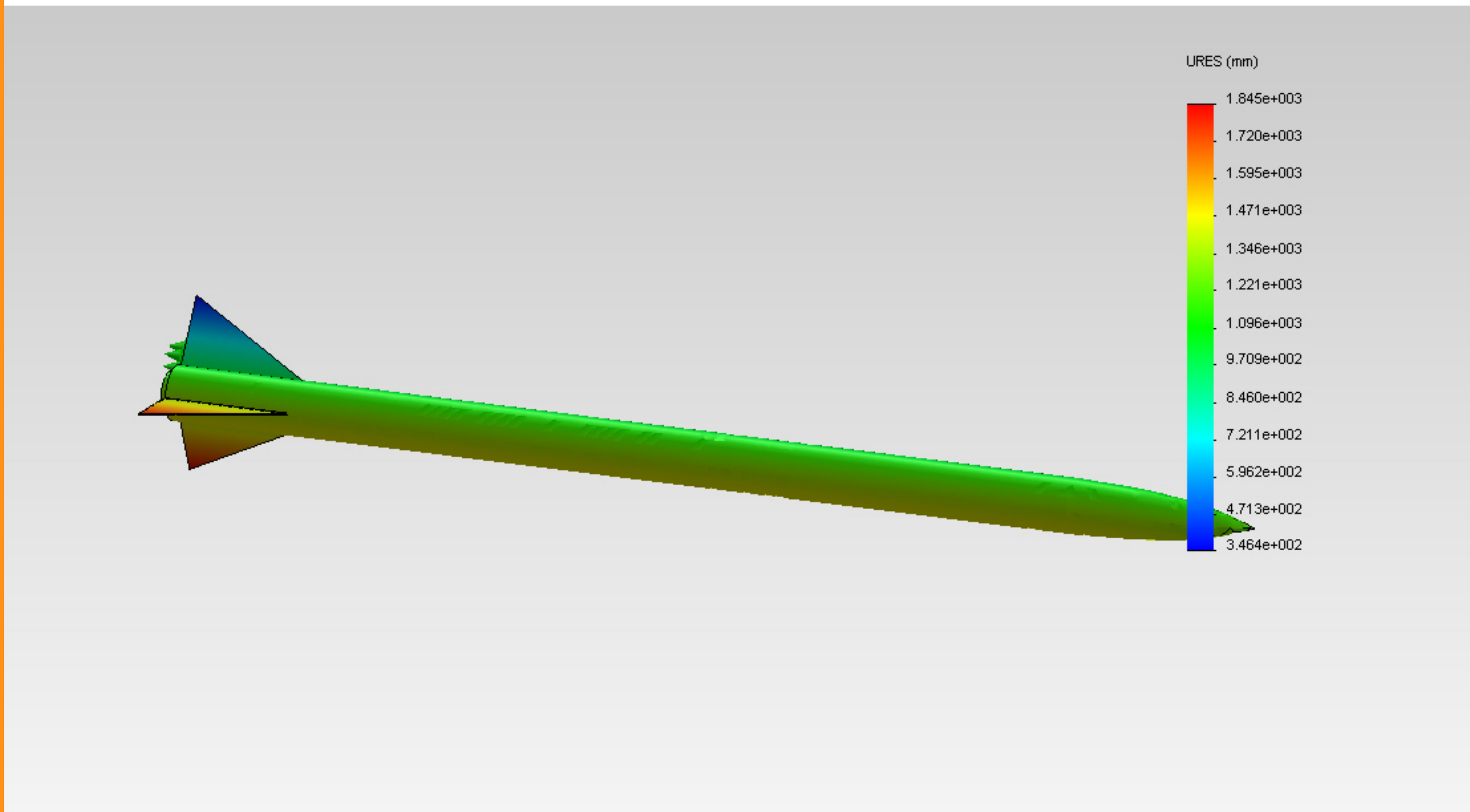


Mode 1: 0 Hz

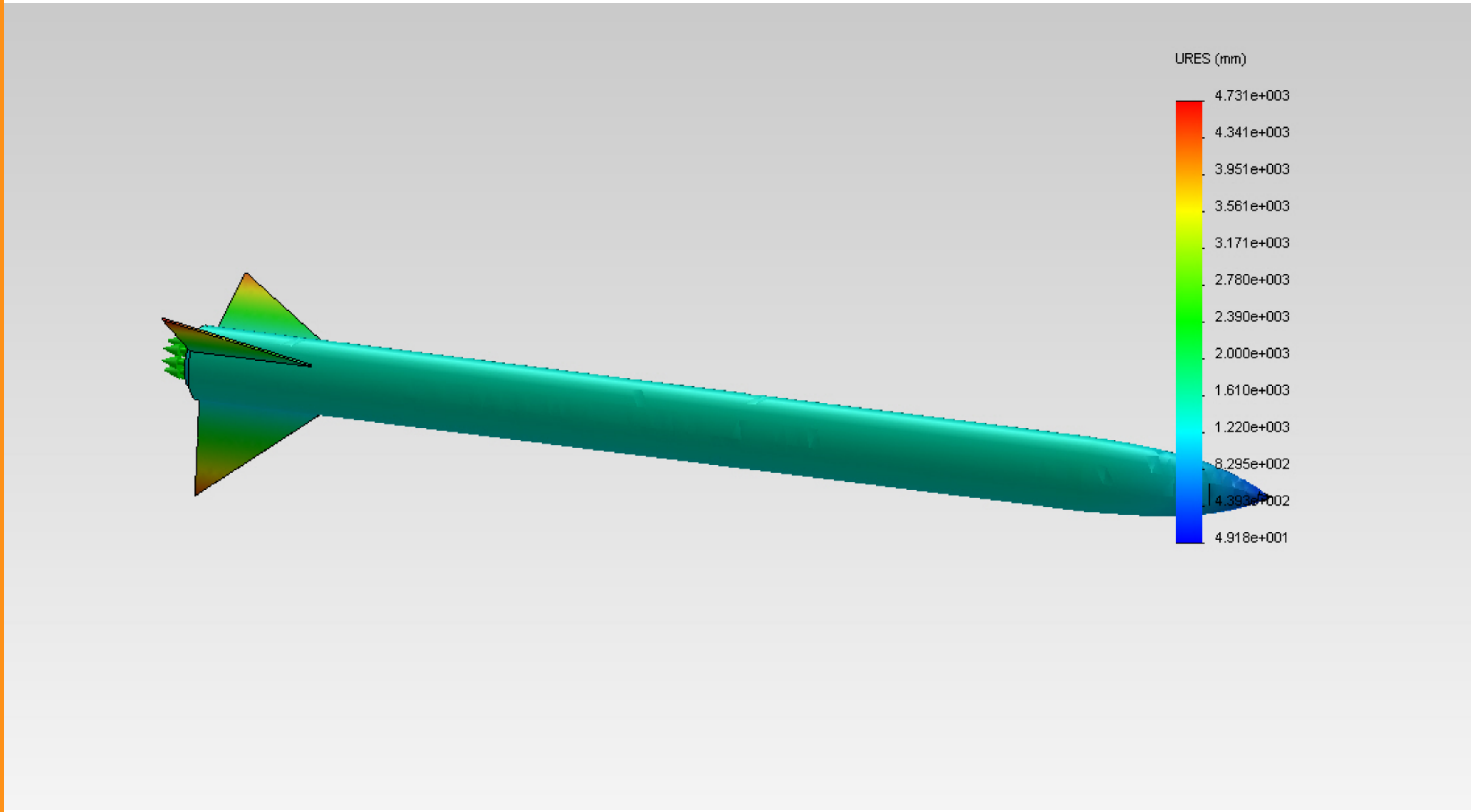


URES (mm)
1.344e+003
1.329e+003
1.314e+003
1.299e+003
1.283e+003
1.268e+003
1.253e+003
1.237e+003
1.222e+003
1.207e+003
1.191e+003
1.176e+003
1.161e+003

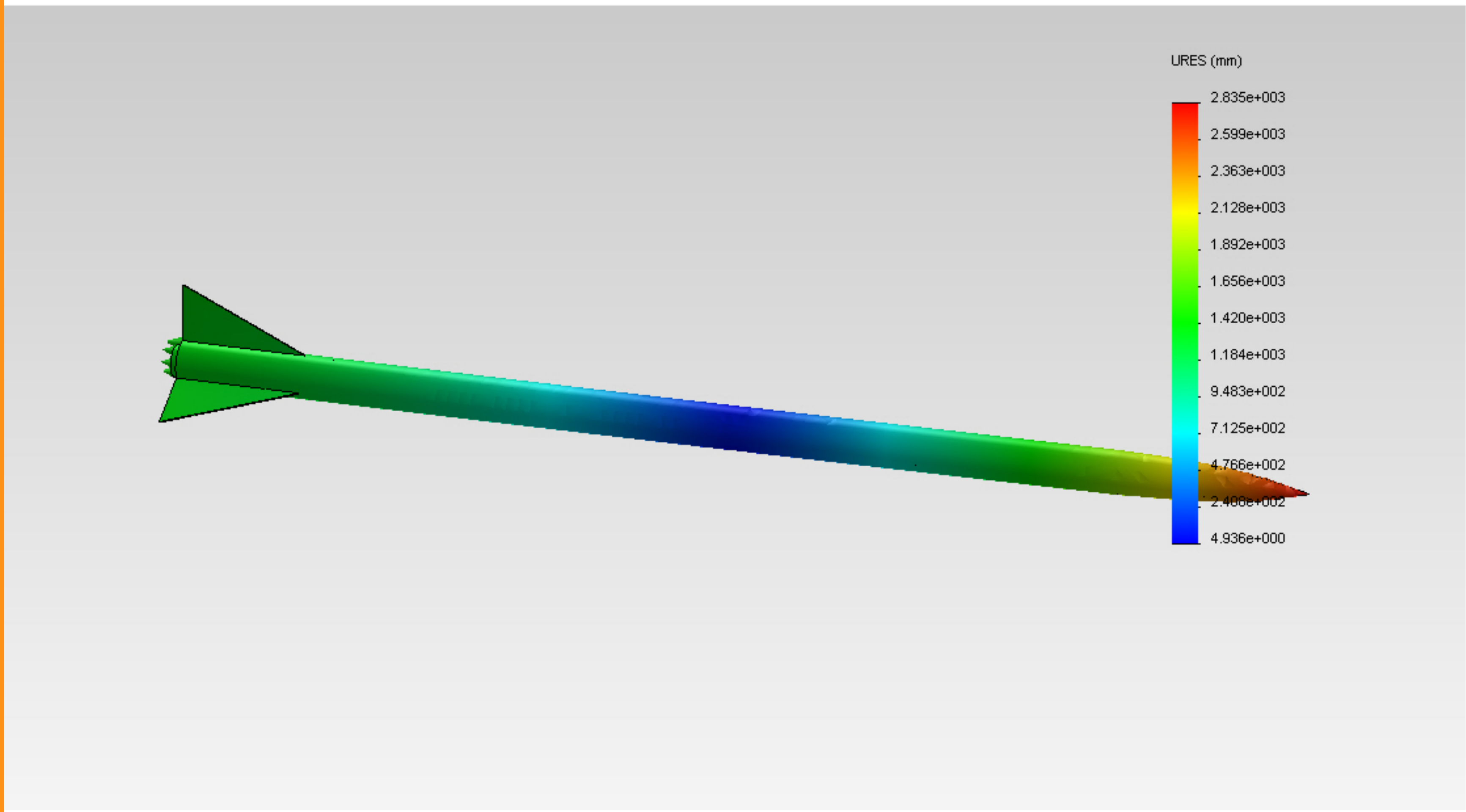
Mode 2: 7.0439E-4 Hz



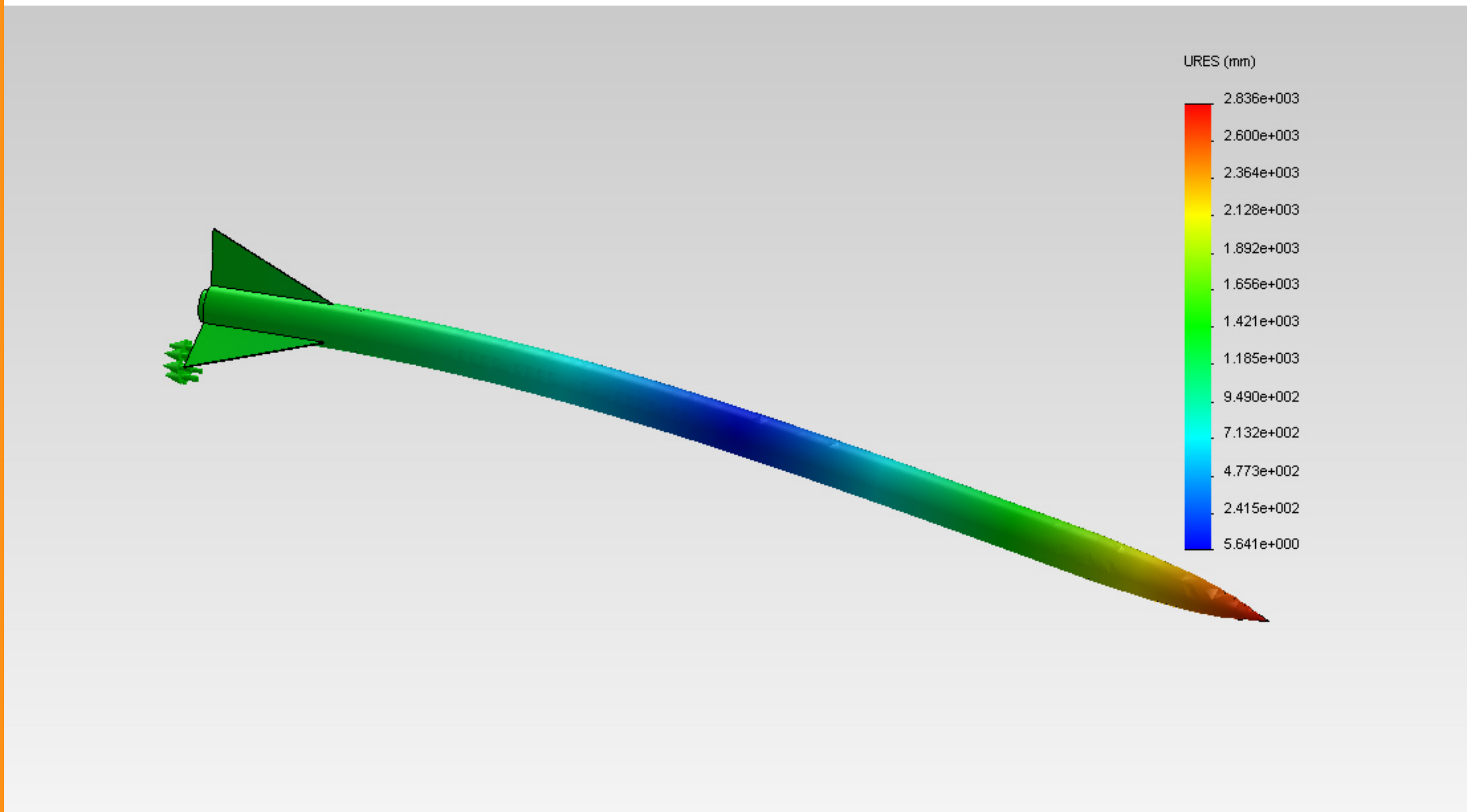
Mode 3: 1.7816E-3 Hz



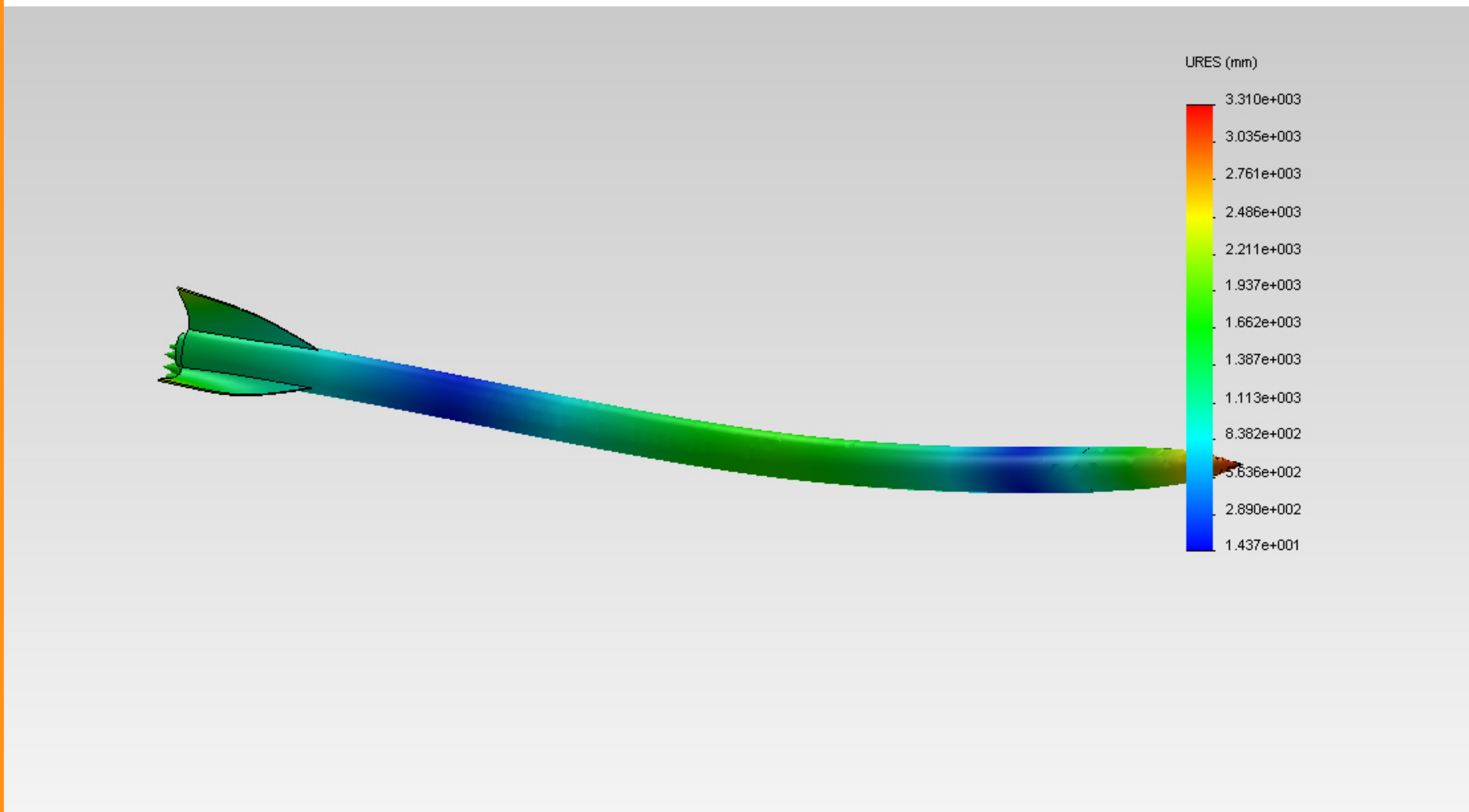
Mode 4: 11.752 Hz



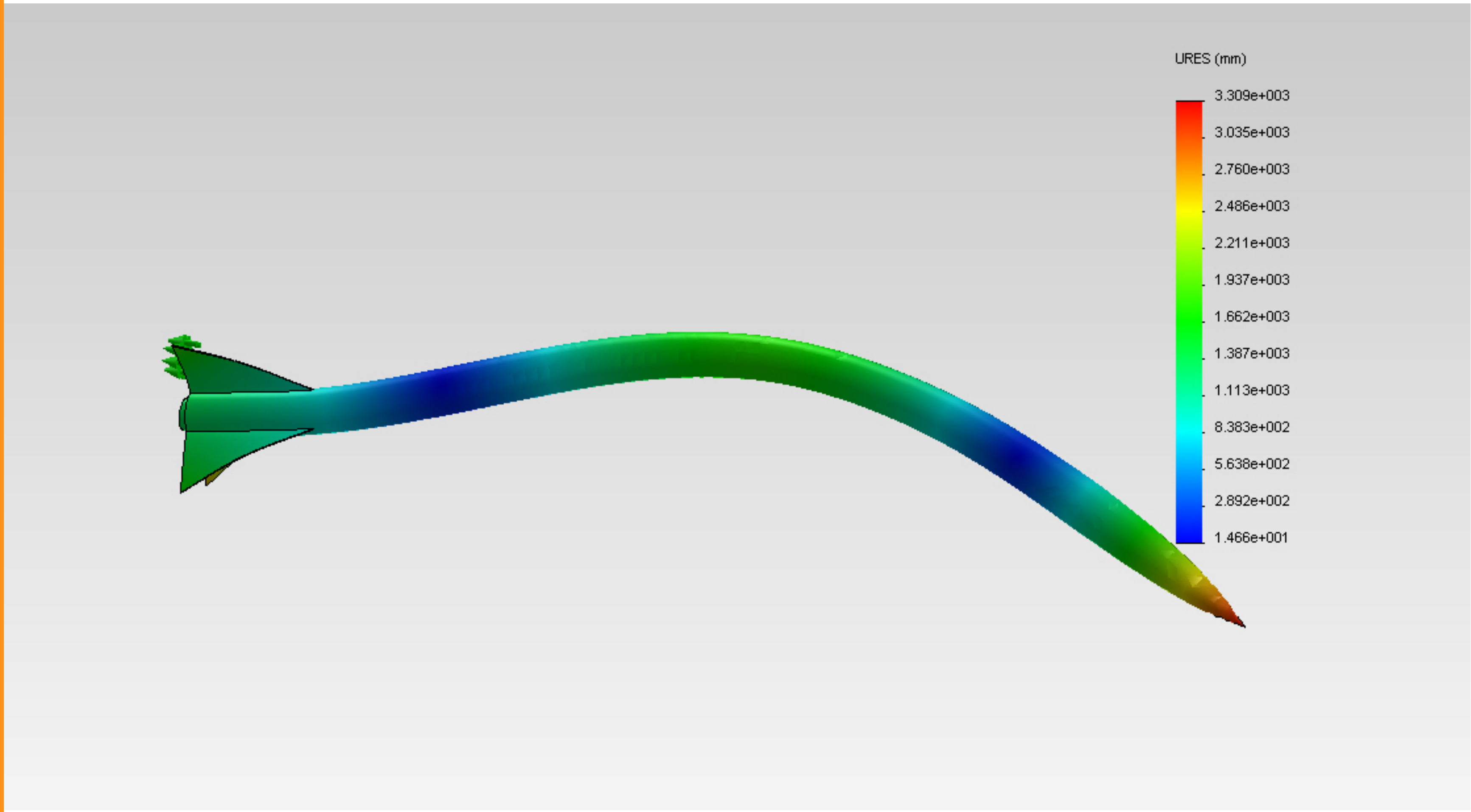
Mode 5: 11.802 Hz



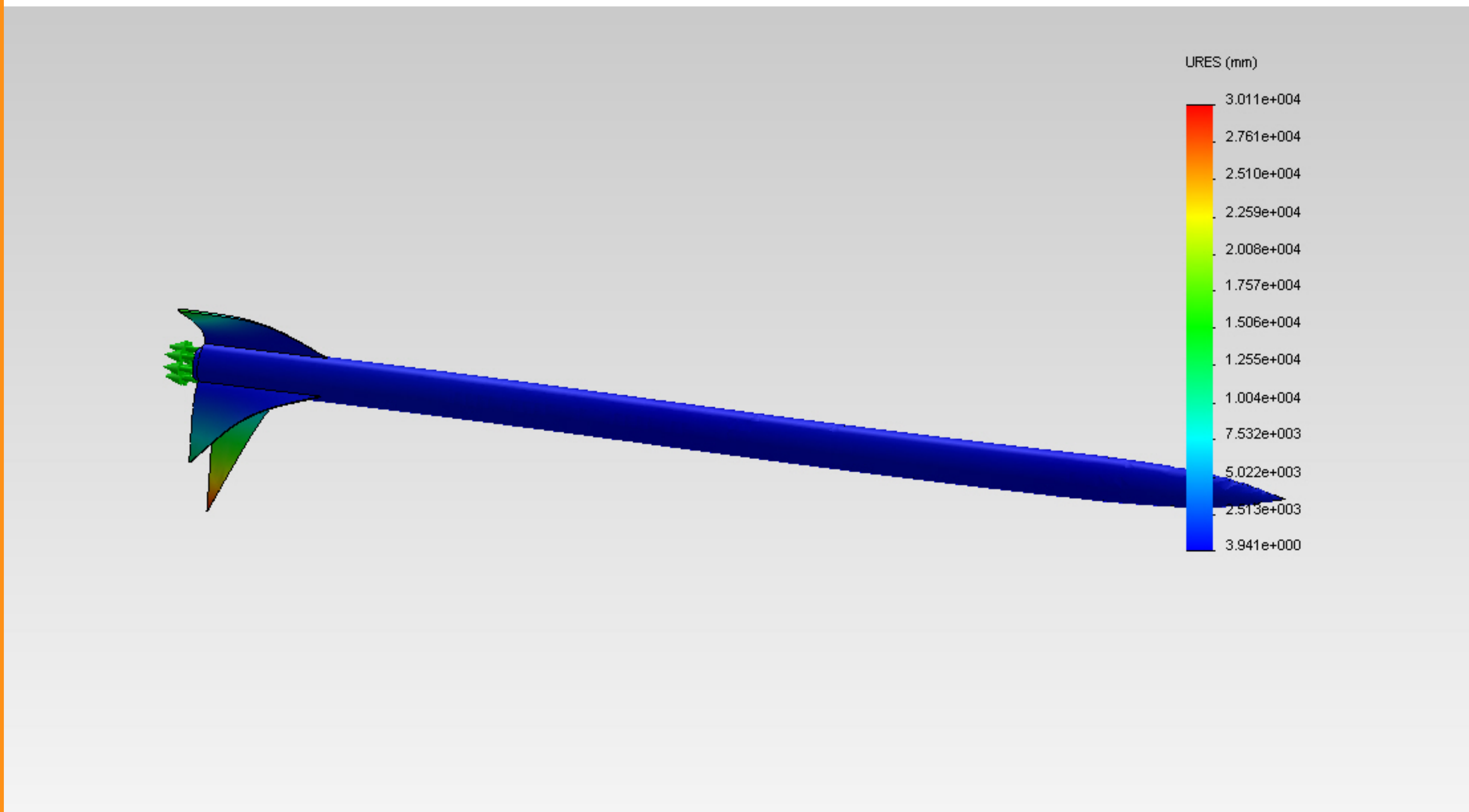
Mode 6: 62.133 Hz



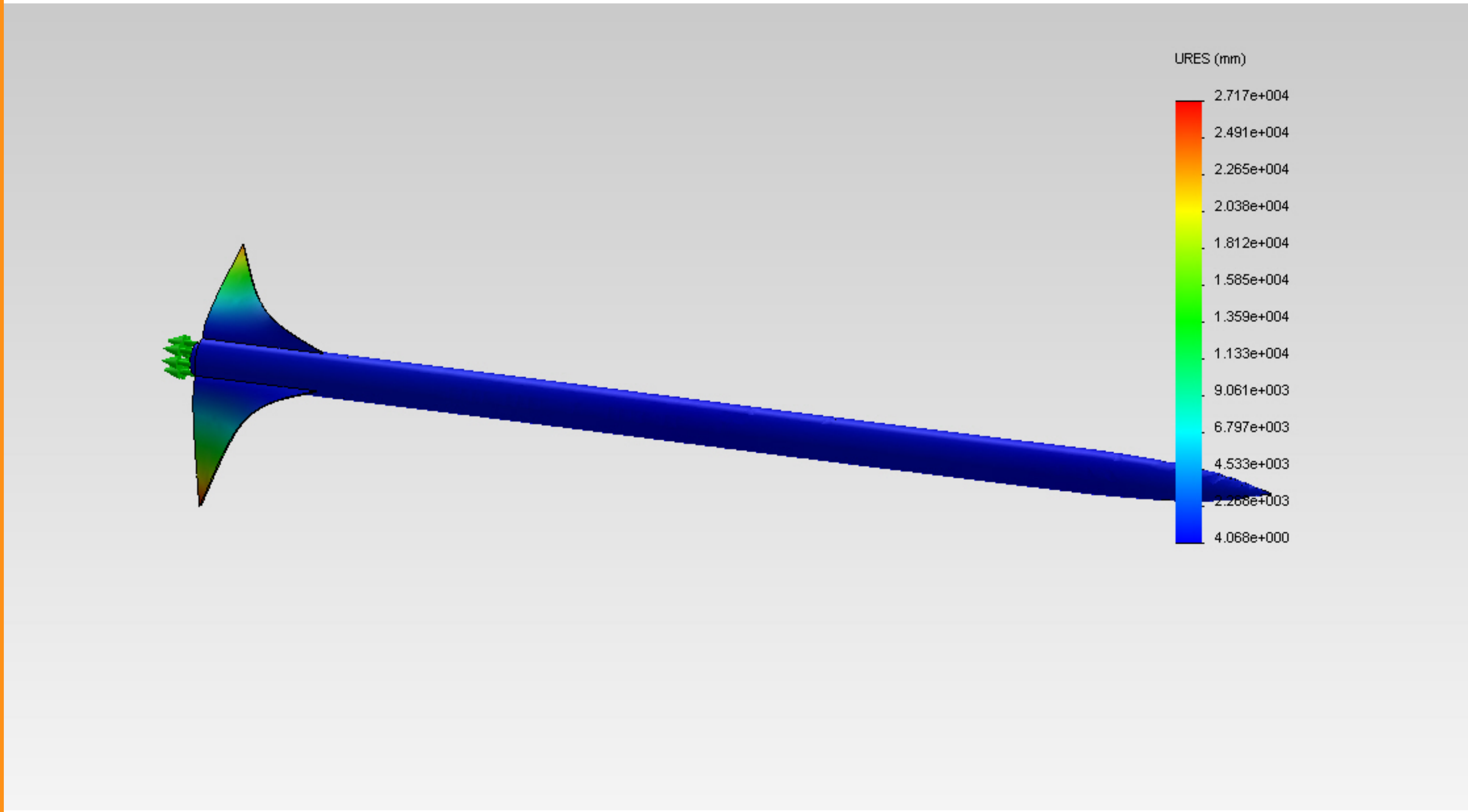
Mode 7: 62.287 Hz



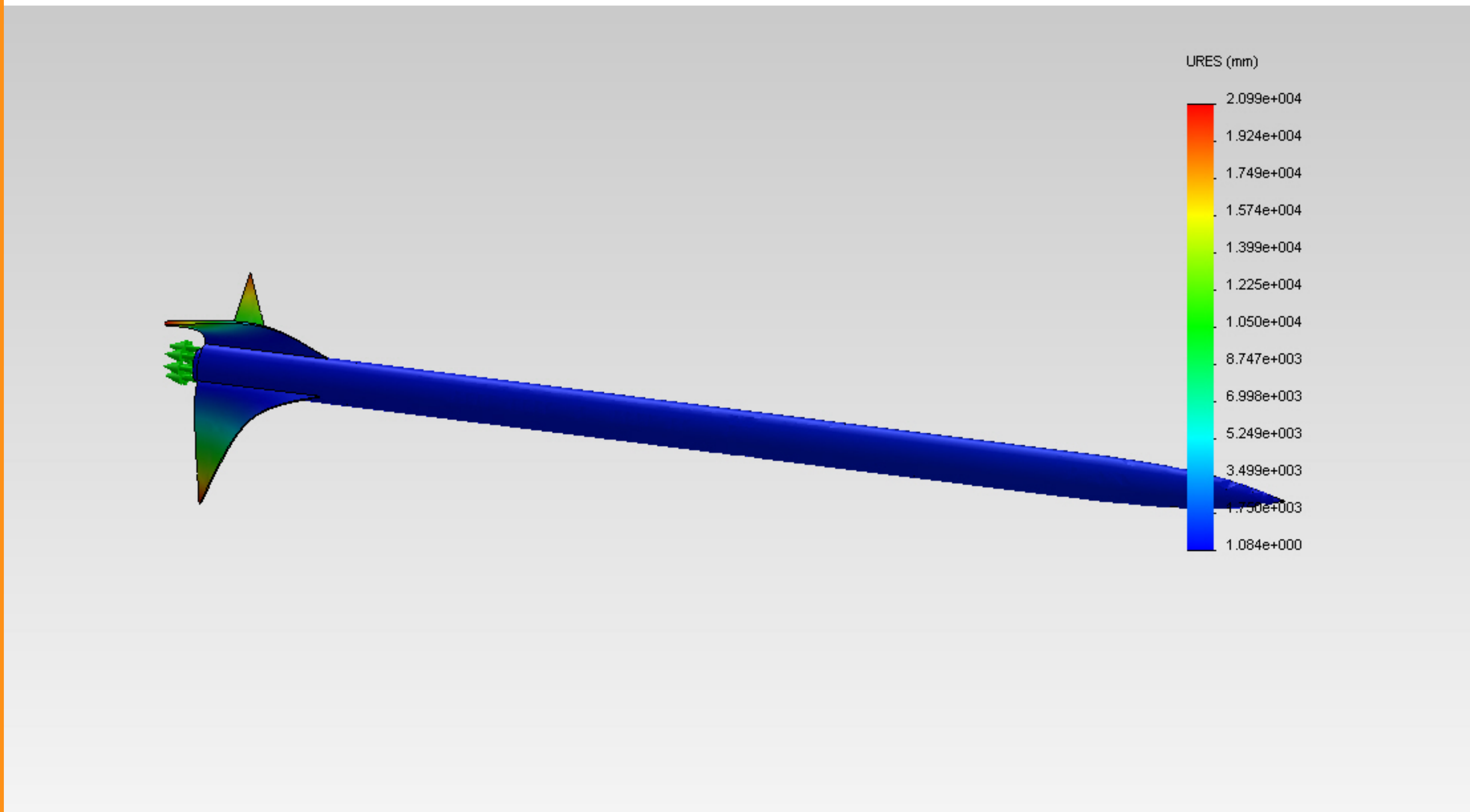
Mode 8: 111.02 Hz



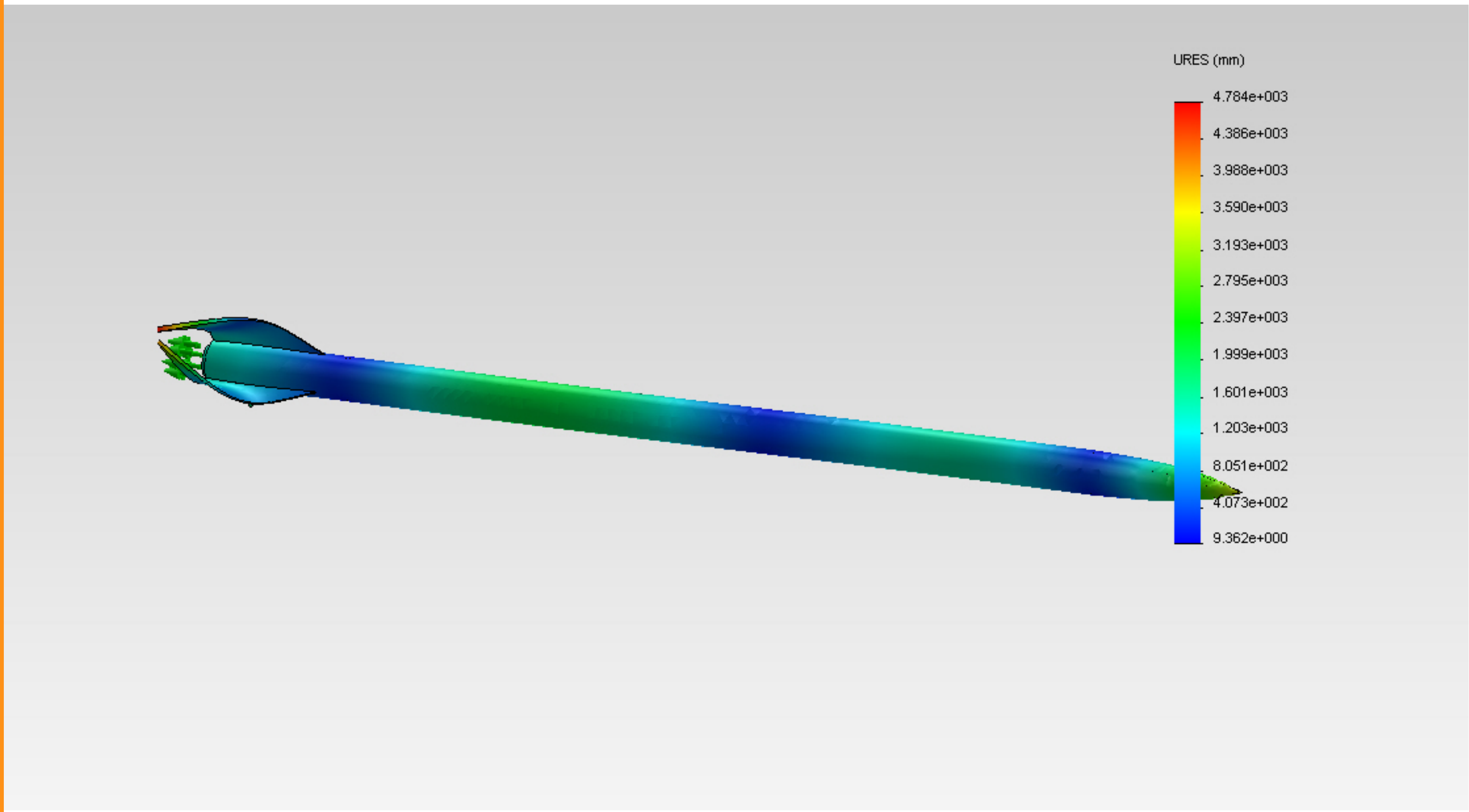
Mode 9: 111.06 Hz



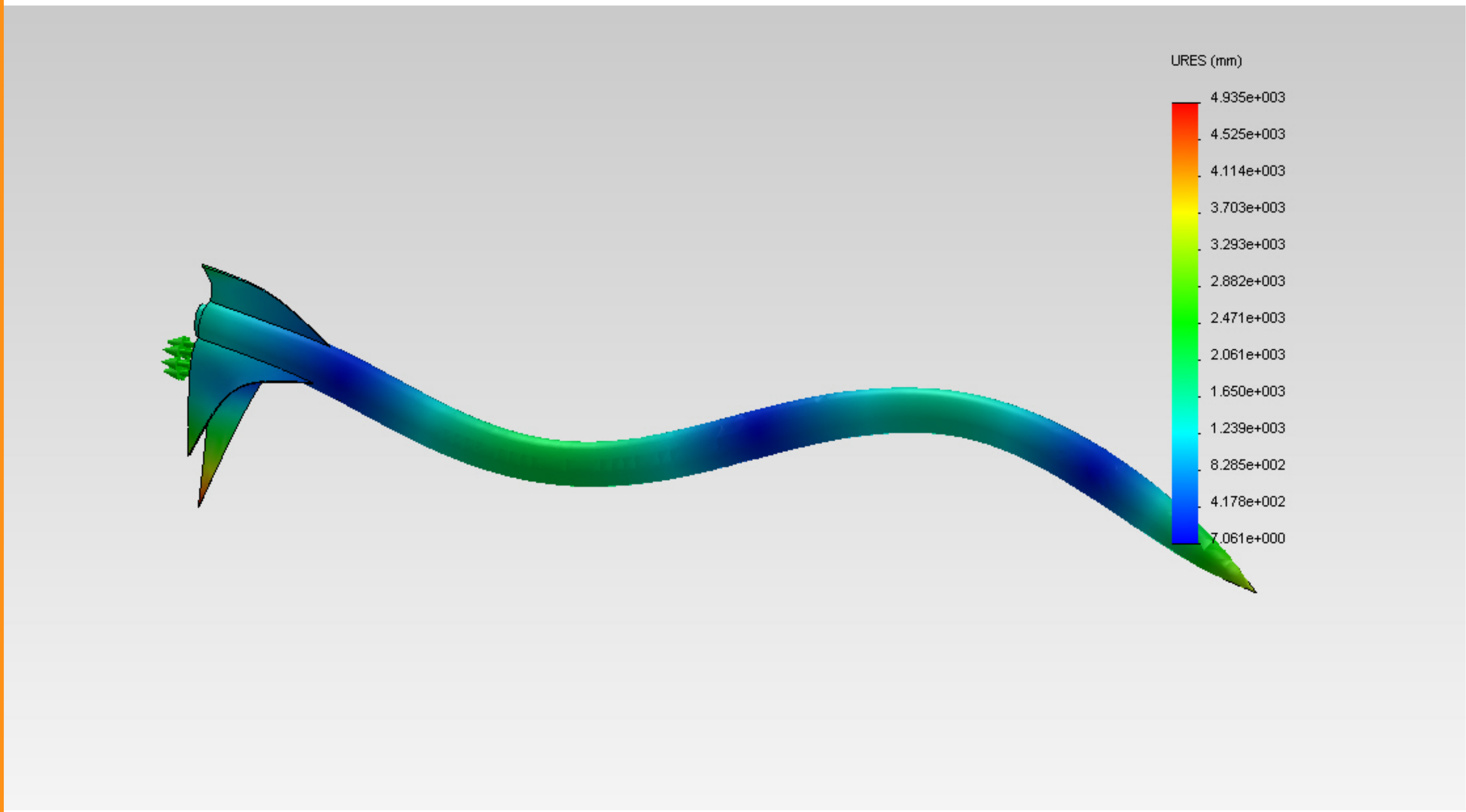
Mode 10: 114.37 Hz



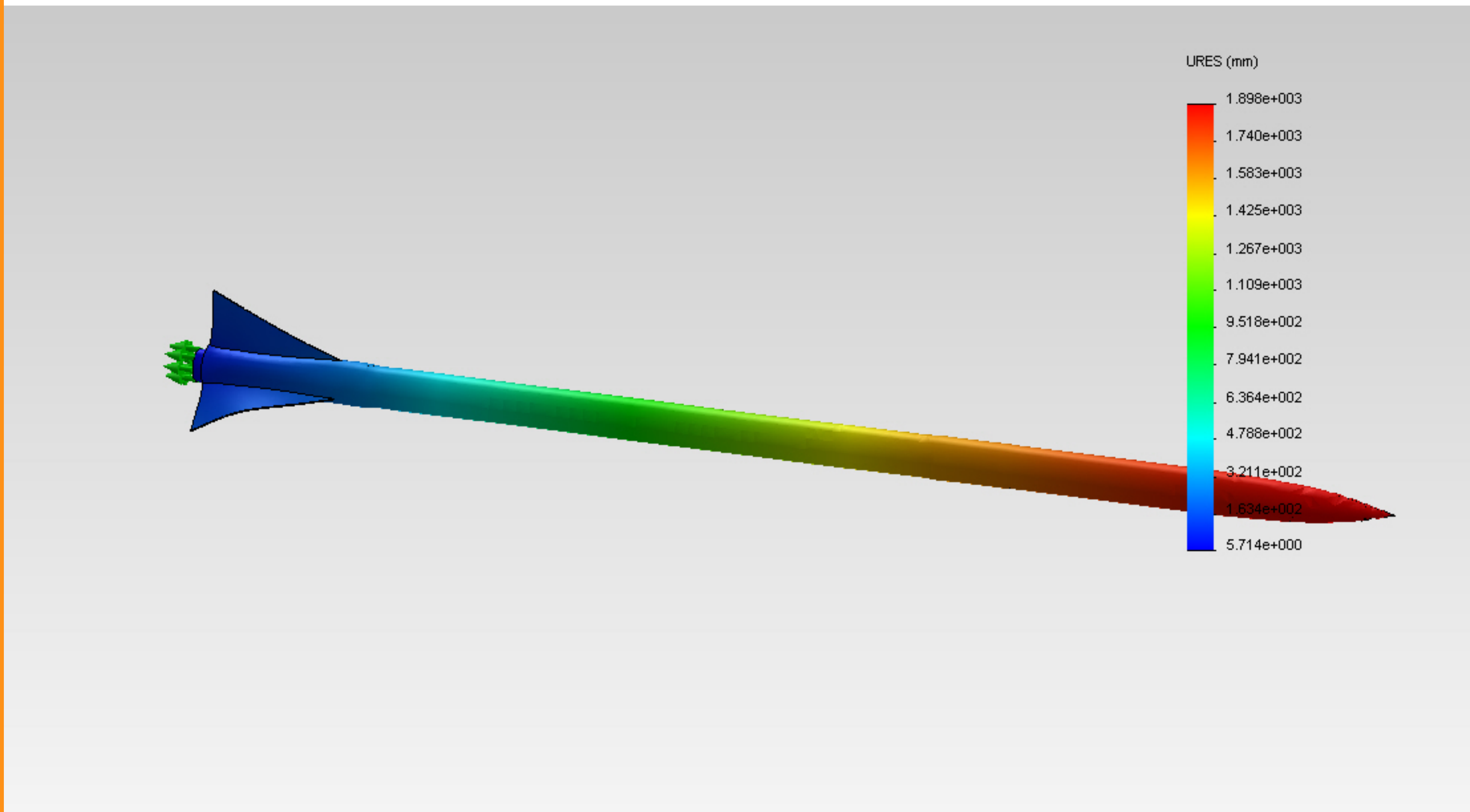
Mode 11: 154.73 Hz



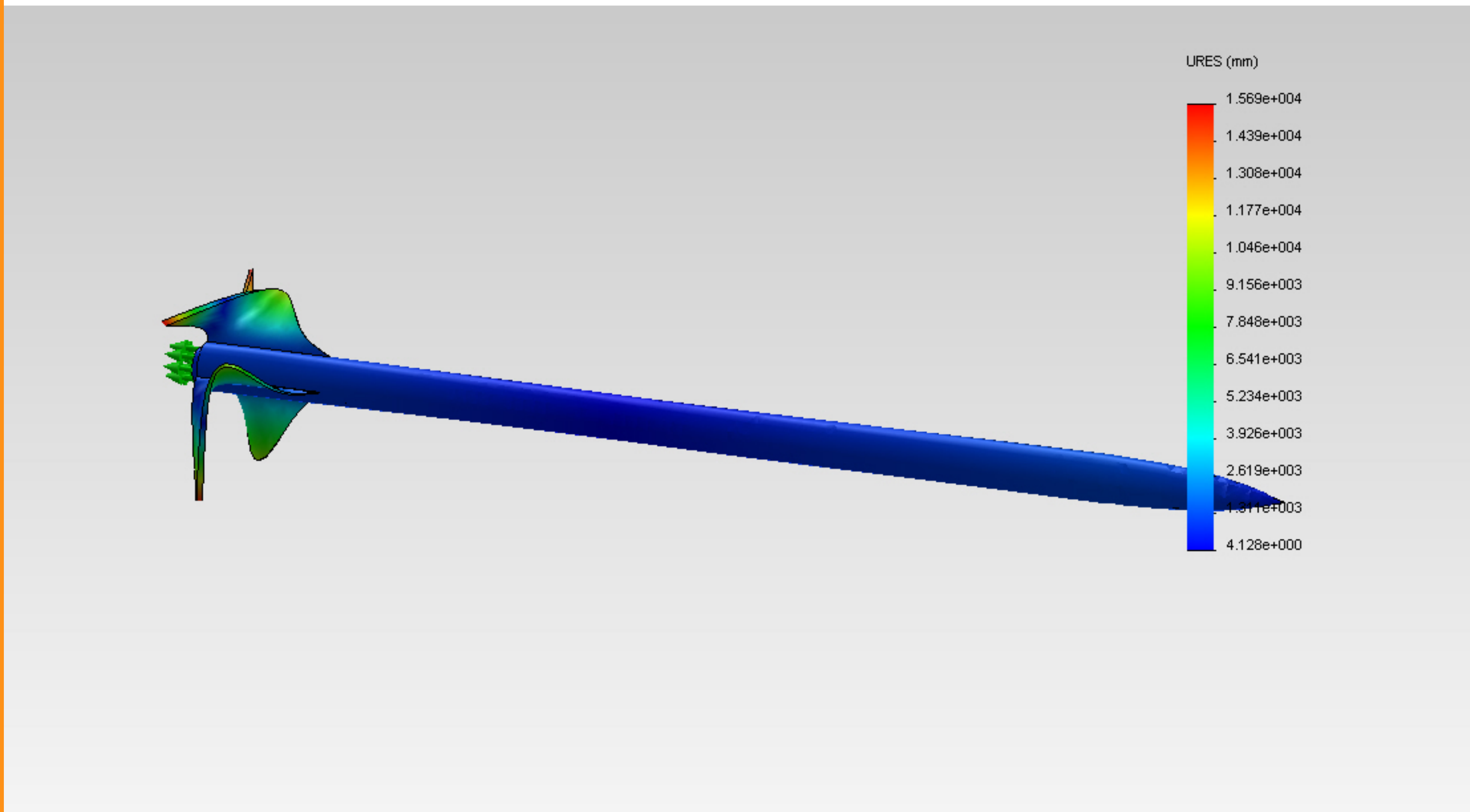
Mode 12: 155.32 Hz



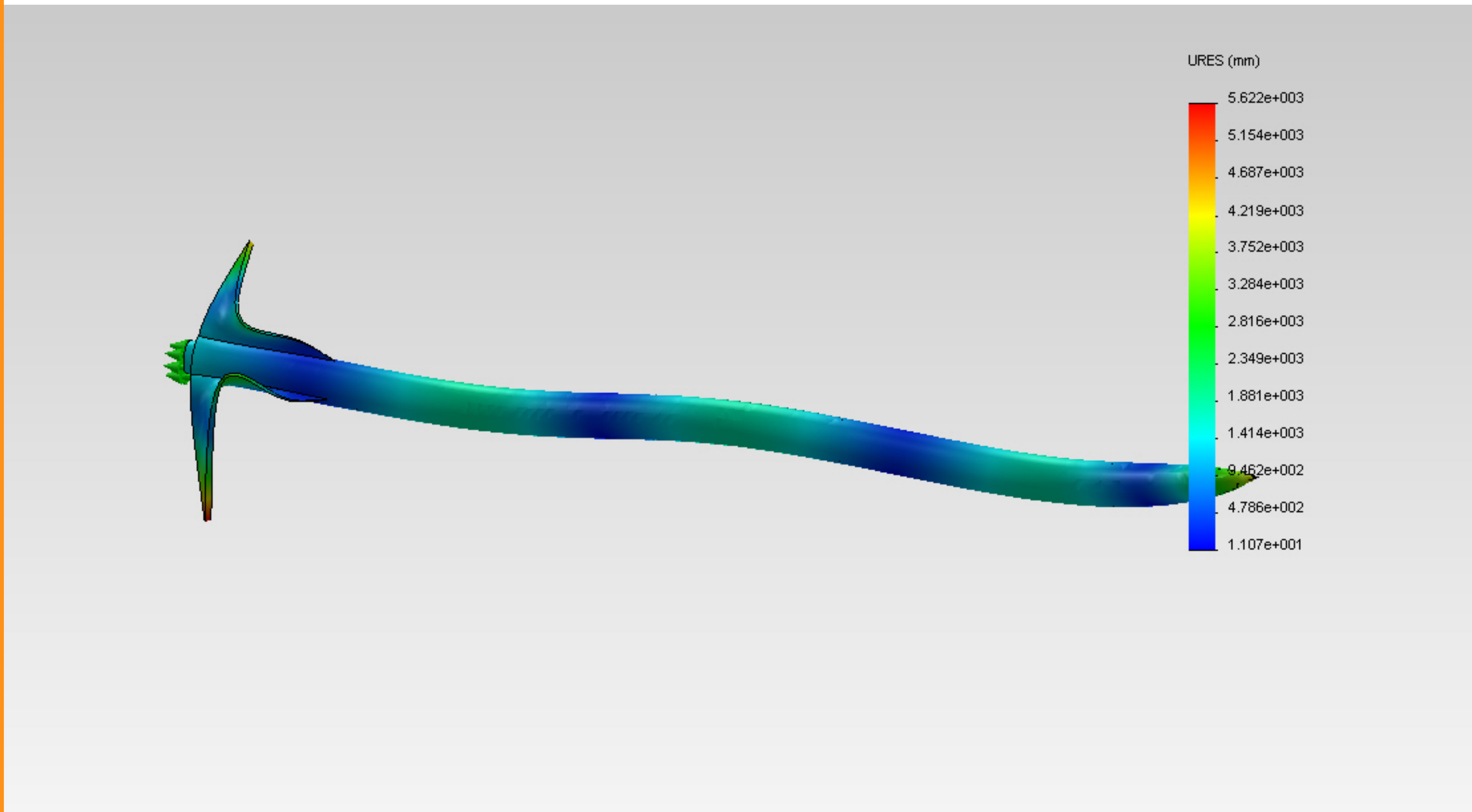
Mode 13: 257.09 Hz



Mode 14: 266.75 Hz



Mode 15: 273.79 Hz



Causes of rocket vibration

- Thrust oscillations
- Noise (pressure waves) due to motor or engine
- Fluid flow phenomena (aerodynamic stress)
  - Wind
  - Turbulence
  - Vortex shedding

Video of flutter

- https://www.youtube.com/watch?v=OhwLojNerMU