E80 Spring 2016

Vibration and System Identification

(Adapted from previous years' lectures)

Prof. Angie Lee

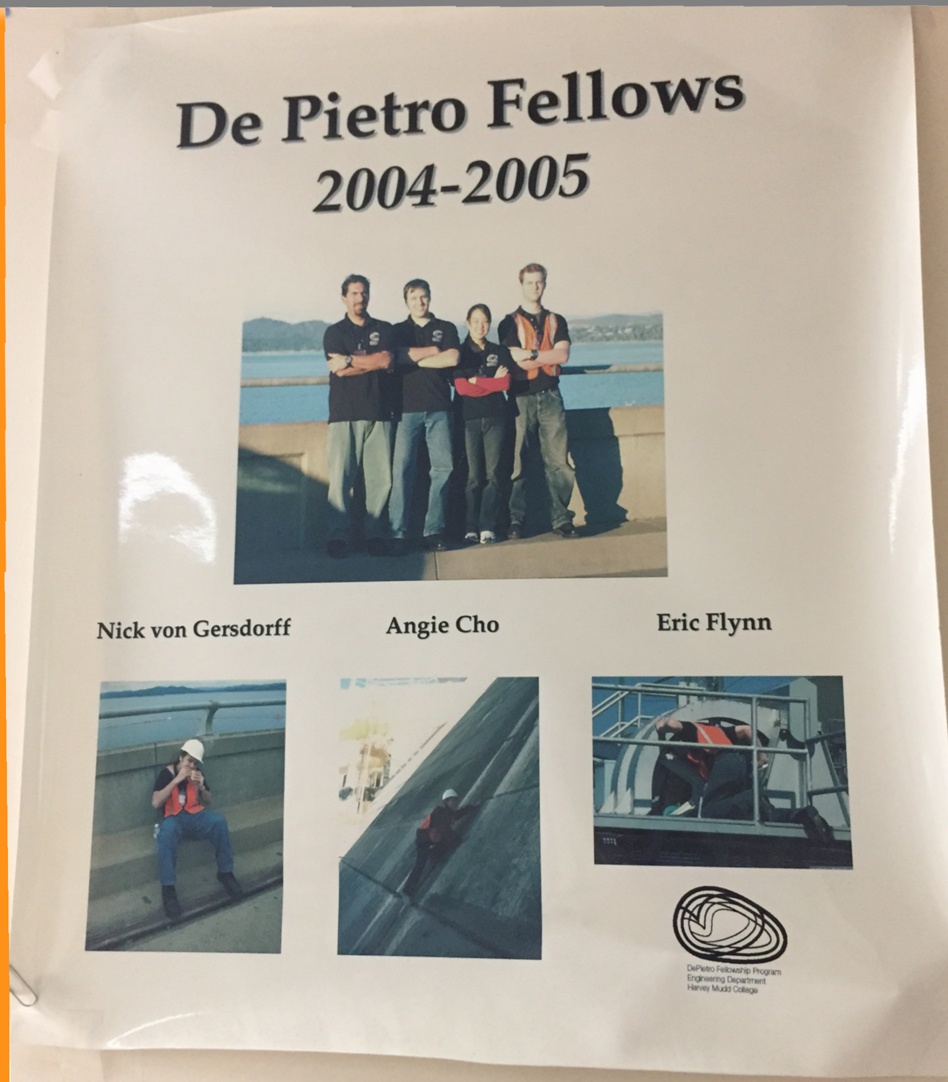
Remaining lectures

The Lecture Schedule

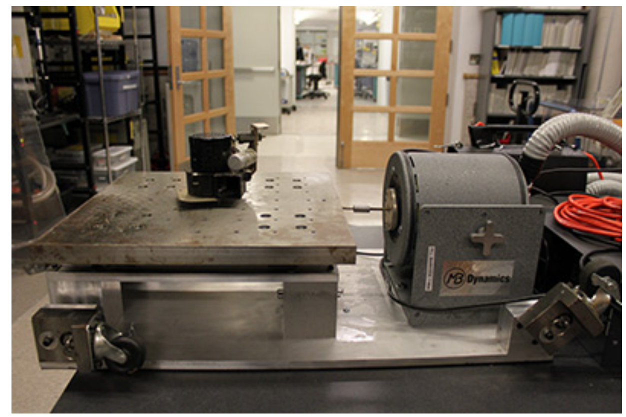
Date	Tuesday	Date	Thursday
19 JAN 2016	Flight Data Basics	21 JAN 2016	Data Fitting and Analysis
26 JAN 2016	LabVIEW & MATLAB	28 JAN 2016	Basic Electrical Measurements
2 FEB 2016	Op-amps & Signal Conditioning	4 FEB 2016	Temperature Measurements
9 FEB 2016	Wind Tunnel & Fluid Measurements	11 FEB 2016	Thrust Measurements & Flight Modeling
16 FEB 2016	Inertial Measurement	18 FEB 2016	Vibration & System ID
23 FEB 2016	Atmospheric Science	25 FEB 2016	Sensors & Transducers
8 MAR 2016	Flight Hardware		
5 APR 2016	Field Operations & Safety		

Start thinking about final project!

Folsam Dam: vibration testing



Shaker: sinusoidal input



http://itll.colorado.edu/test_measurement_equipment/vibration_testing/

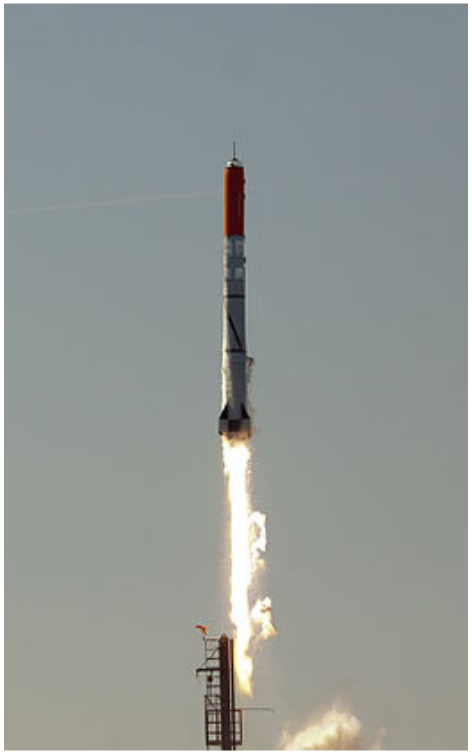
Cold gas thruster: impulse input

Lecture outline

- Interesting examples of rocket vibrations
- Intro to system identification and modal vibration
- Vibration analysis (math model, computational model)
- Vibration testing (experiment)
- Application: cantilever beam
- Application: rocket
- Fun video

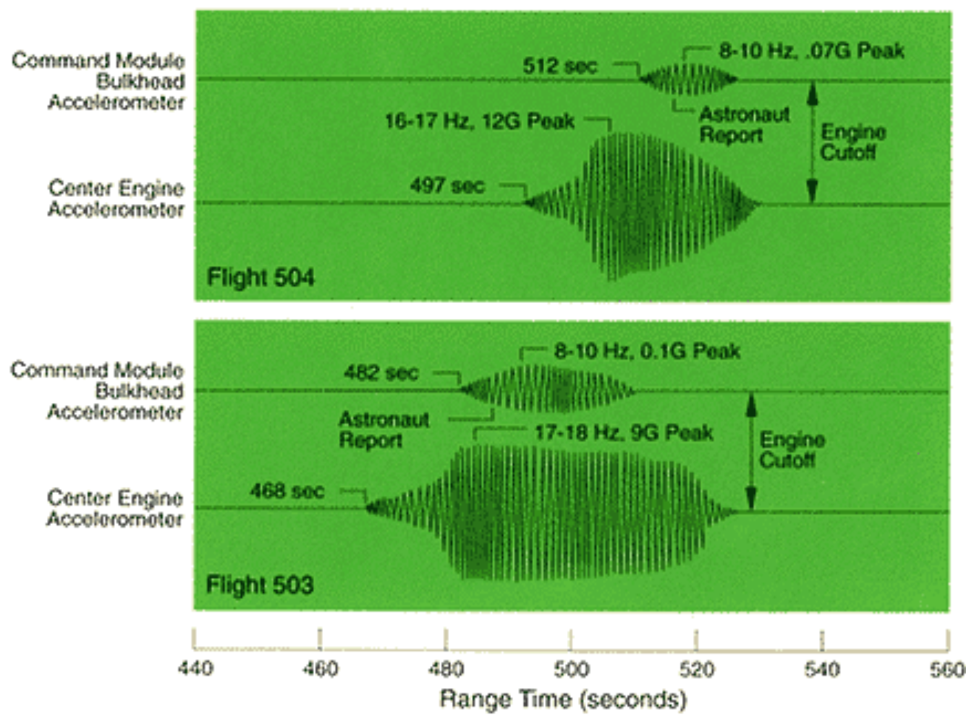
Example of rocket vibration

- HEAT1X-Tycho Brahe inaugural flight
- Pilot's POV – 9 Hz oscillation
- http://www.youtube.com/watch?v=-rASHRBo9Rg&feature=player_embedded



Example of rocket vibration

Saturn rocket



"Pain was directly associated with motion of the eyeballs and testicles, as well as from internal heating that resulted from sloshing of the brain and viscera. The vibration frequency was also in the range of normal brain waves, adding confusion to decision making, hand and arm movement, and even speech."

Jim Fenwich on Pogo oscillations

http://www.pwrengineering.com/articles/pogo.htm

Example of rocket vibration

Space shuttle main engine turbopumps

"The high-pressure pumps rotated at speeds reaching 36,000 rpm on the fuel side and 24,000 rpm on the oxidizer side. At these speeds, minor faults were exacerbated and could rapidly propagate to catastrophic engine failure."

"…the vibration spectral data contained potential failure indicators in the form of discrete rotordynamic spectral signatures. These signatures were prime indicators of turbomachinery health…"

"Wings in Orbit" edited by Wayne Hale and Helen Lane

Example of rocket vibration

Rocket failure, March 2012

“While the lower stages of the North Korean rocket continued to function for several minutes, resonance at the top of the launch vehicle resulted in ‘**catastrophic disassembly** of the third stage at Max Q,’ said Charles Vick, senior technical and space policy analyst at GlobalSecurity.org. ‘The vibrations just tore it apart.’”

http://www.nytimes.com/2012/04/13/world/asia/north-korea-launches-rocket-defying-world-warnings.html?pagewanted=all&_r=0

http://www.eetimes.com/electronics-news/4370955/Severe-vibrations-likely-brought-down-N--Korean-rocket

Causes of rocket vibration

- Thrust oscillations
- Noise (pressure waves) due to motor or engine
- Fluid flow phenomena (aerodynamic stress)
  - Wind
  - Turbulence
  - Vortex shedding

Question

Why might we be interested in rocket vibrations?

Intro to vibration analysis

- **System identification:**

  building mathematical models of dynamical systems based on observed input-output data

- **Modal analysis:**

  characterization of vibrational mode shapes and corresponding frequencies of a physical system

- **Dynamic load and response:**

  load (input) applied dynamically (varying over time)

  the response associated with this load is the <u>dynamic response</u> of the system

BASIC IDEA/GOAL

Determine the modal properties of a system: natural frequencies and modal shapes

Analysis of dynamic loading

How do you approach the problem of analyzing a structure with dynamic loading?

(a) Model your physical problem

Geometry, kinematics, material, loading

(b) Derive governing equations (mostly differential equations)

(c) Solve the equations

(d) Interpret results and refine and repeat!

What else can you do to characterize a structure?

Take measurements!

Experimental studies to validate a model or help develop a model

A simple model: spring-mass-damper system

- Around a resonance frequency, you can model as

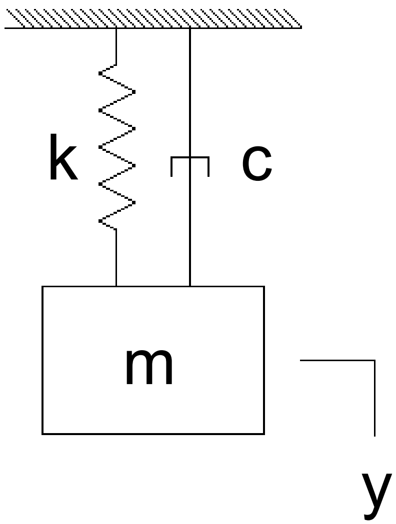
$$m_e \ddot{y} = f - ky - c\dot{y}$$

$$m\ \ddot{y} + c\dot{y} + ky = f$$

$$\ddot{y} + \frac{c}{m_e}\dot{y} + \frac{k}{m_e}y = \frac{f}{m_e}$$

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = f / m_e$$

$$\omega_n = \sqrt{\frac{k}{m_e}} \qquad \zeta = \frac{c}{2\sqrt{m_e k}}$$



Frequency response function (FRF)

- Position

$$\frac{Y}{F} = \frac{\dfrac{1}{m_e}\left(\dfrac{1}{\omega_n}\right)^2}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2 + 2\zeta\dfrac{\omega}{\omega_n}j}$$

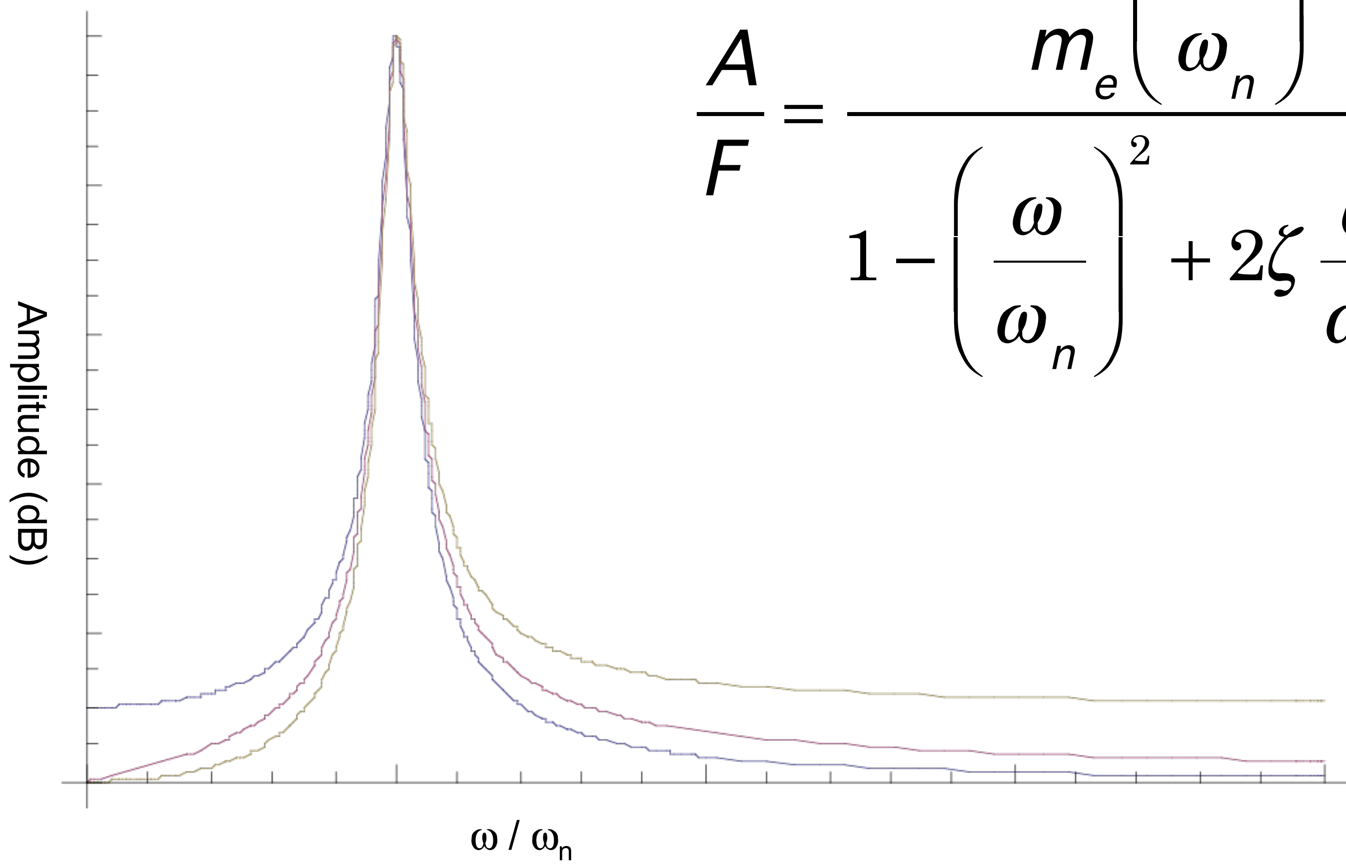
- Velocity

$$\frac{V}{F} = \frac{j\omega\dfrac{1}{m_e}\left(\dfrac{1}{\omega_n}\right)^2}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2 + 2\zeta\dfrac{\omega}{\omega_n}j}$$

Frequency response function (FRF)

- Acceleration

$$\frac{A}{F} = \frac{-\frac{1}{m_e}\left(\frac{\omega}{\omega_n}\right)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2 + 2\zeta\frac{\omega}{\omega_n}j}$$

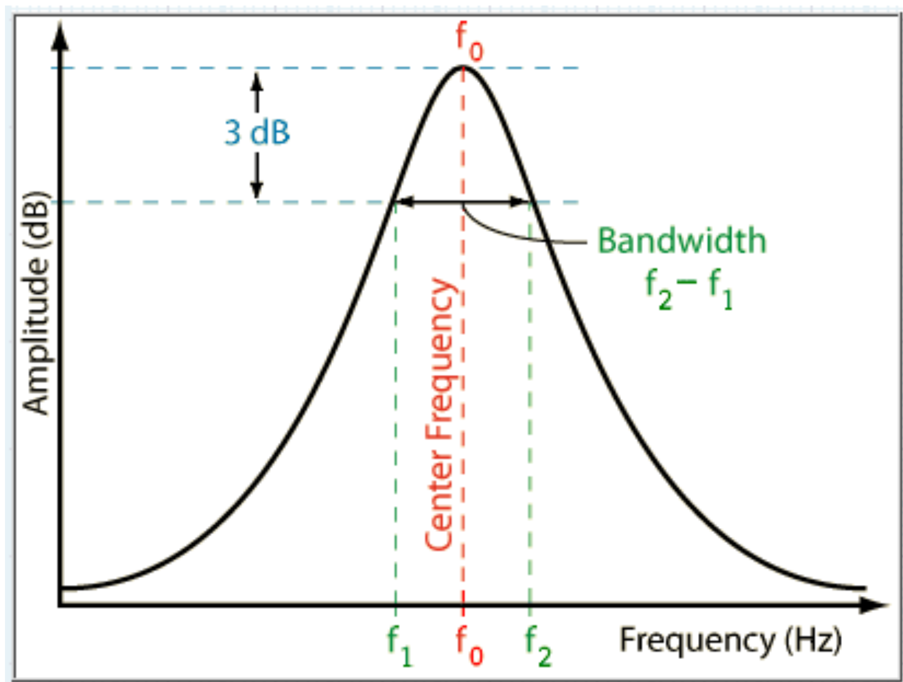


Damping coefficient

- From the peak $\omega_r = \omega_n \sqrt{1-\zeta^2}$
- From the half-power bandwidth $\Delta\omega = \omega_{+hp} - \omega_{-hp}$

$$Q = \frac{\omega_r}{\Delta\omega}$$

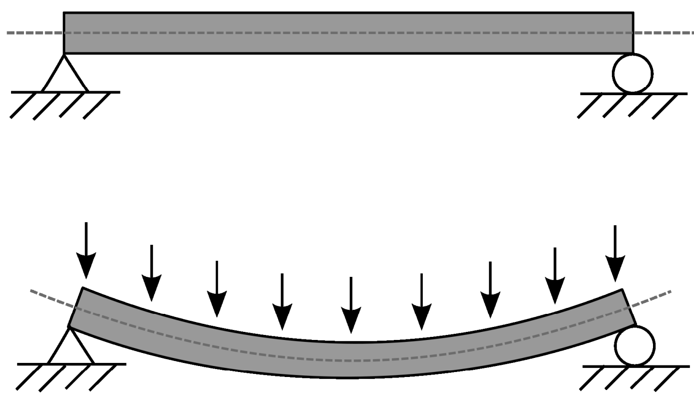
$$\zeta = \frac{1}{2Q}$$



http://www.sengpielaudio.com/calculator-cutoffFrequencies.htm

Beams

- We can extend this idea from the spring-mass-damper example to a more complicated structural element, such as a beam
- Beams are one of the most important components in structural engineering
  - Examples of beams: bridges, walkways, rockets, …

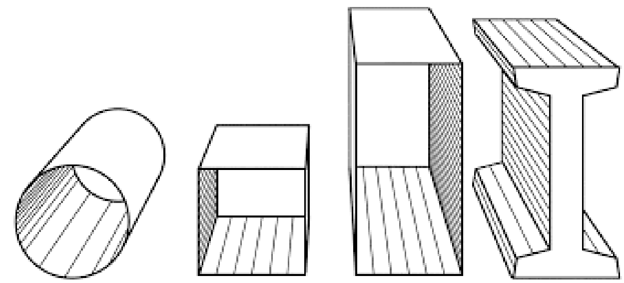


https://en.wikipedia.org/wiki/Beam_(structure)

Beams

Some important properties and characteristics of a beam are:

1) Cross sectional area: A, I
2) Length: L
3) Material: E, ρ
4) Supports (boundary conditions)



http://cnx.org/contents/mu6YpDEI@1/Beams-pillarsstruts-crossbars-

Type	Boundary Conditions
Pinned or hinged end ($x = 0$, $x = L$)	Left end: $w(0) = 0, \quad -EI\frac{d^2w(0)}{dx^2} = 0$ Right end: $w(L) = 0, \quad EI\frac{d^2w(L)}{dx^2} = 0$
Clamped or fixed end ($x = 0$, $x = L$)	Left end: $w(0) = 0, \quad \frac{dw(0)}{dx} = 0$ Right end: $w(L) = 0, \quad \frac{dw(L)}{dx} = 0$
Free end ($x = 0$, $x = L$)	Left end: $-EI\frac{d^2w(0)}{dx^2} = 0, \quad EI\frac{d^3w(0)}{dx^3} = 0$ Right end: $EI\frac{d^2w(L)}{dx^2} = 0, \quad -EI\frac{d^3w(L)}{dx^3} = 0$
Sliding or guided end ($x = 0$, $x = L$)	Left end: $\frac{dw(0)}{dx} = 0, \quad EI\frac{d^3w(0)}{dx^3} = 0$ Right end: $\frac{dw(L)}{dx} = 0, \quad -EI\frac{d^3w(L)}{dx^3} = 0$

https://www.ecomputingx.com/demo1.jsp

Experiments: vibration testing

- Lab tests
  - Shaker tests

    https://www.youtube.com/watch?v=o8H_NT7Ziao

  - Impact hammer tests

    https://www.youtube.com/watch?v=tBRjPN8m6zE

Cantilever beam (rotation lab)

- Mathematical model

  to obtain natural frequencies and modal shapes

- Computational model

  example: SolidWorks model

- Experimental data

  tap test in lab using strain gauges

Cantilever vibration modes

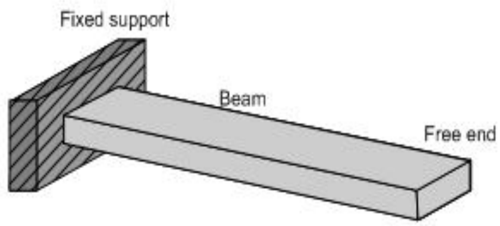


Fig. 4.1 (a): A cantilever beam

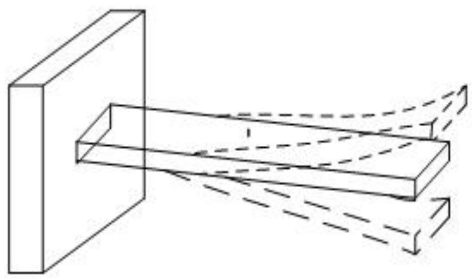
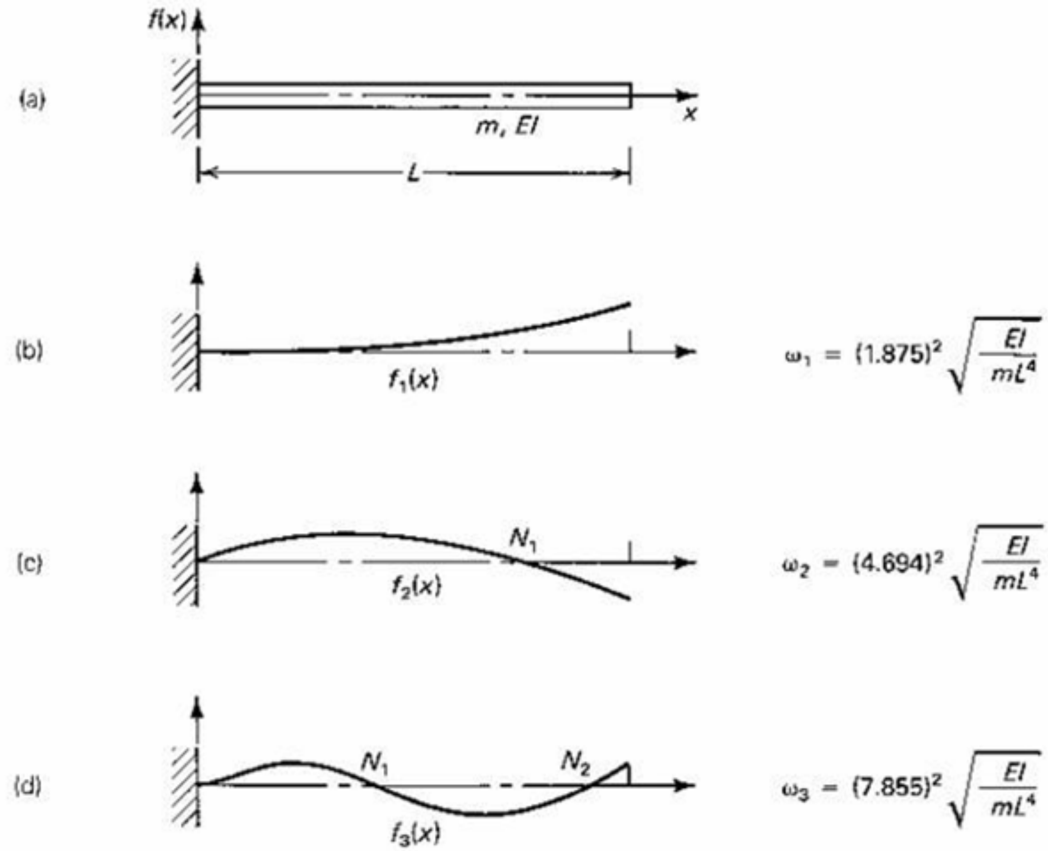


Fig. 4.1 (b): The beam under free vibration

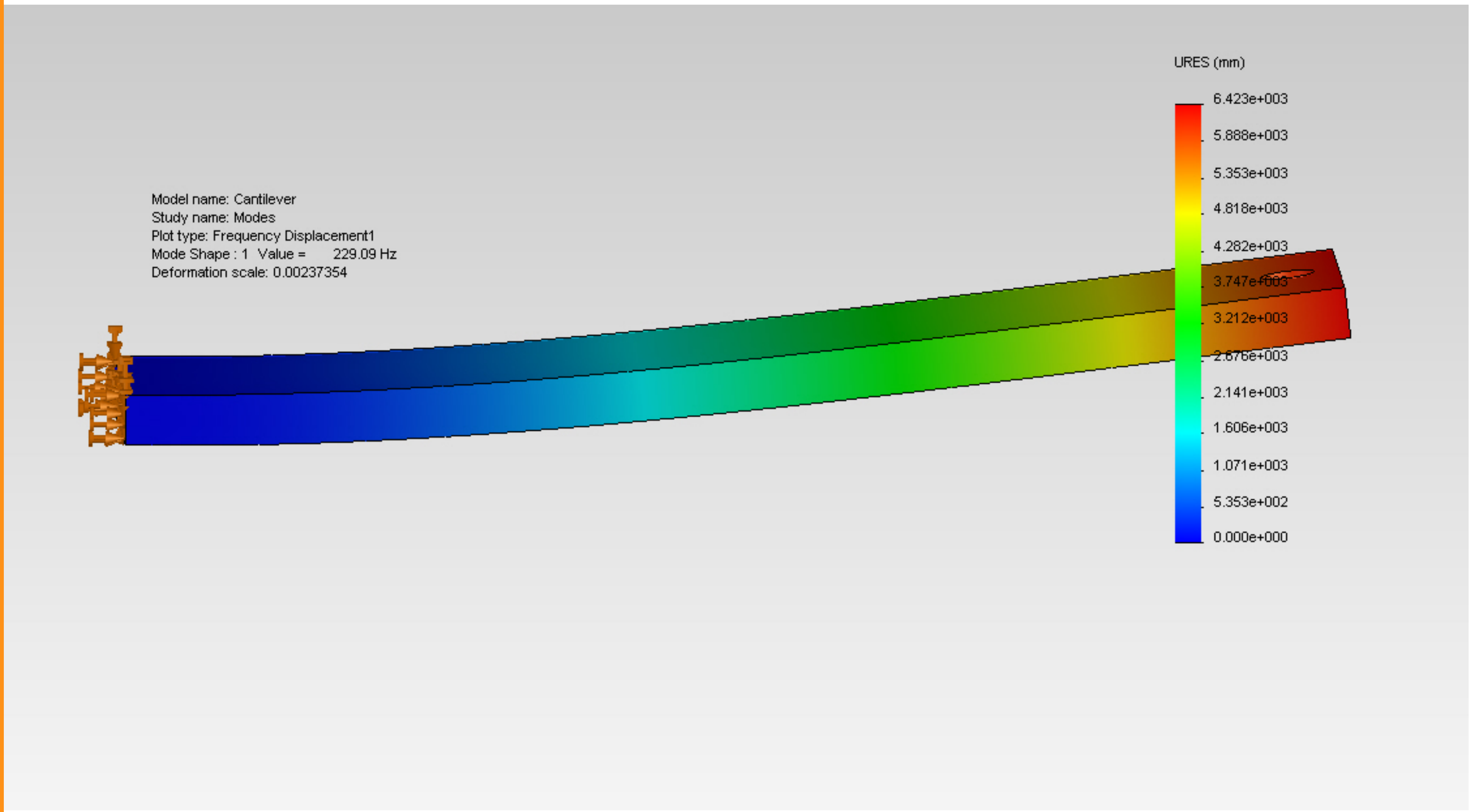


http://iitg.vlab.co.in/?sub=62&brch=175&sim=1080&cnt=1

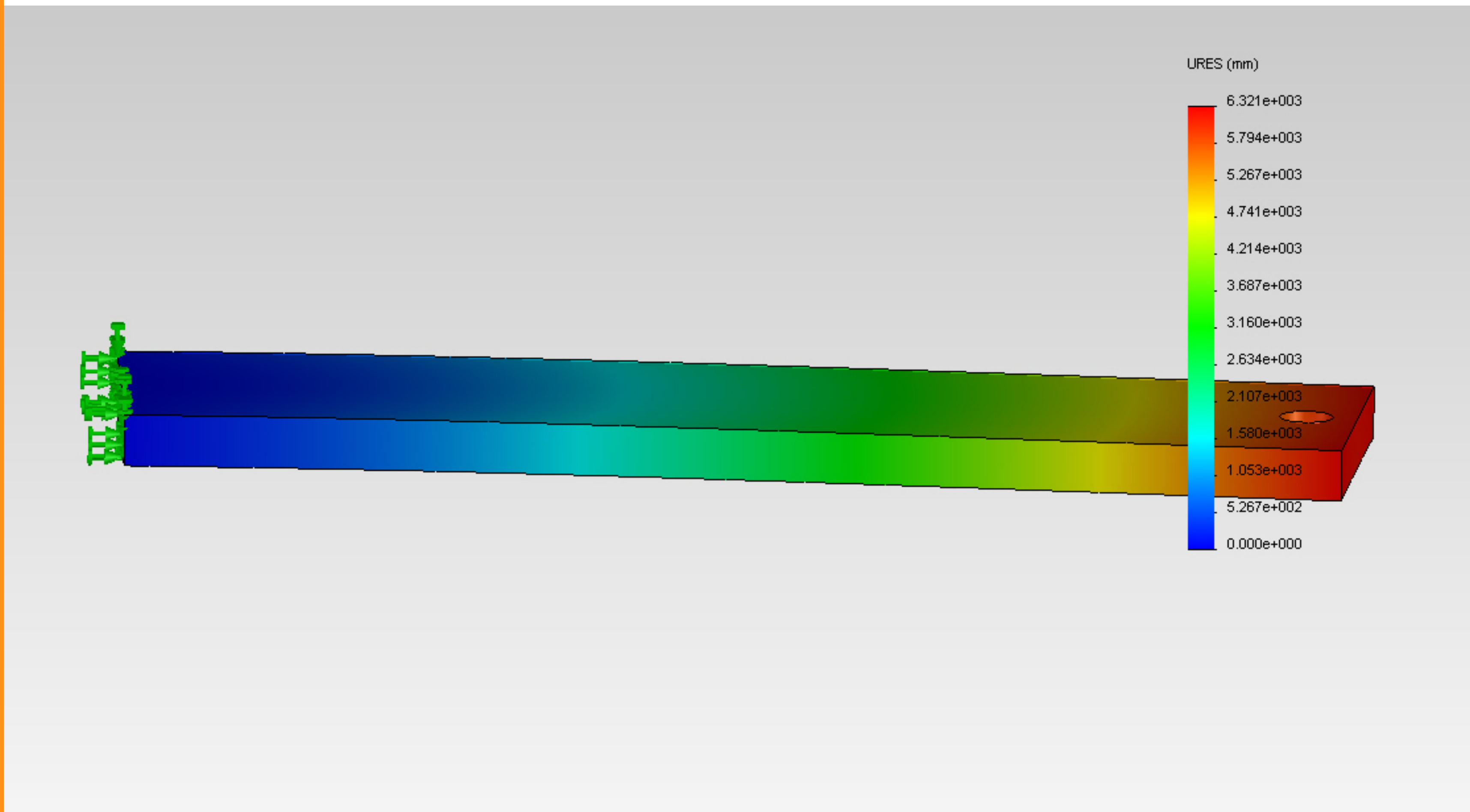
Fig. 4.3: The first three undamped natural frequencies and mode shape of cantilever beam

https://www.youtube.com/watch?v=kun62B7VUg8

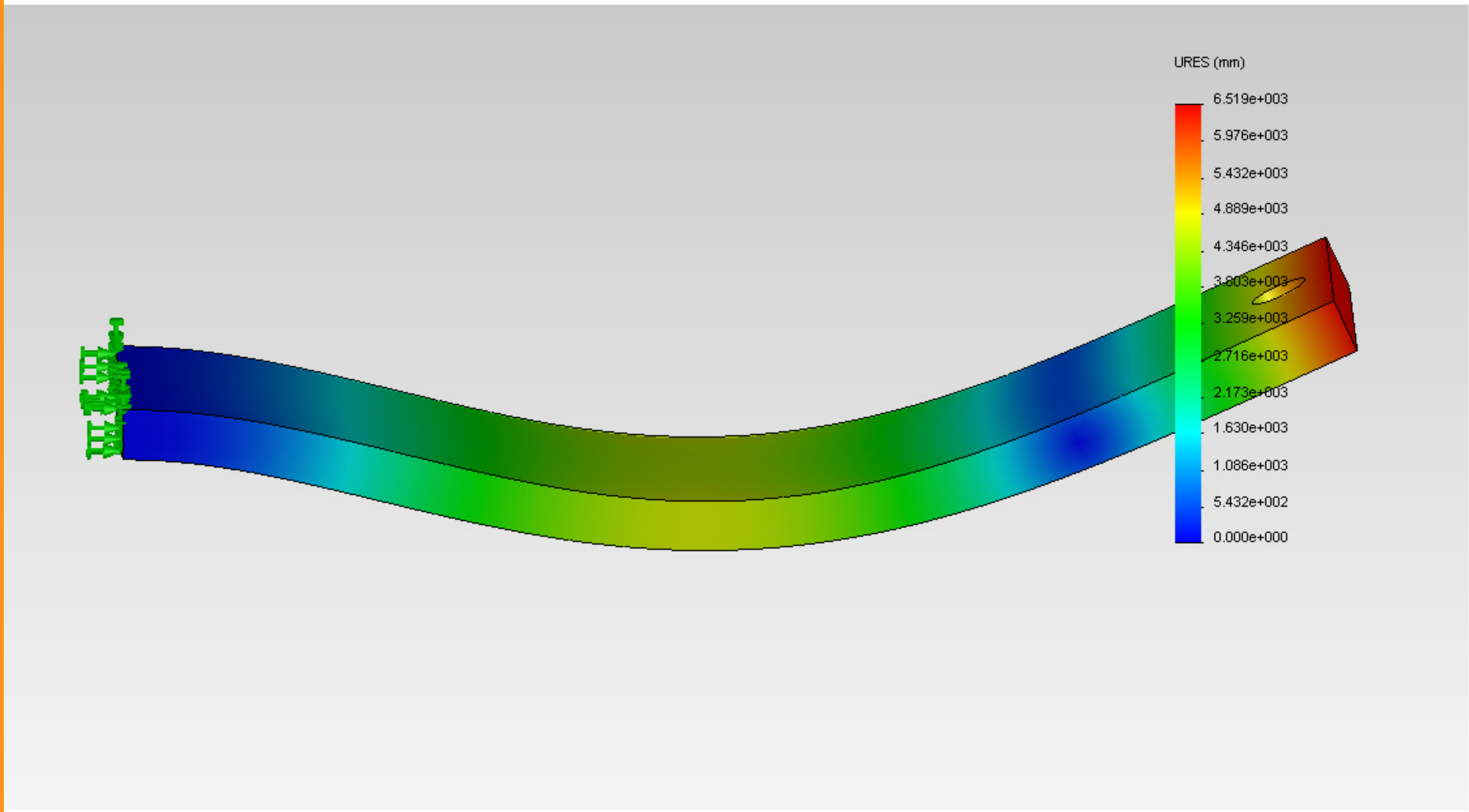
Cantilever: computational model
Mode 1: 299.09 Hz



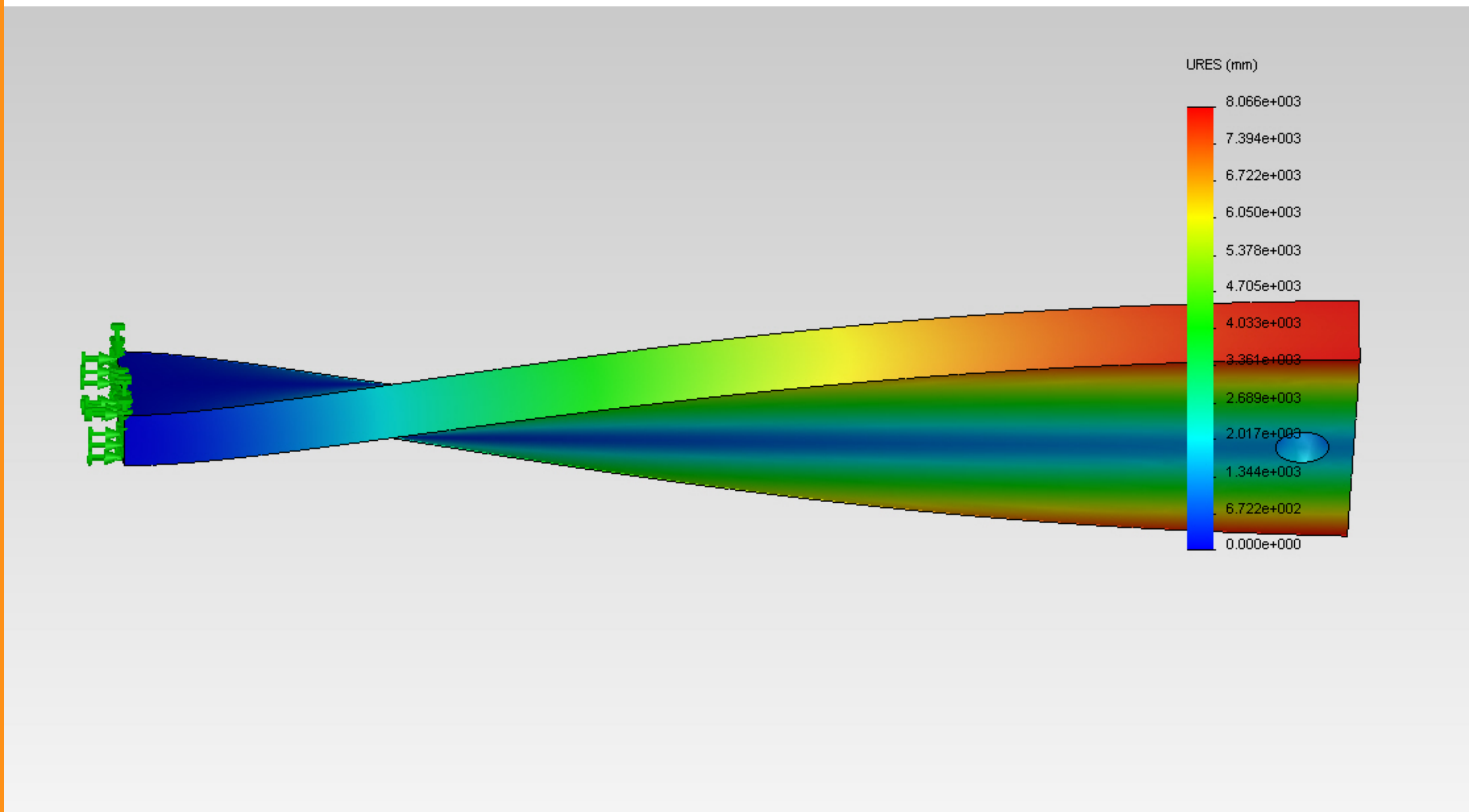
Mode 2: 1297.9 Hz



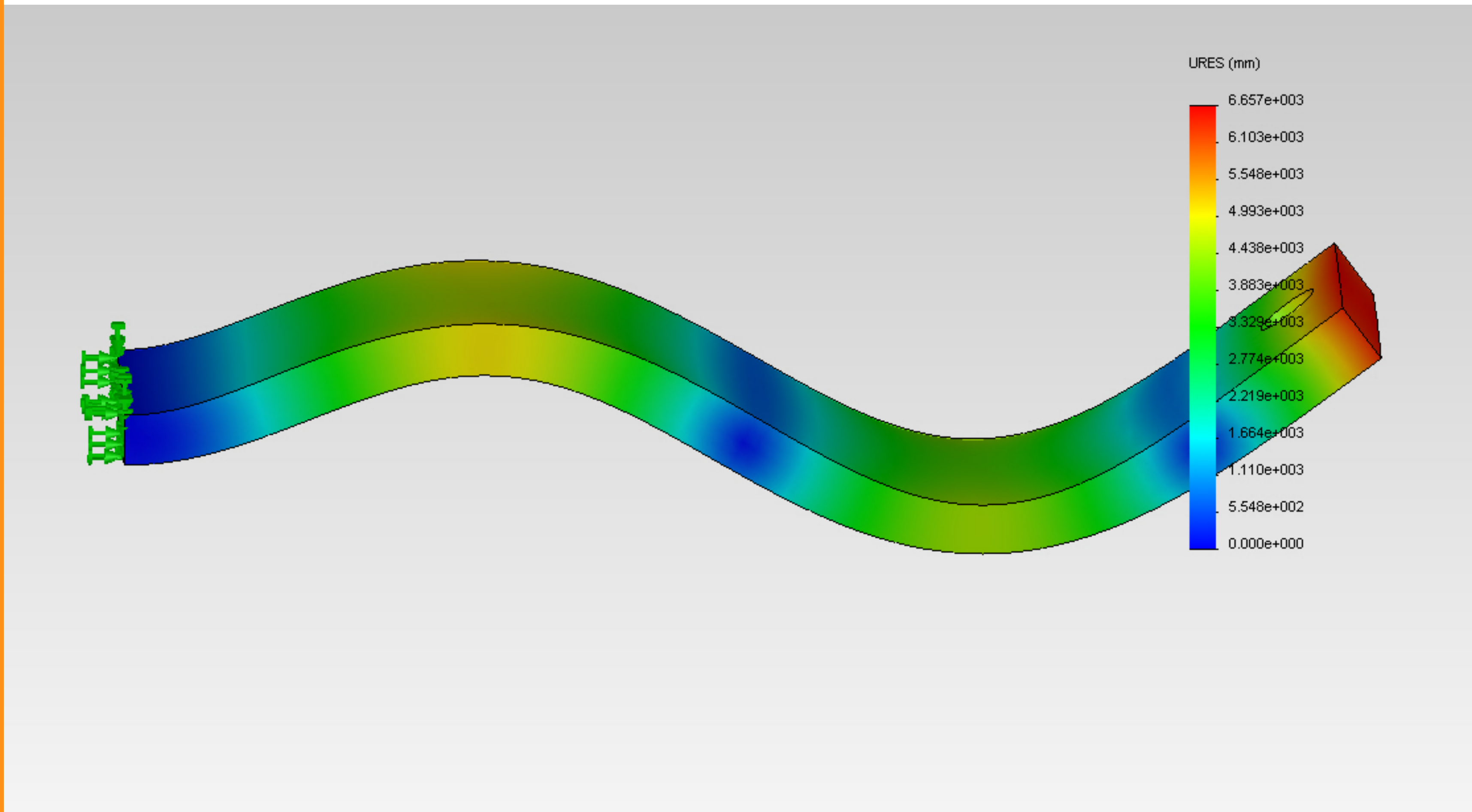
Mode 3: 1417.6 Hz



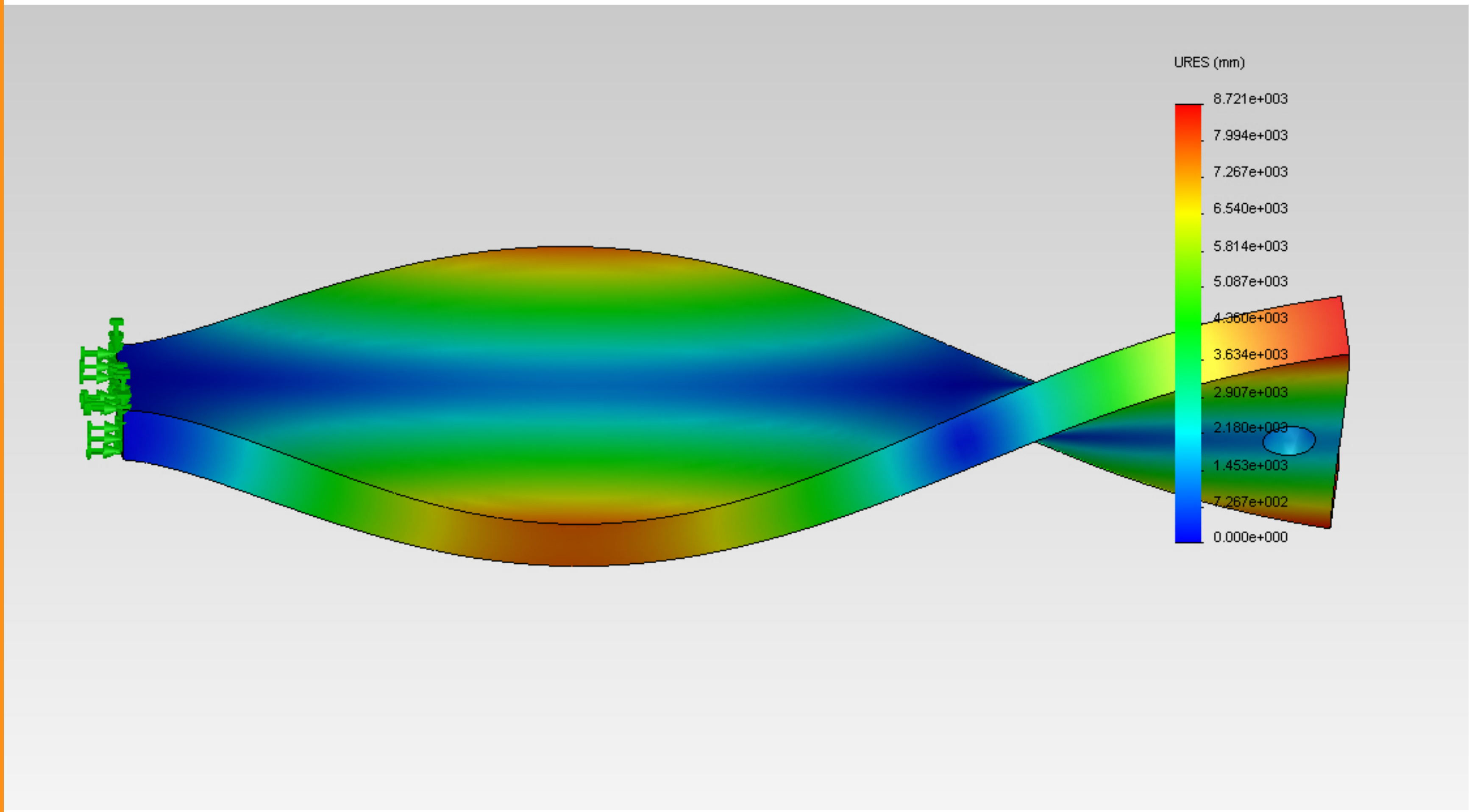
Mode 4: 1679.3 Hz



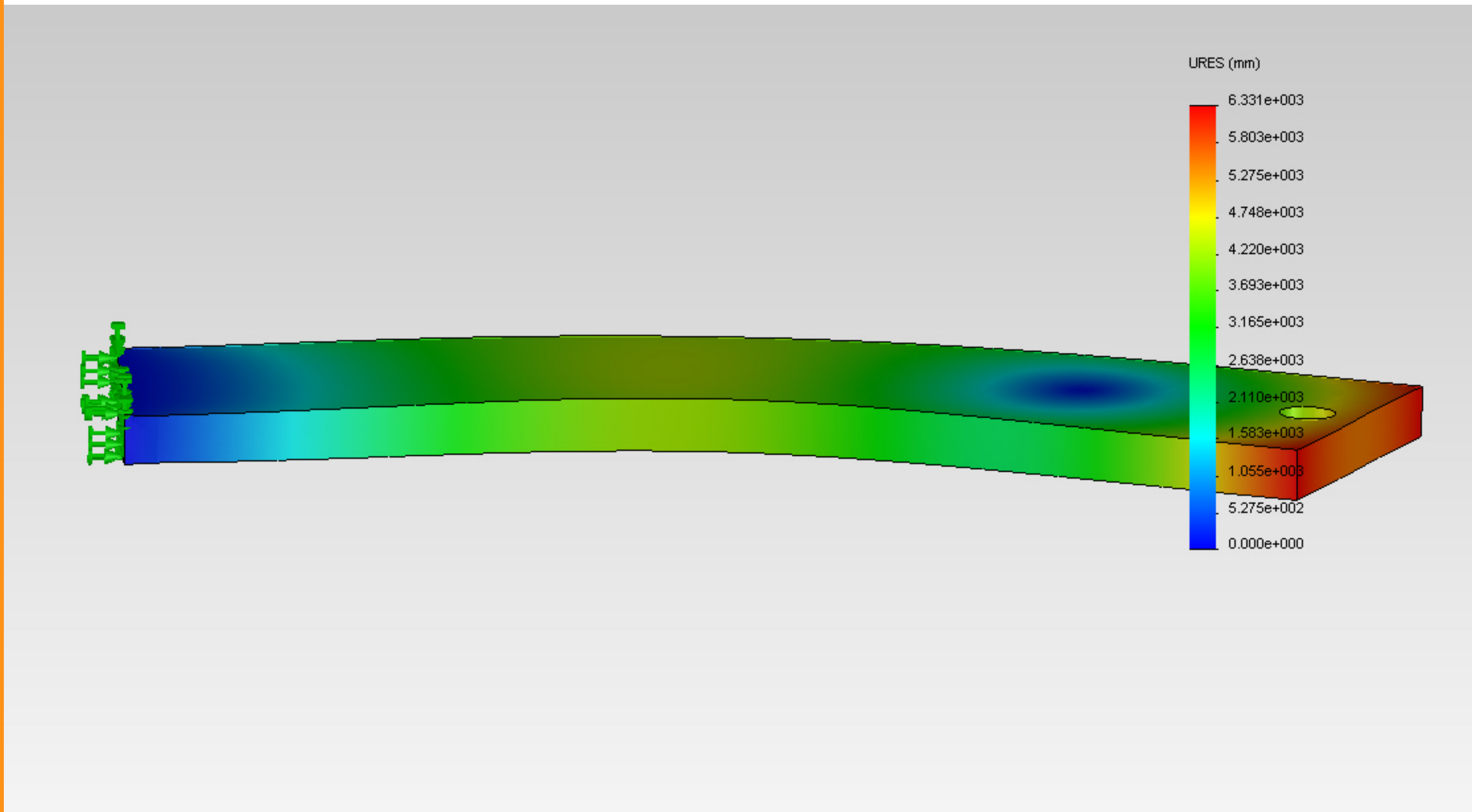
Mode 5: 3917.6 Hz



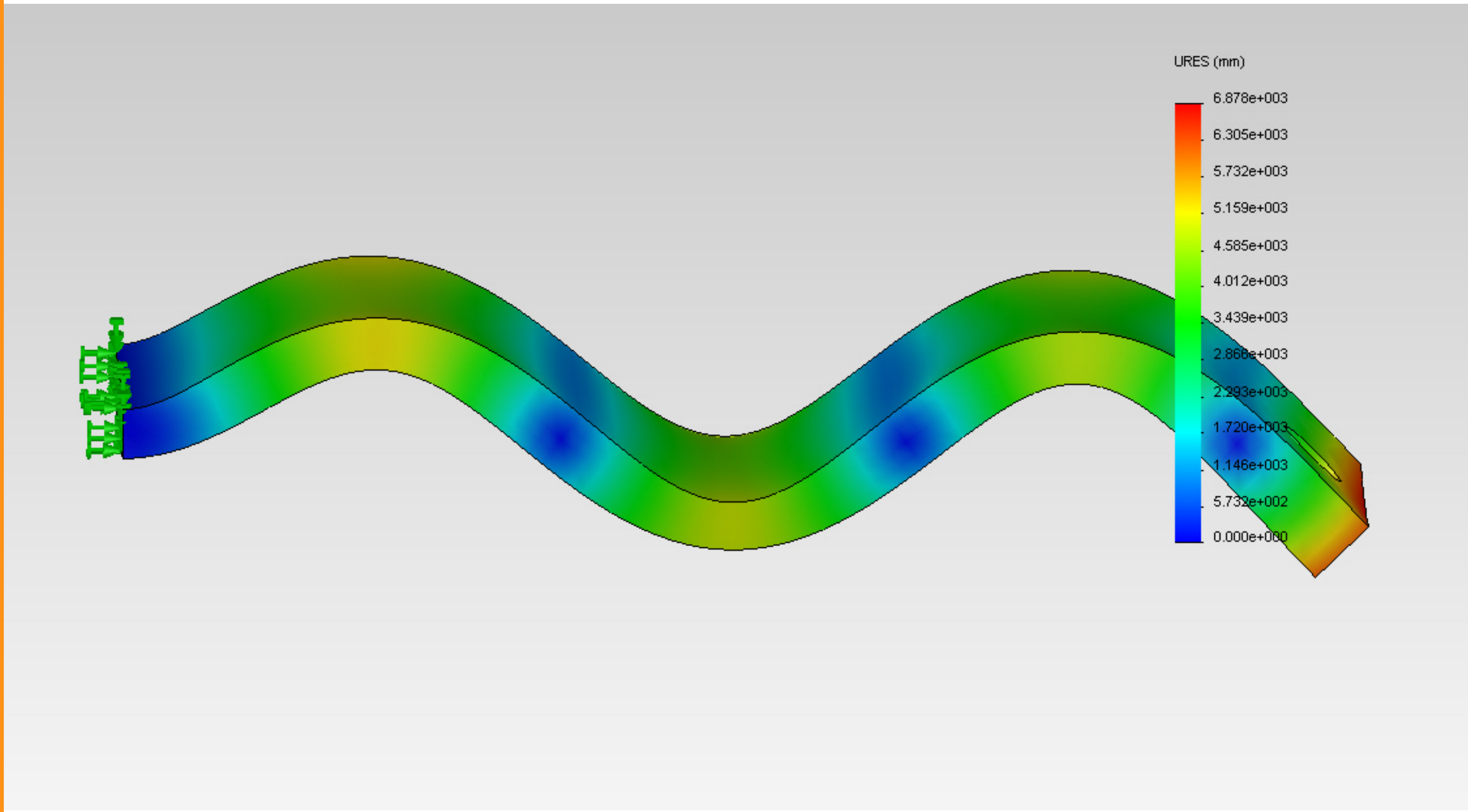
Mode 6: 5149.6 Hz



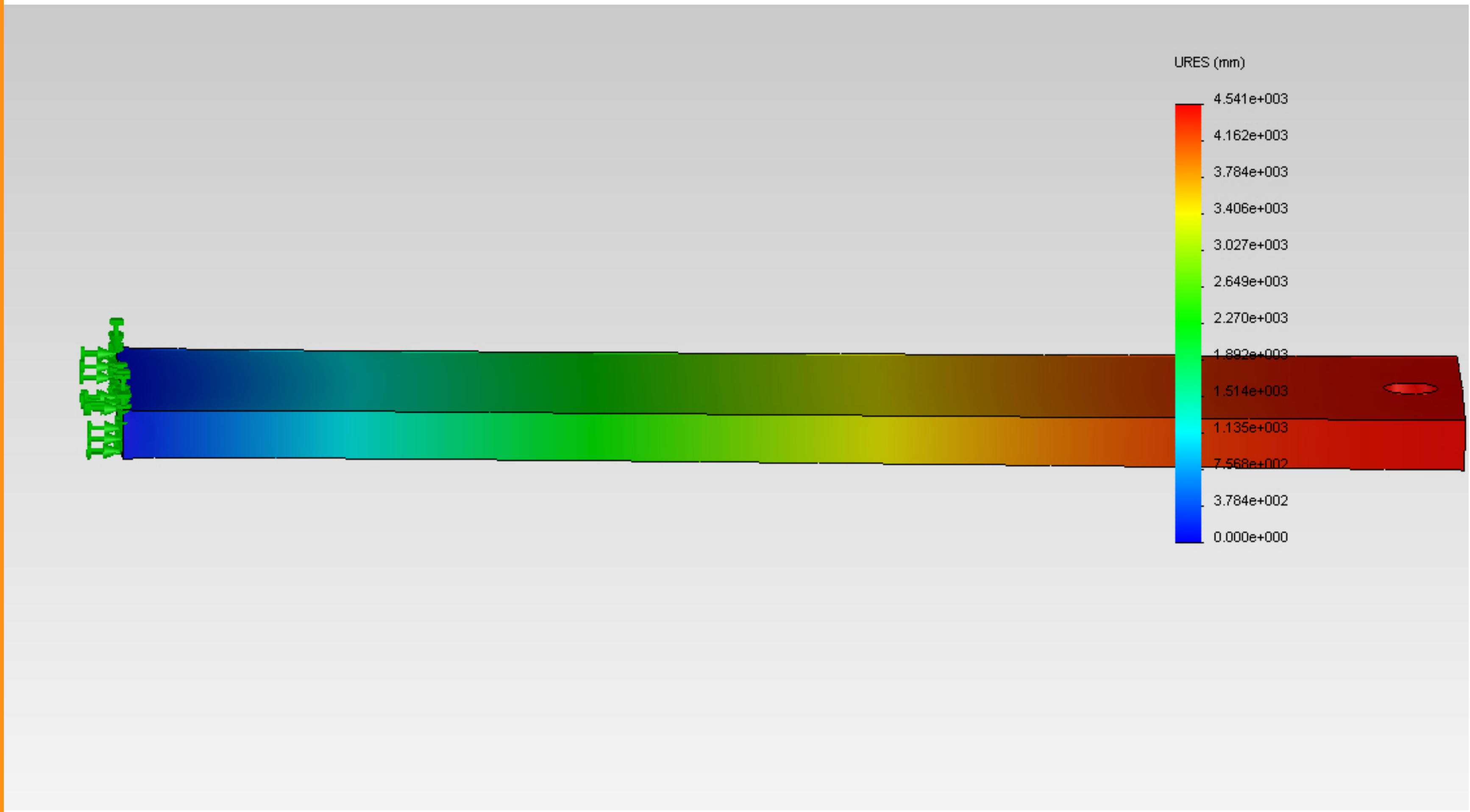
Mode 7: 6538.1 Hz



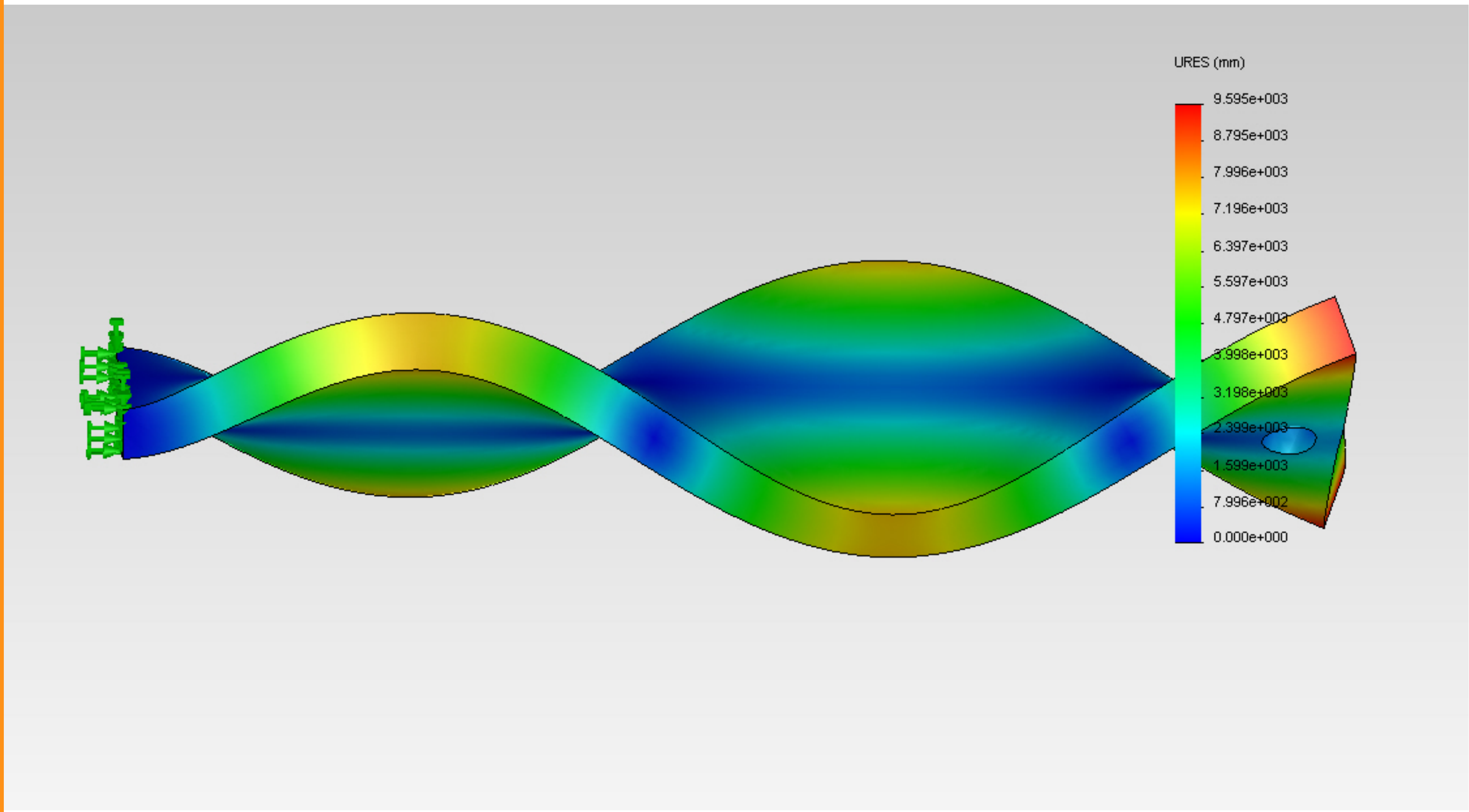
Mode 8: 7545.1 Hz



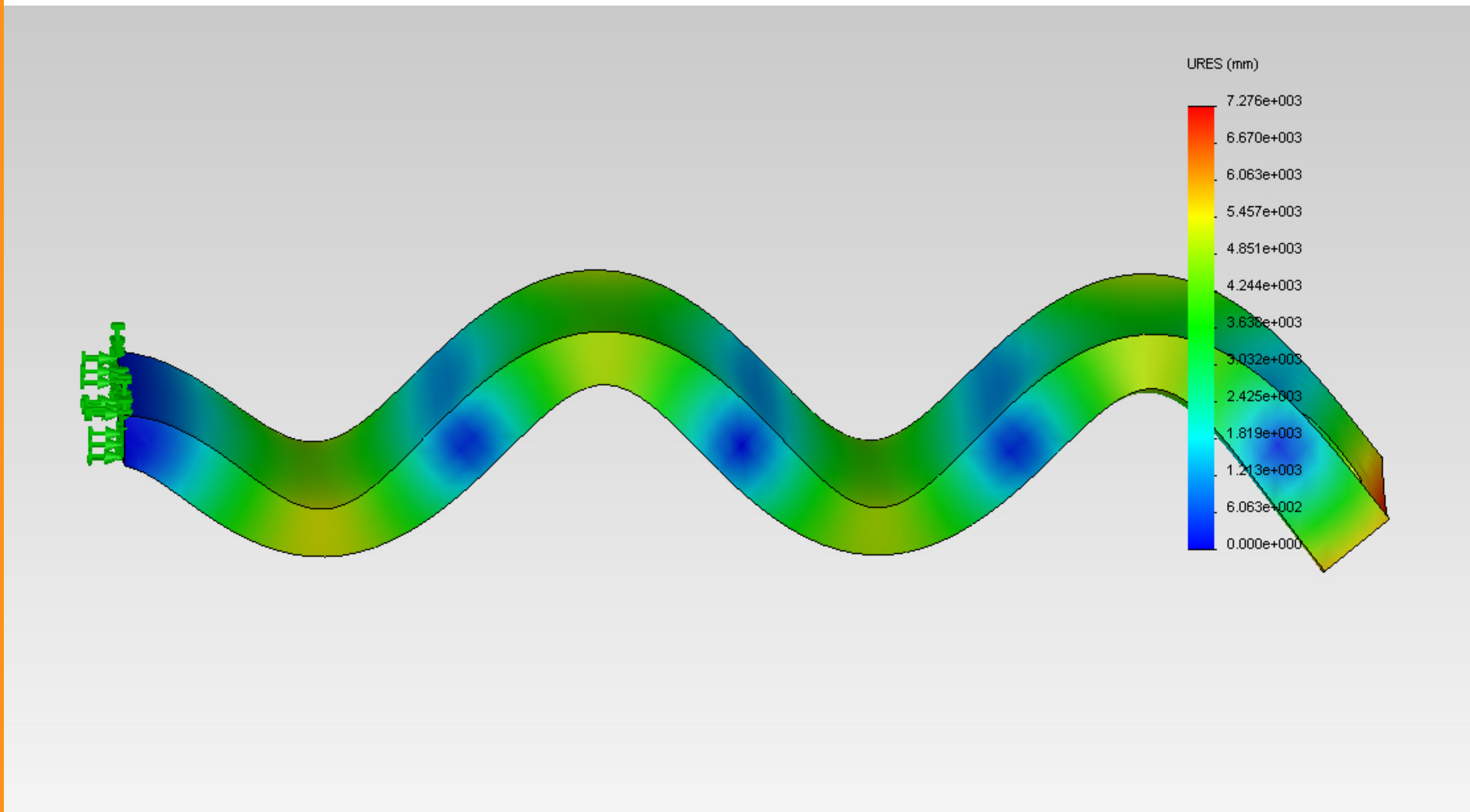
Mode 9: 8377.9 Hz



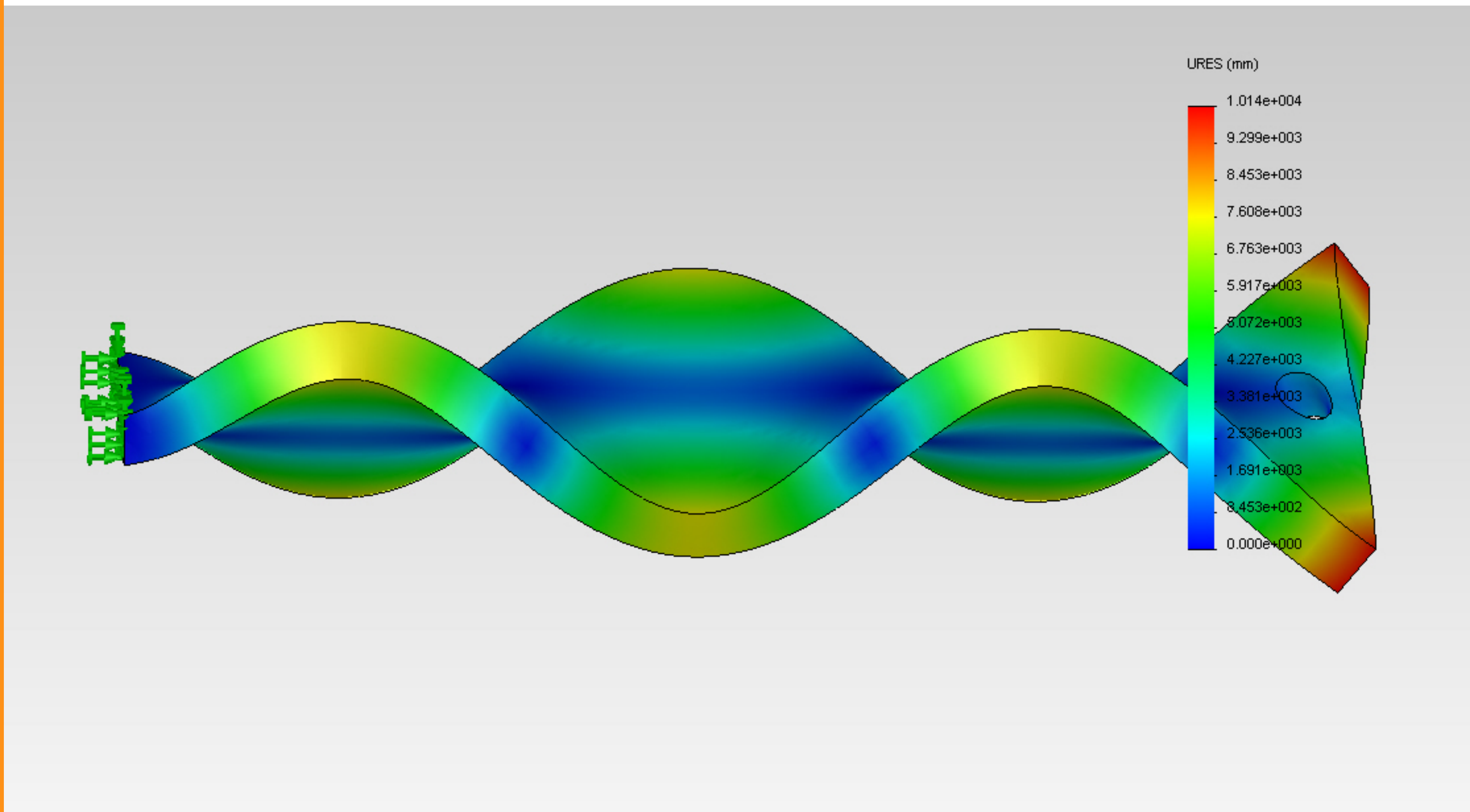
Mode 10: 8933.4 Hz



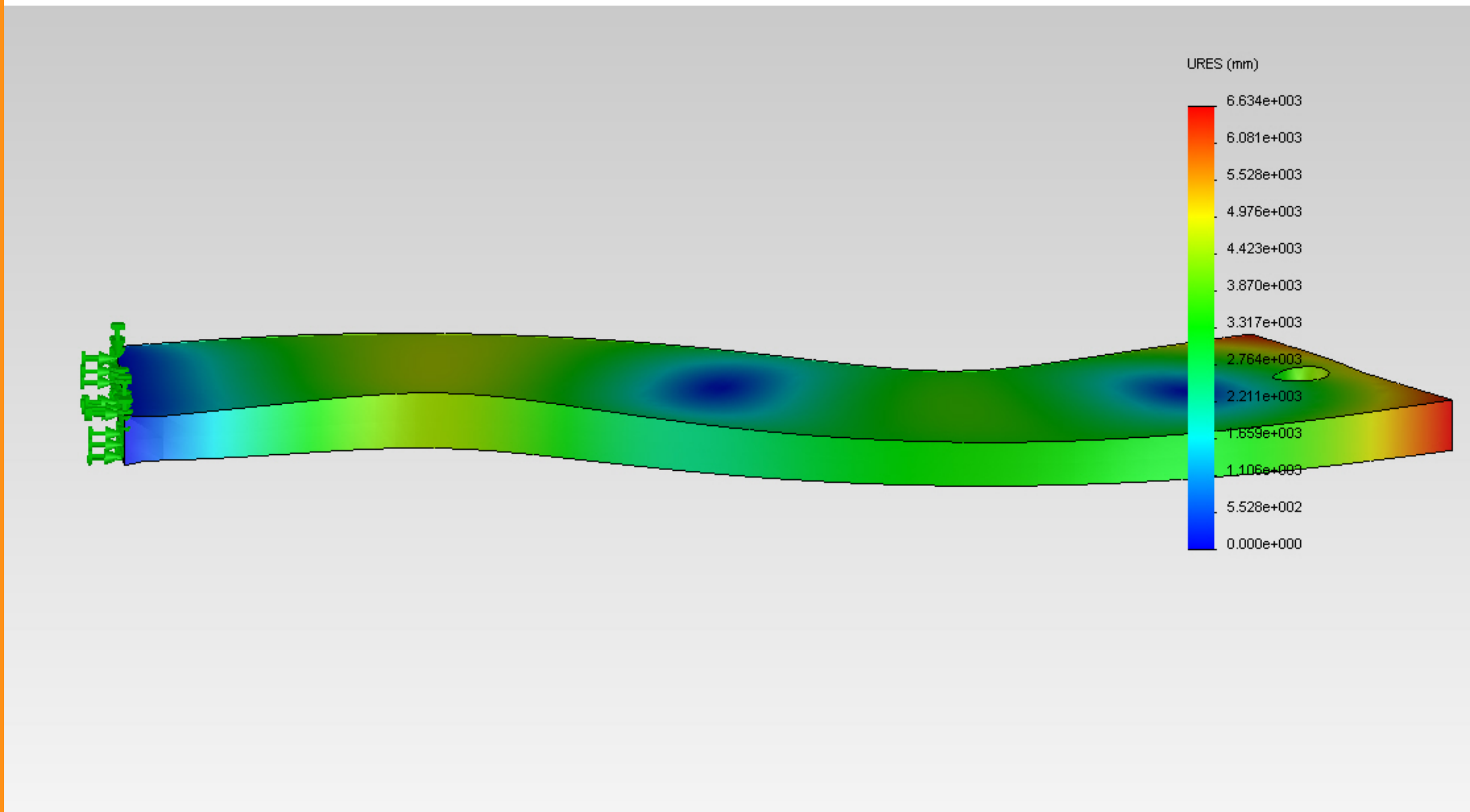
Mode 11: 12199 Hz



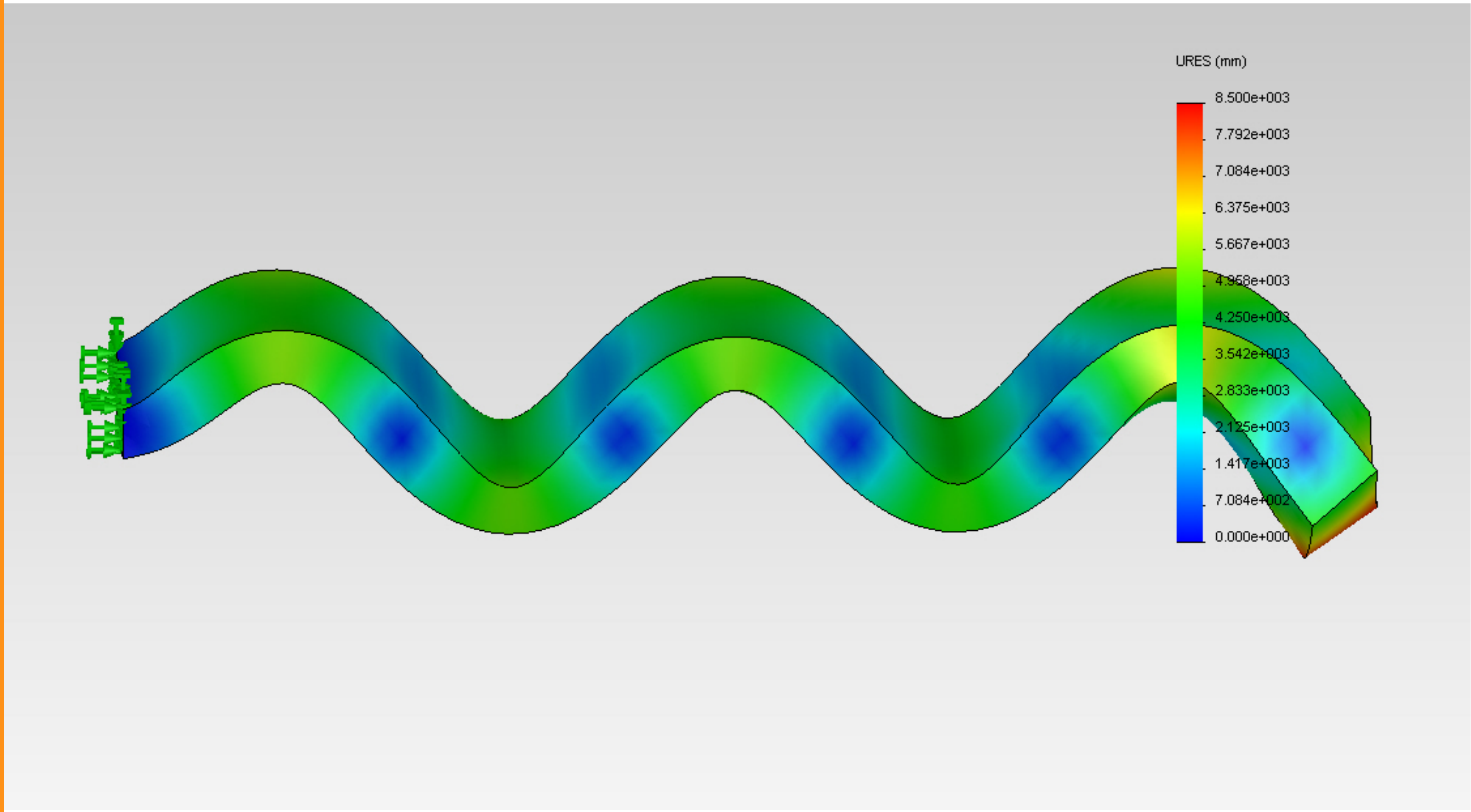
Mode 12: 13198 Hz



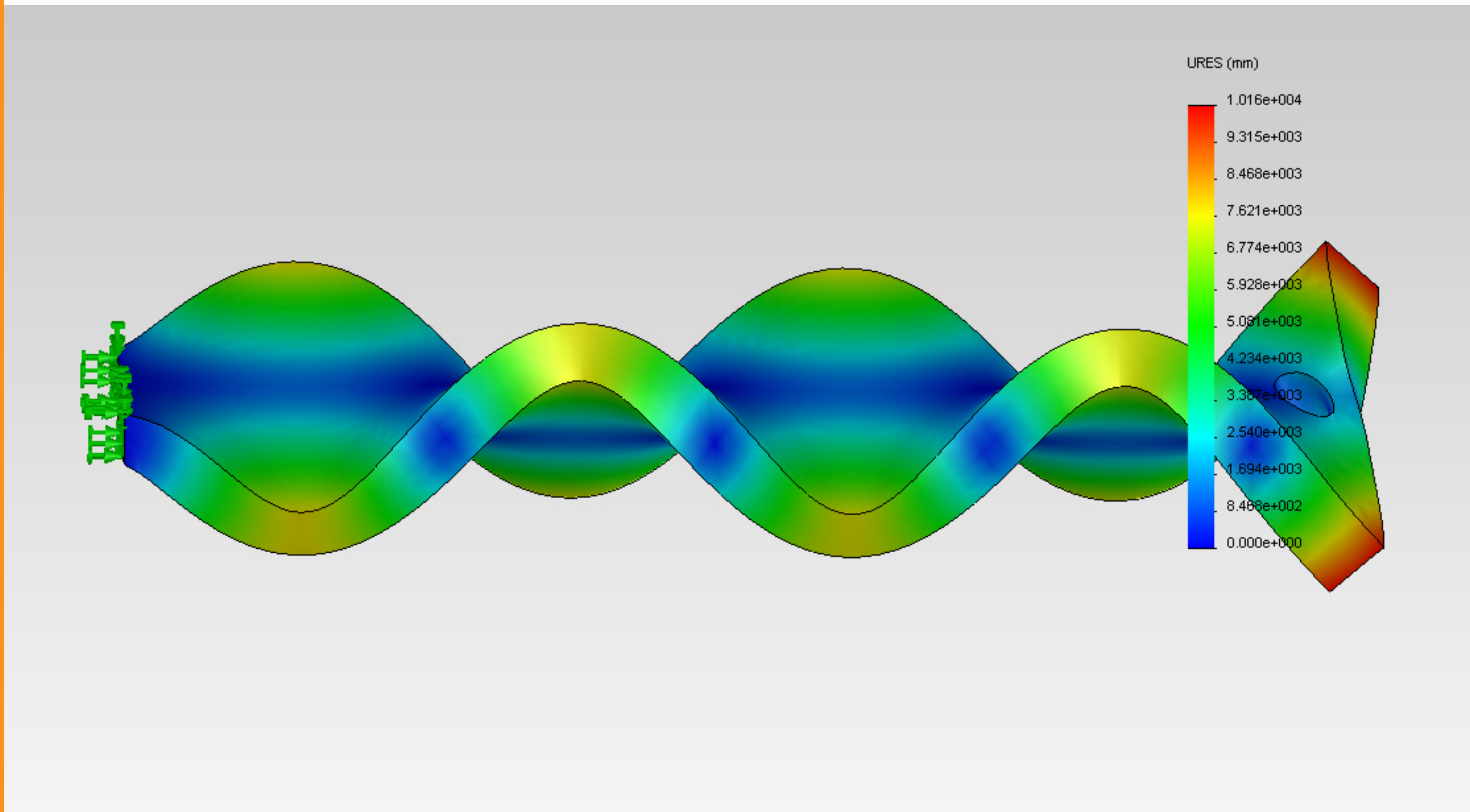
Mode 13: 14941 Hz



Mode 14: 17714 Hz



Mode 15: 18072 Hz



Cantilever: experiment

- Sensors: piezoelectric dynamic strain gauges
- Obtain data in time domain and in frequency domain
- Compare to analytical natural frequencies
- Build a low-pass filter to help analyze frequency data

Rocket (final project)

- Mathematical model

  to obtain natural frequencies and modal shapes

- Computational model

  example: SolidWorks model

- Experimental data

  - tap test in ***lab*** using impact hammer and accelerometers

  - sensors during ***flight***

Rocket: mathematical model

- general solution for a free-free beam is:

$$y(x,t) = \sum_{n=1}^{\infty} \left(A_n \sin \omega_n t + B_n \cos \omega_n t\right) \sin \frac{n\pi x}{L}$$

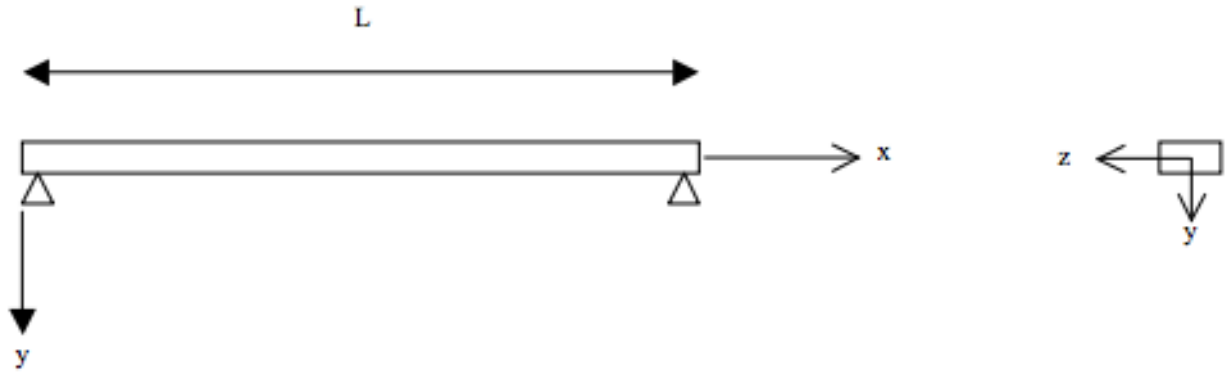
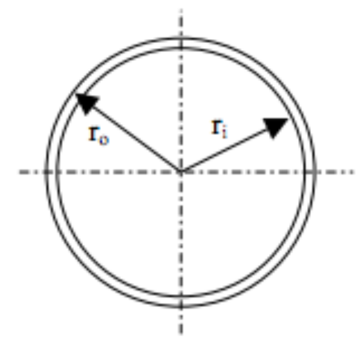


Figure 1: Schematic of a free-free beam

- calculations of a beam's cross sectional properties

$$A = \pi(r_o^2 - r_i^2)$$

$$I = \frac{1}{4}\pi(r_o^4 - r_i^4)$$



Cross section of a hollow cylinder

"Structural Dynamics, The theory and applications," Joseph W. Tedesco, Addison Wesley, Longman Inc., 1999

Rocket: mathematical model

- natural frequencies are:

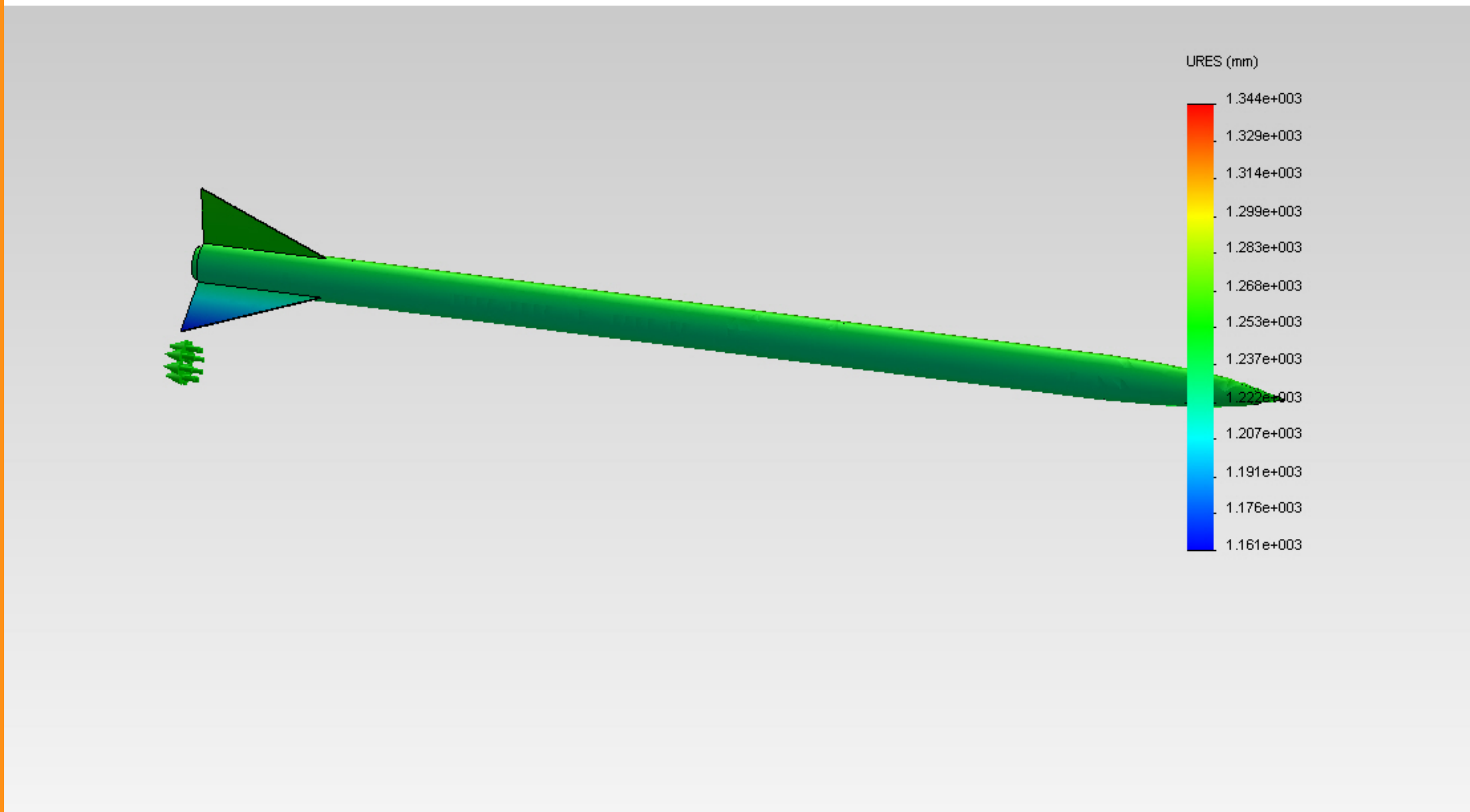
$$\omega_n = \beta_n^{\ 2}\sqrt{\frac{EI_z}{\rho A}} = (\beta_n L)^2\sqrt{\frac{EI_z}{\rho A L^4}}$$

Boundary Conditions	**Frequency Equations**	$\beta_1 L$	$\beta_2 L$	$\beta_2 L$
Pinned-pinned	$\sin \beta L = 0$	3.141	6.282	9.423
Fixed-free	$\cos \beta L \cosh \beta L + 1 = 0$	1.875	4.694	7.855
Fixed-pinned (and pinned-free)	$\tan \beta L = \tanh \beta L$	3.927	7.069	10.210
Fixed-fixed (and free-free)	$\cos \beta L \cosh \beta L = 1$	4.730	7.853	10.996
Fixed-sliding (and free-sliding)	$\tan \beta L + \tanh \beta L = 1$	2.365	5.498	8.639

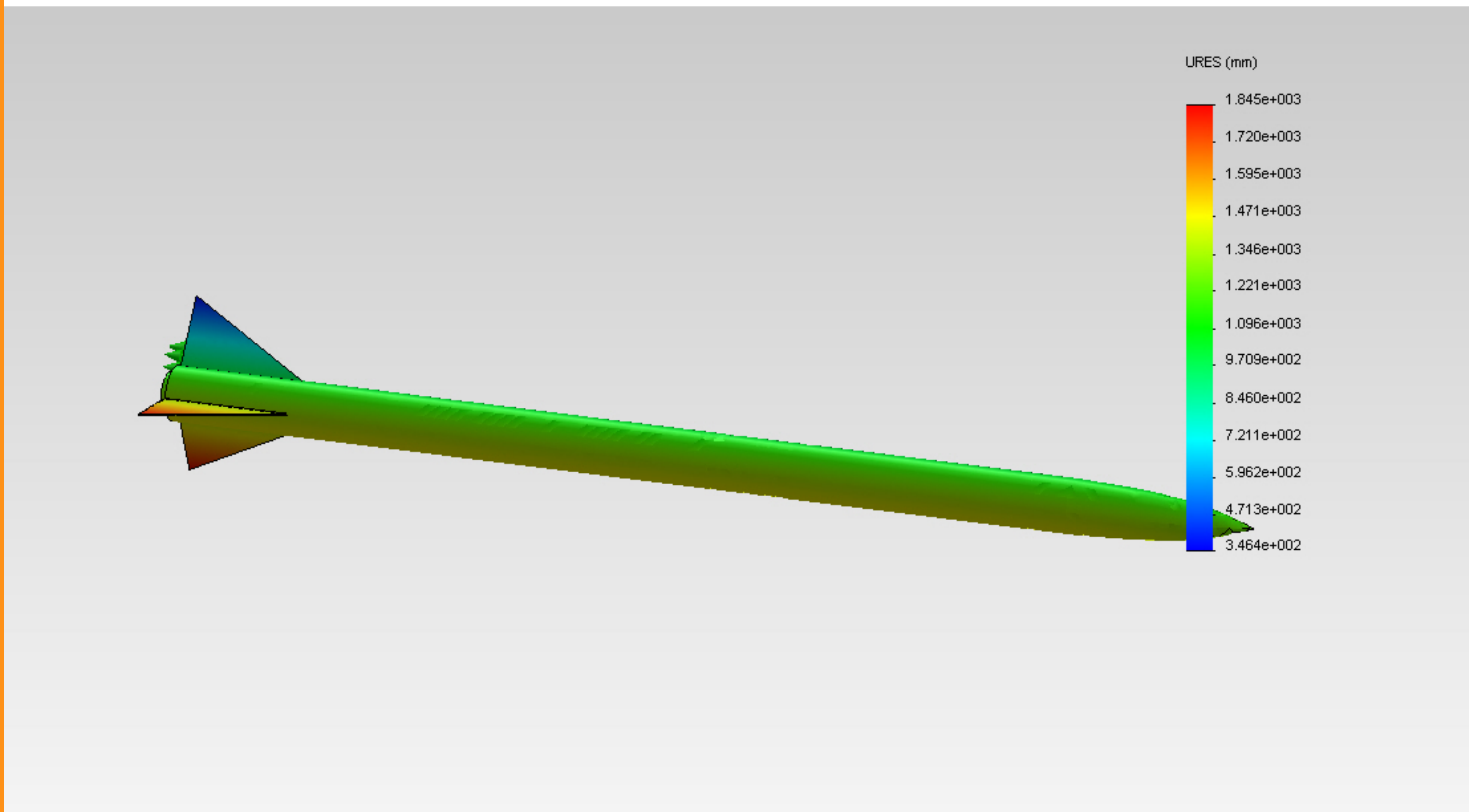
Table 1: Natural Frequencies for Single-Span Beams

"Structural Dynamics, The theory and applications," Joseph W. Tedesco, Addison Wesley, Longman Inc., 1999

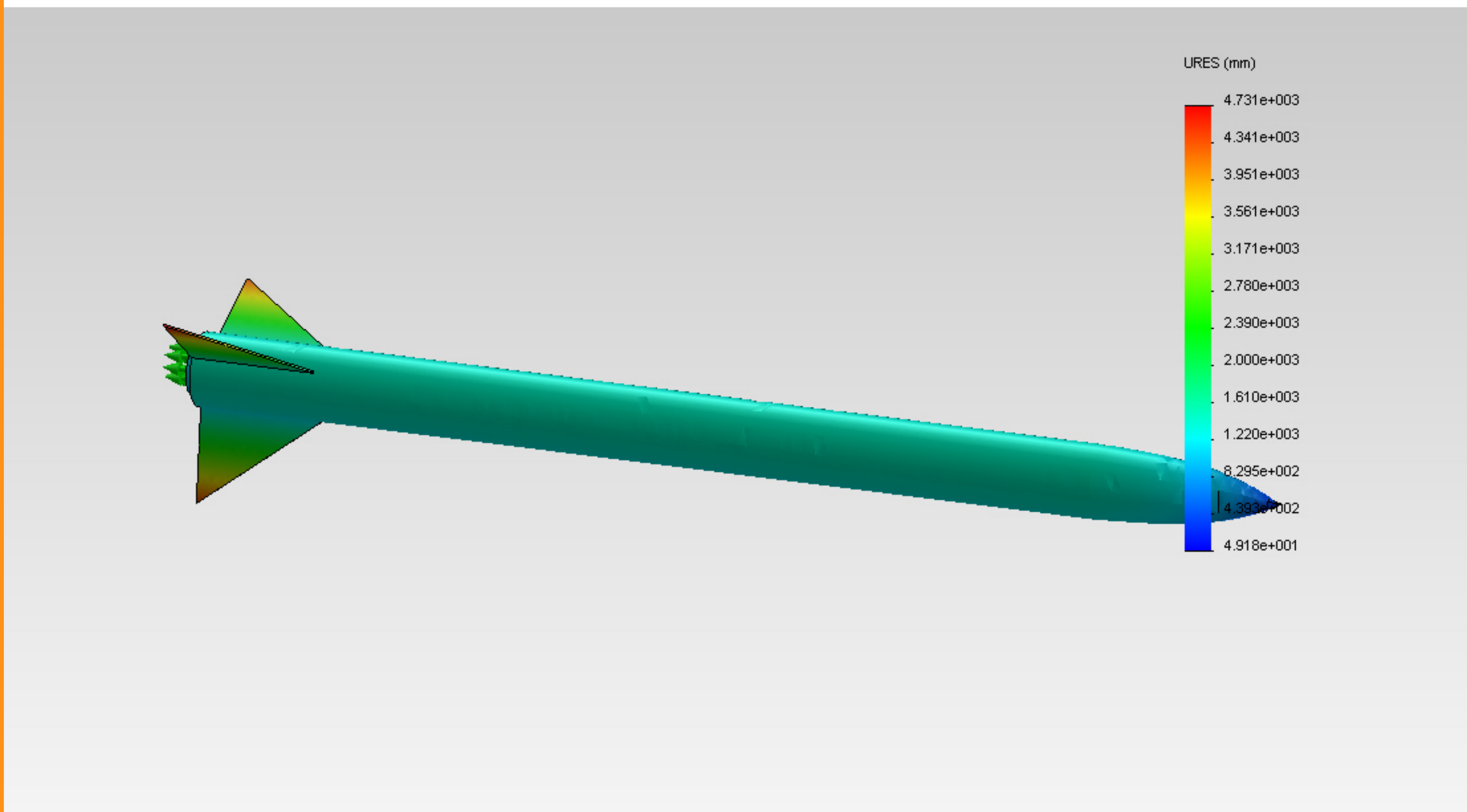
Rocket: computational model
Mode 1: 0 Hz



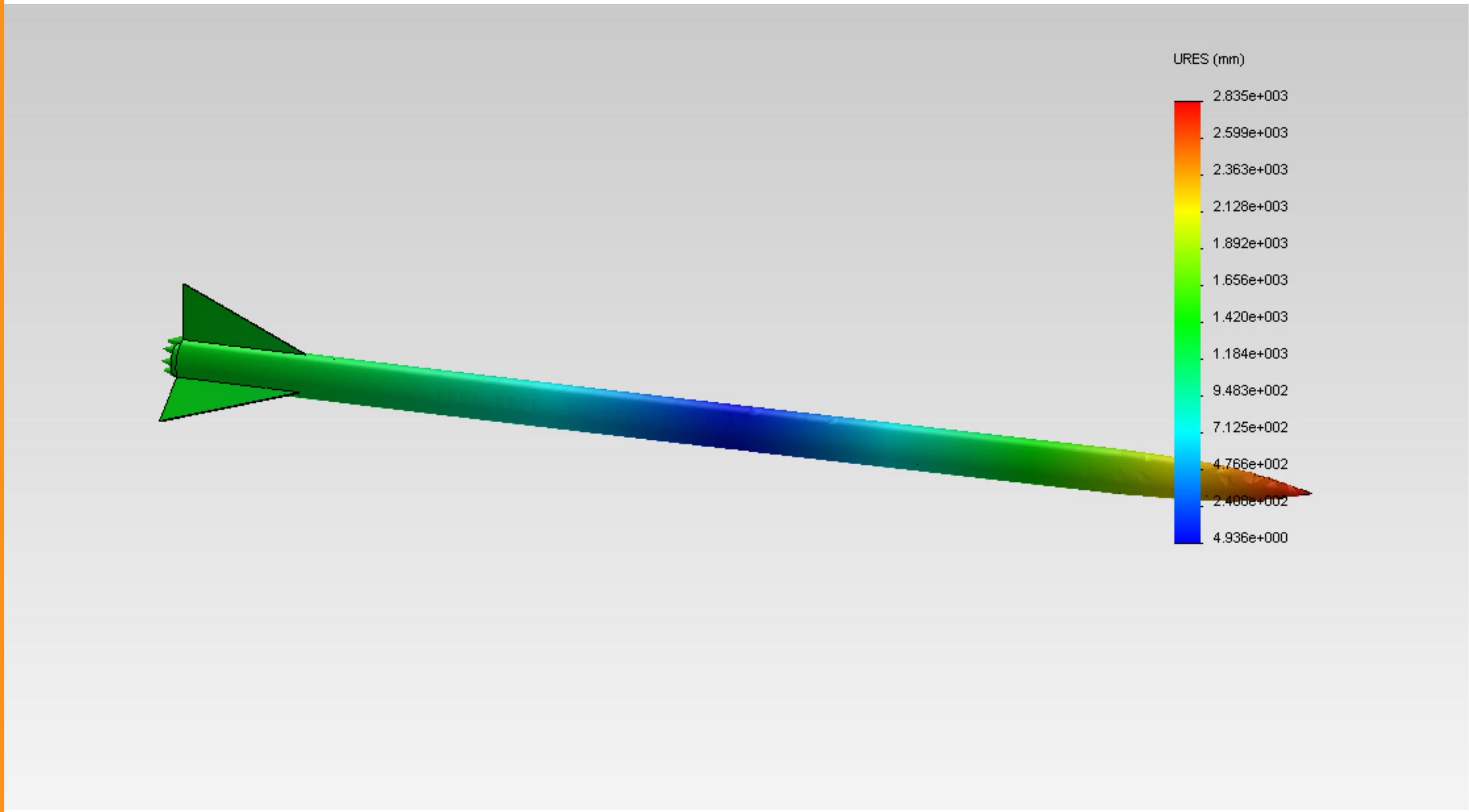
Mode 2: 7.0439E-4 Hz



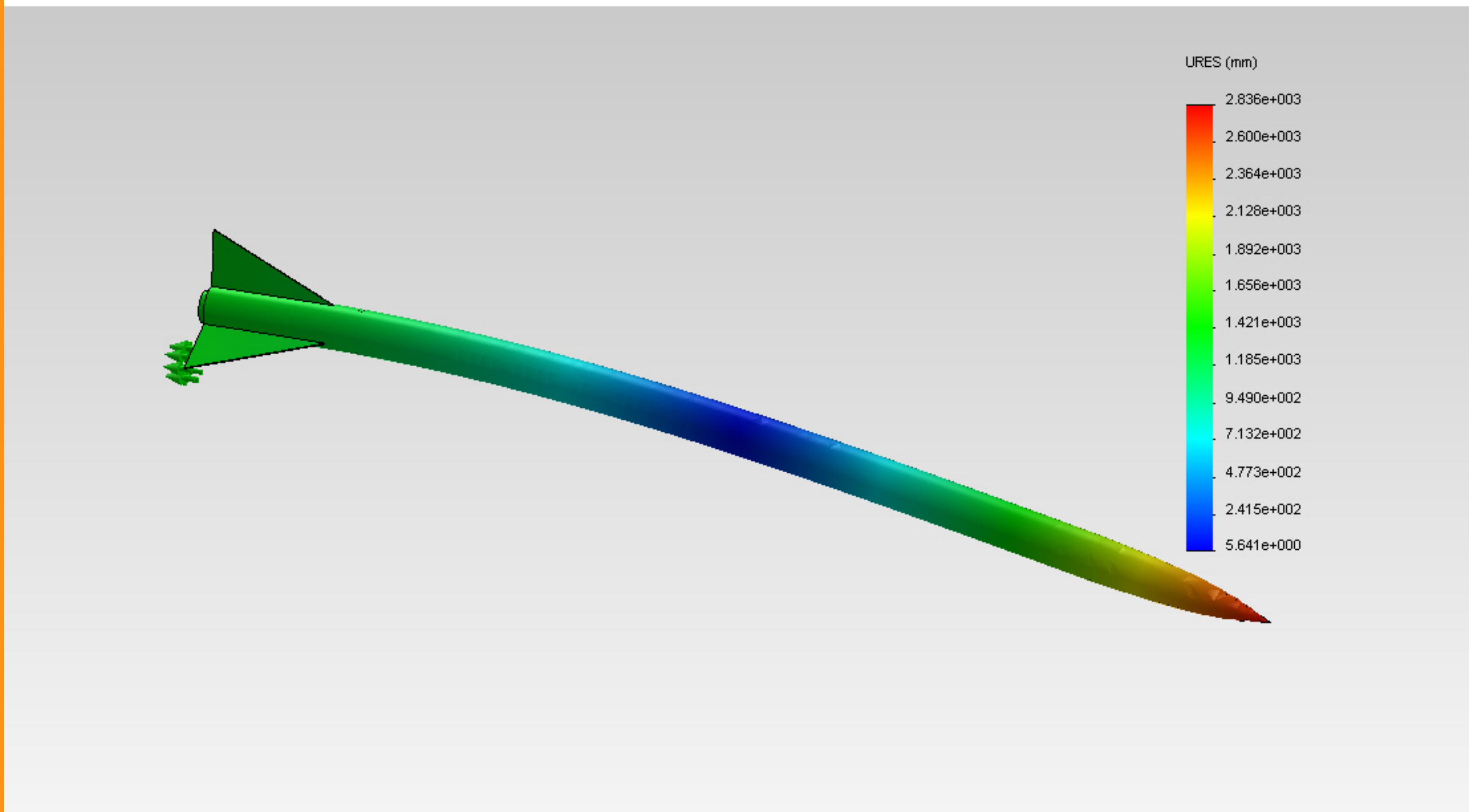
Mode 3: 1.7816E-3 Hz



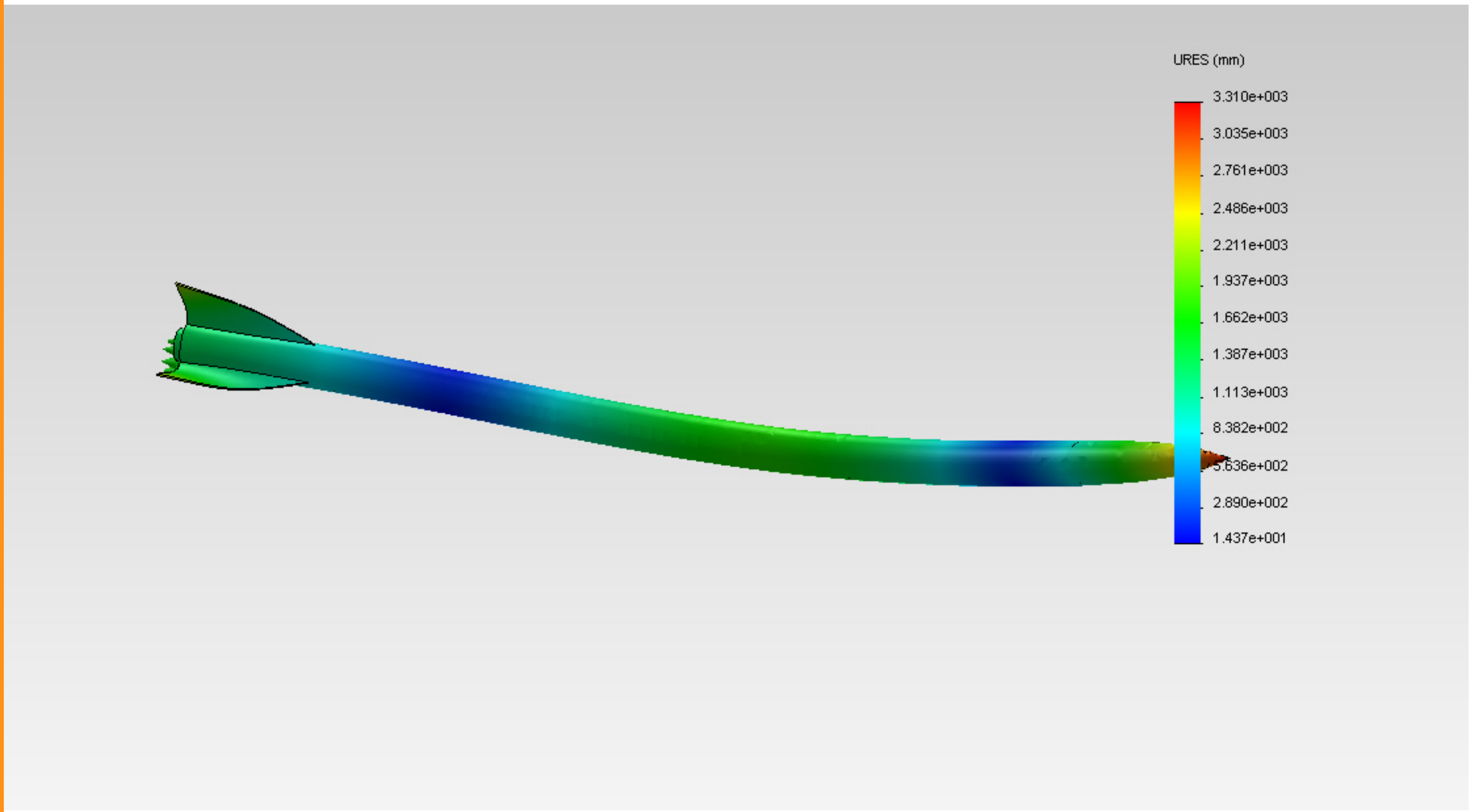
Mode 4: 11.752 Hz



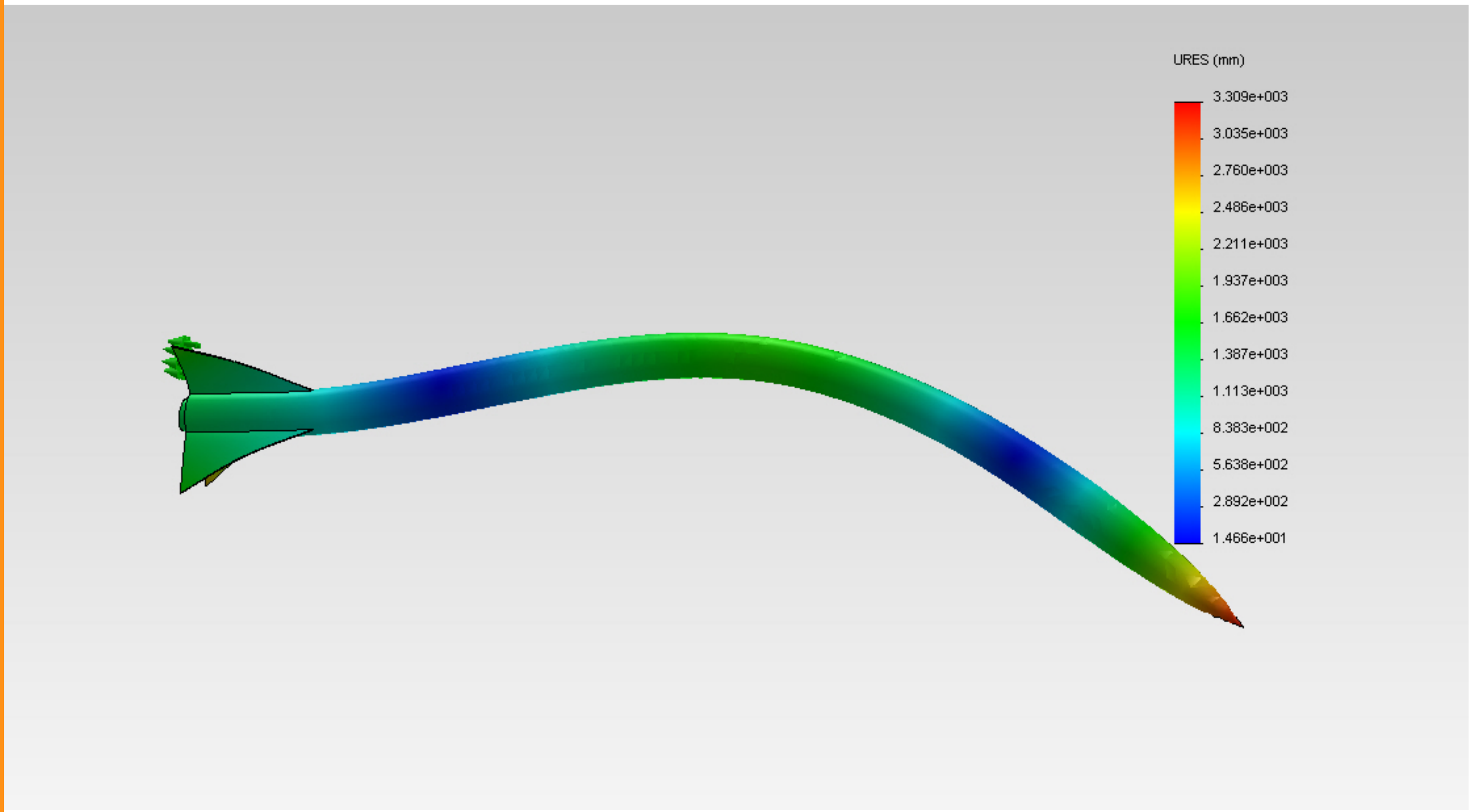
Mode 5: 11.802 Hz



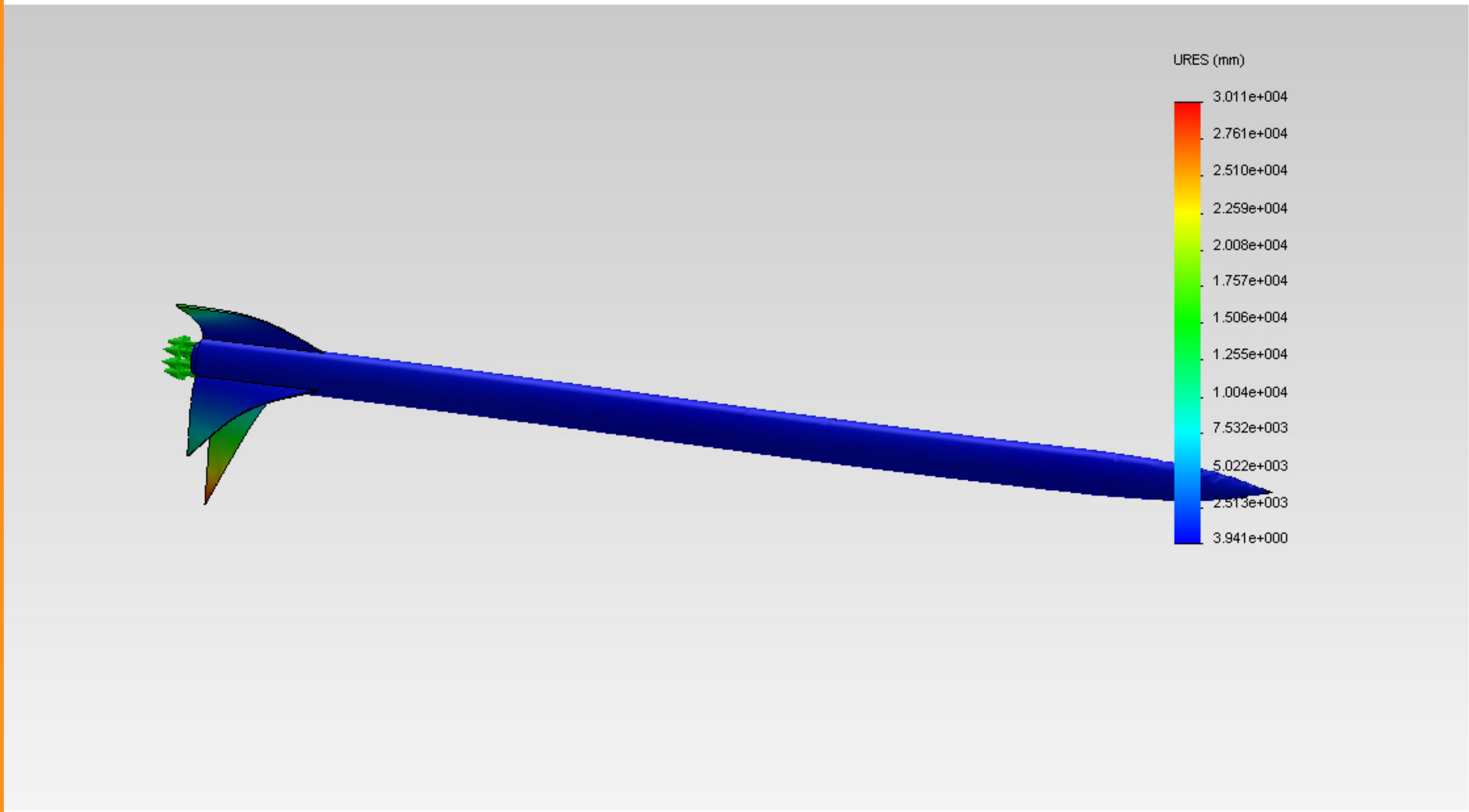
Mode 6: 62.133 Hz



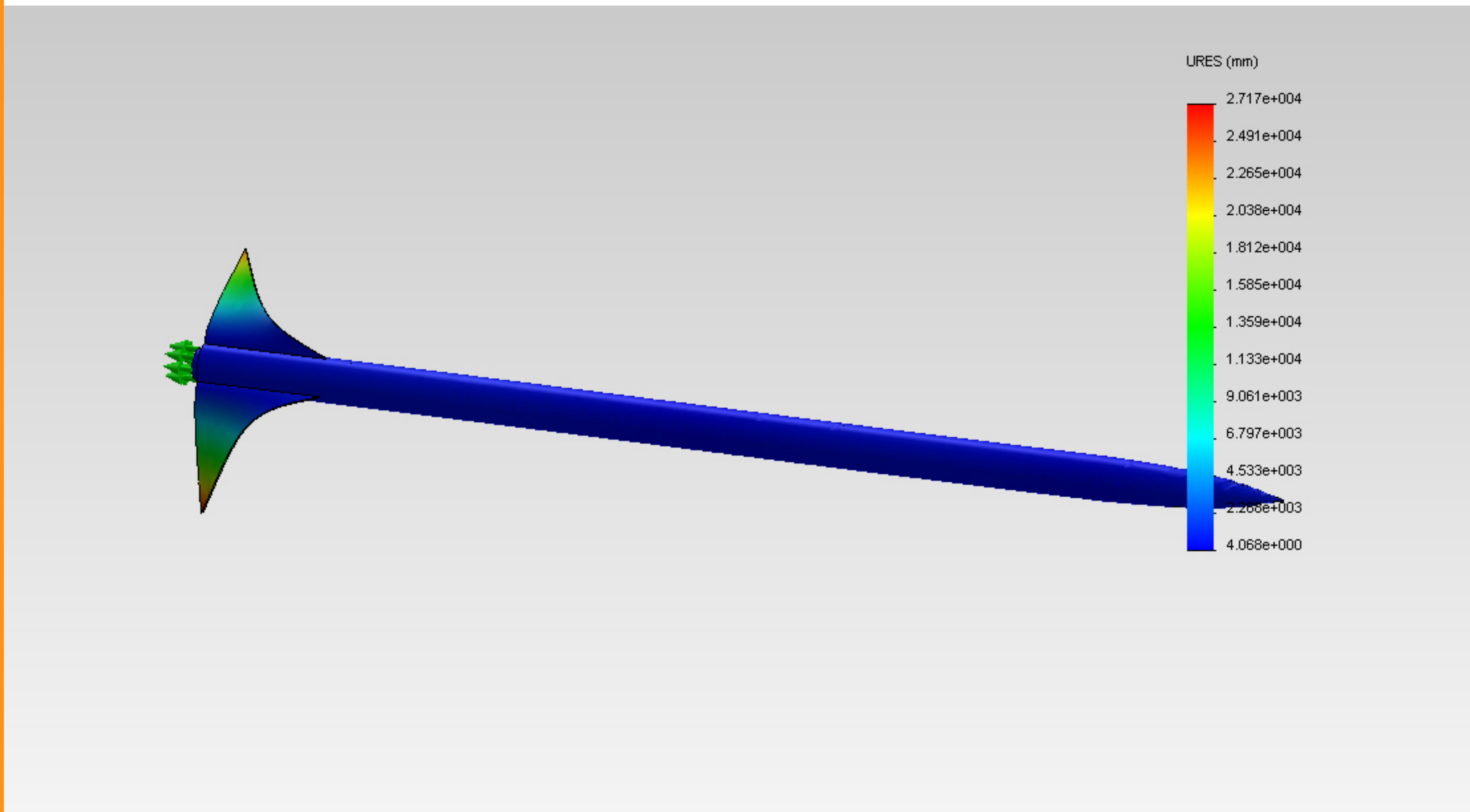
Mode 7: 62.287 Hz



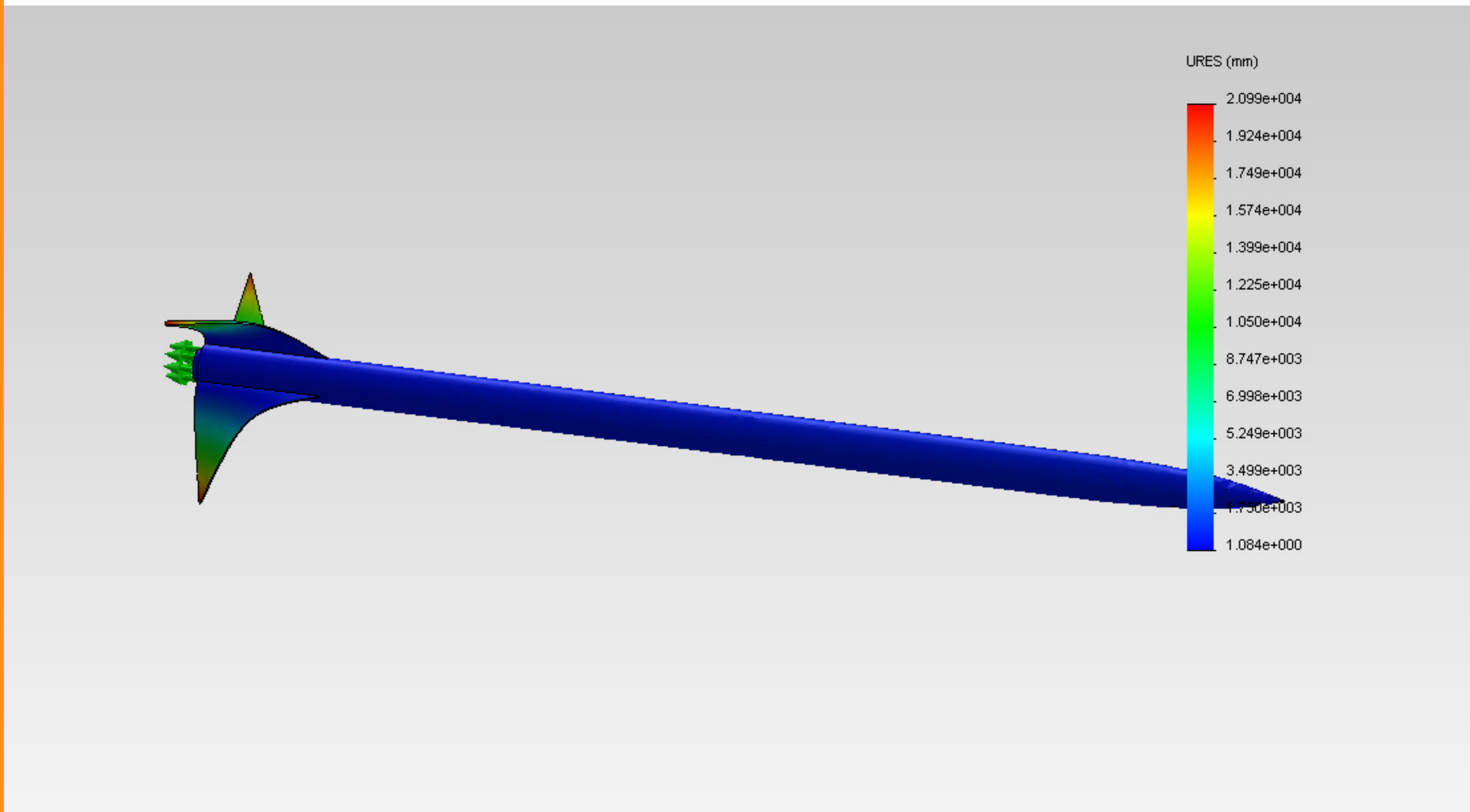
Mode 8: 111.02 Hz



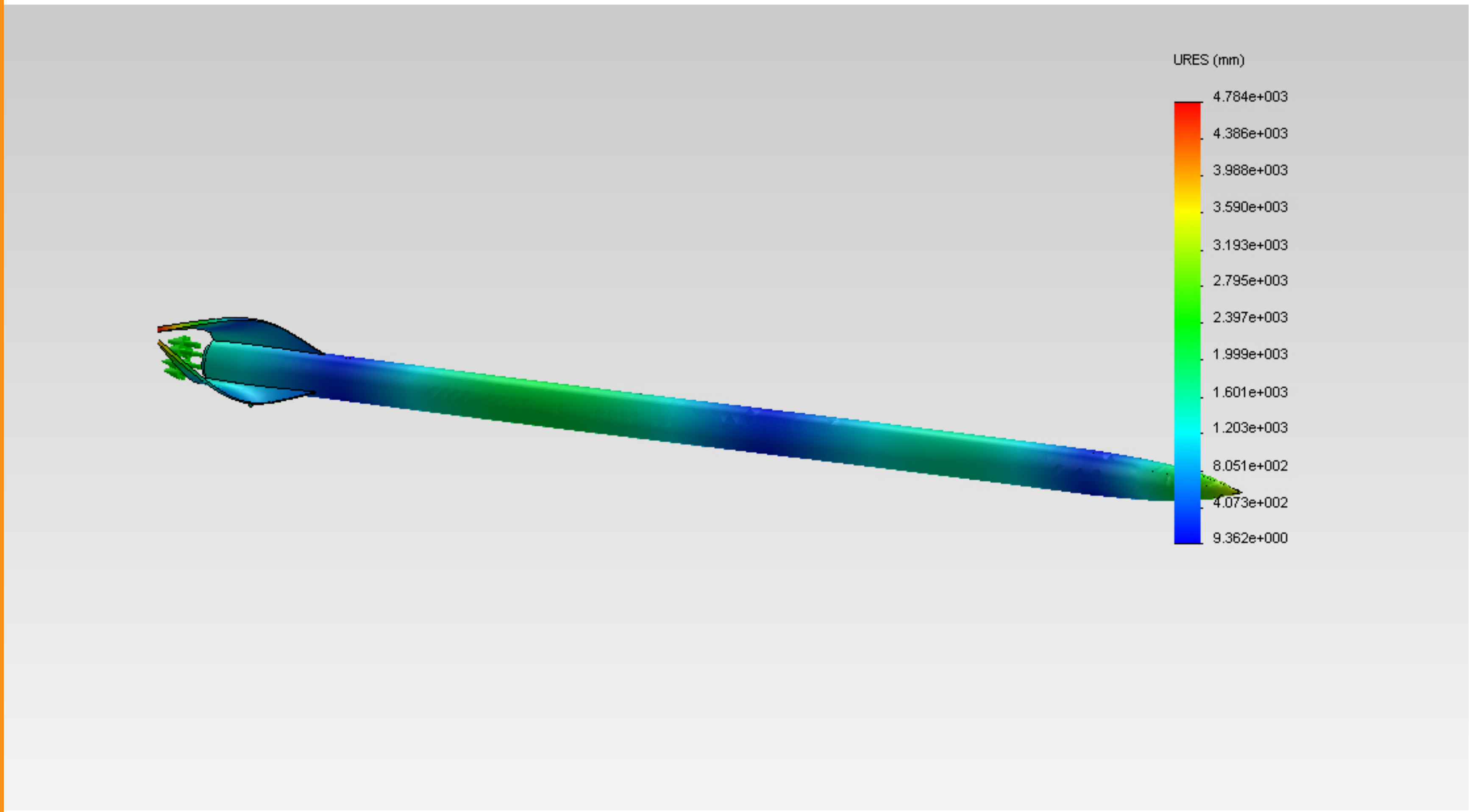
Mode 9: 111.06 Hz



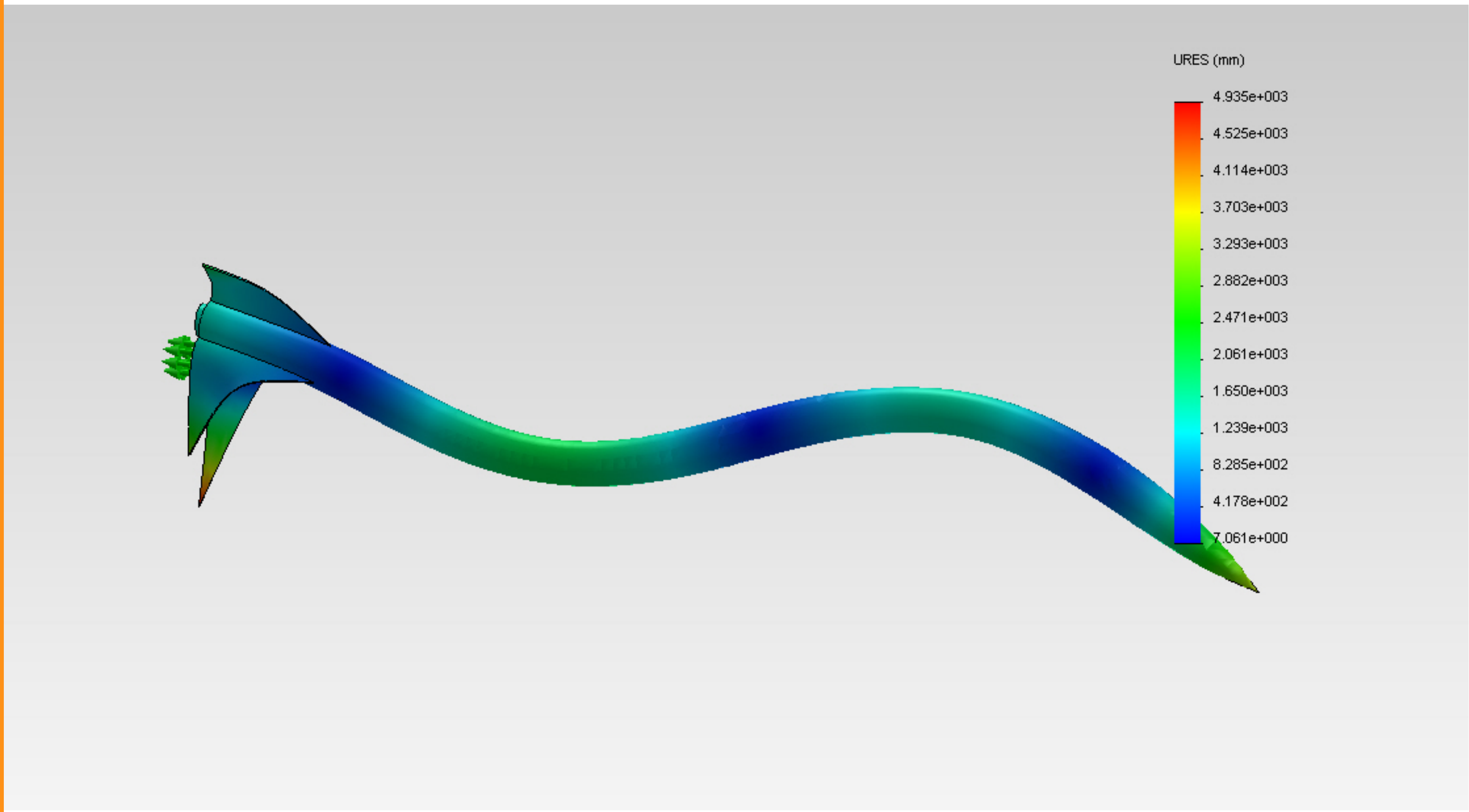
Mode 10: 114.37 Hz



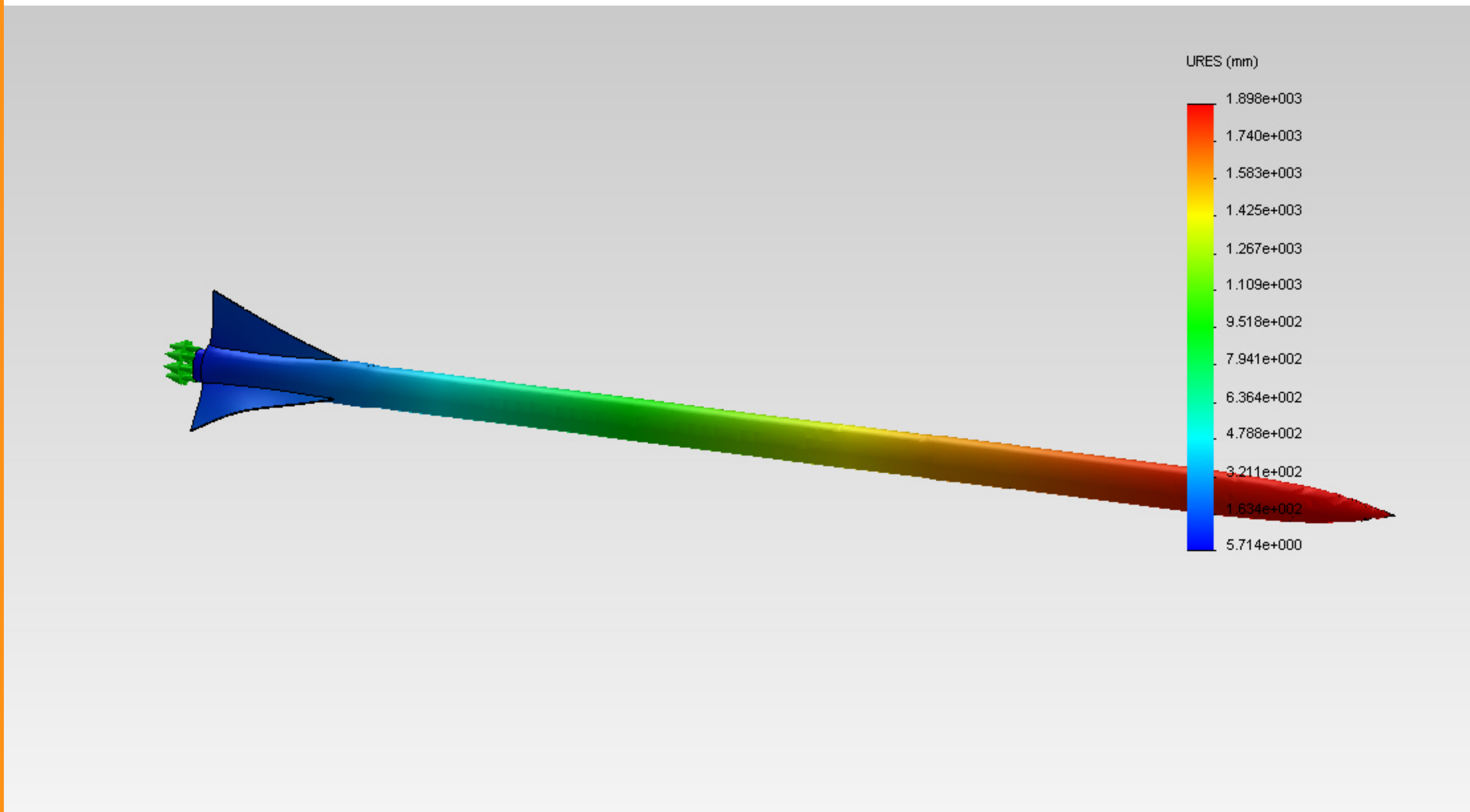
Mode 11: 154.73 Hz



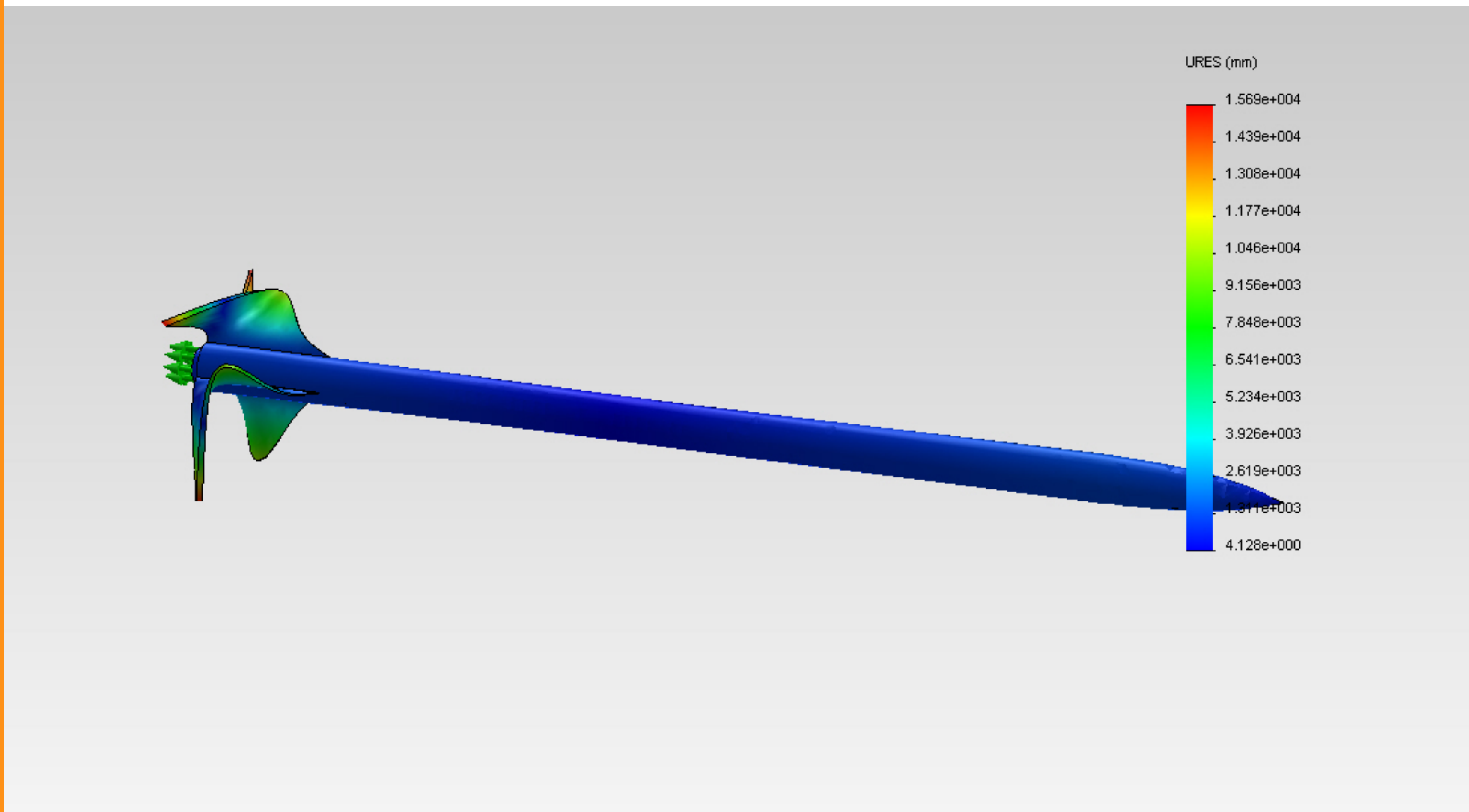
Mode 12: 155.32 Hz



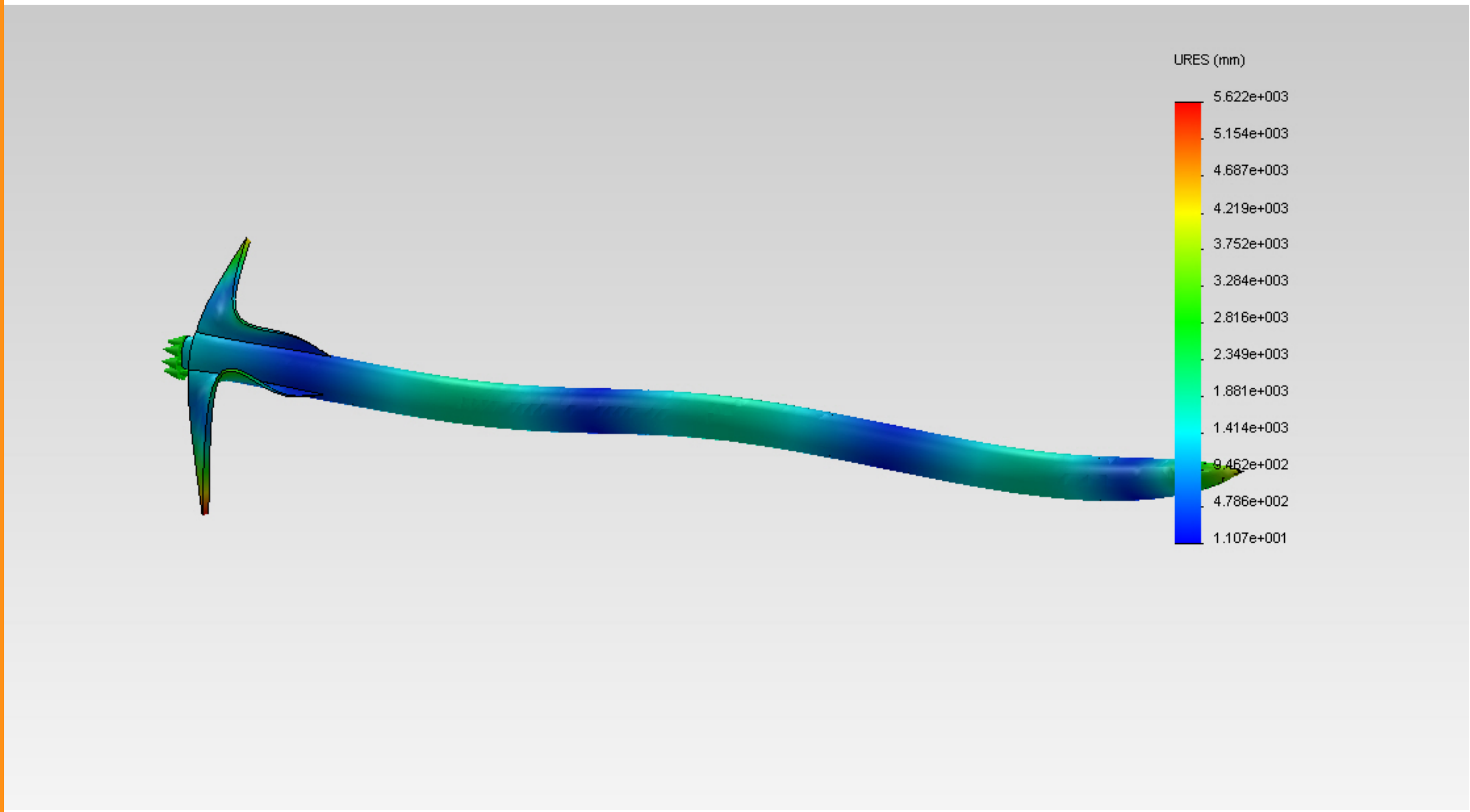
Mode 13: 257.09 Hz



Mode 14: 266.75 Hz

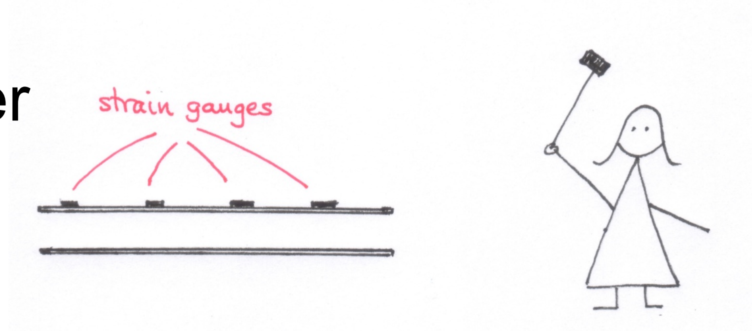


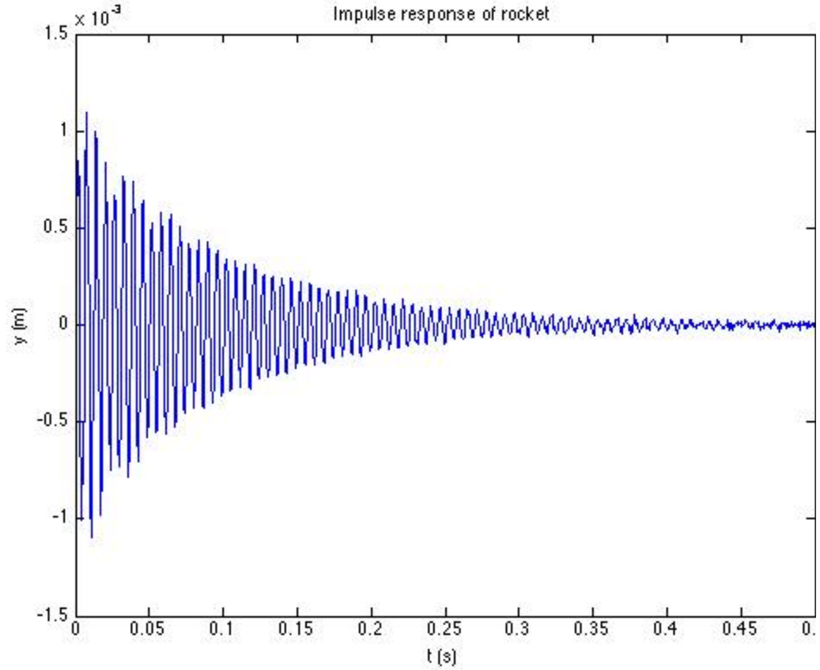
Mode 15: 273.79 Hz

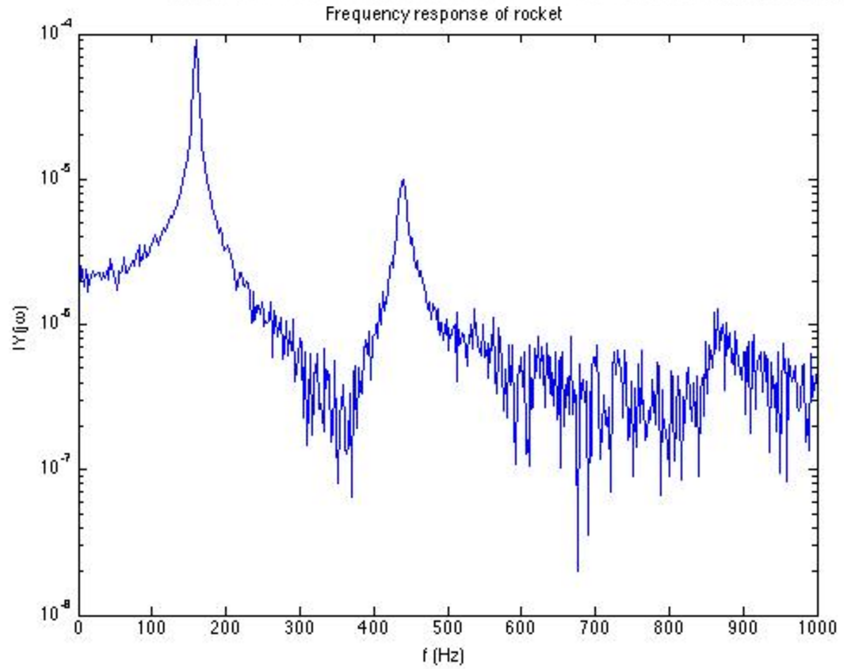


Rocket: experimental data

In ***lab***: tap test with impact hammer
"TapTestFRF.vi"







https://www.youtube.com/watch?v=XkmgMkDKAyU

Rocket: experimental data

During ***flight***: get time data from sensors

Post flight: analyze data in frequency domain
compare to tap test results
compare to model predictions
(from analytical model and/or
computational model)

Question

Why might we be interested in rocket vibrations?

- To characterize rocket's natural frequencies and modal shapes
- Avoid dead spots (nodes) to optimize sensor placement
- Design a vibration isolator to minimize vibrations in payload
- Validate your model

Video of flutter

- https://www.youtube.com/watch?v=OhwLojNerMU