Lecture 9
Inertial Measurement

Feb. 19, 2013

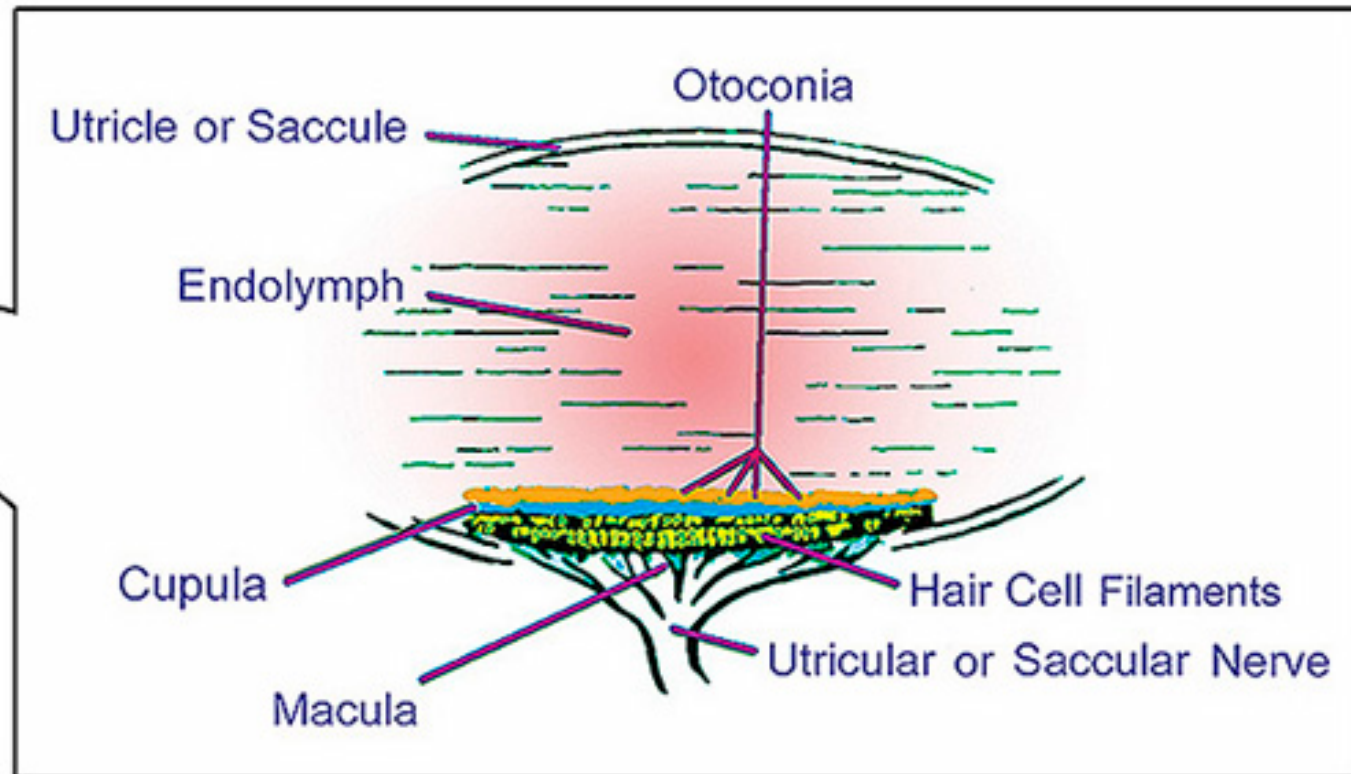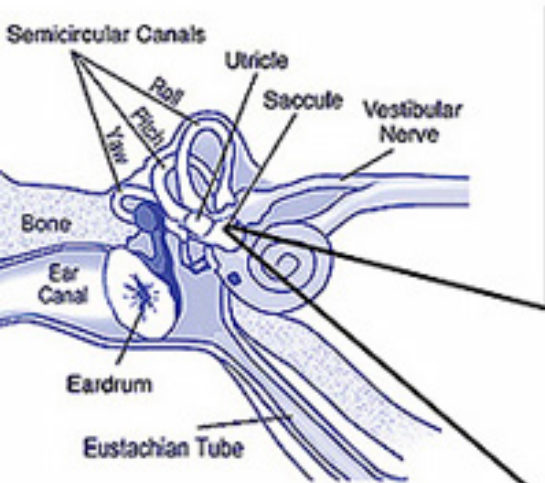Christopher M. Clark
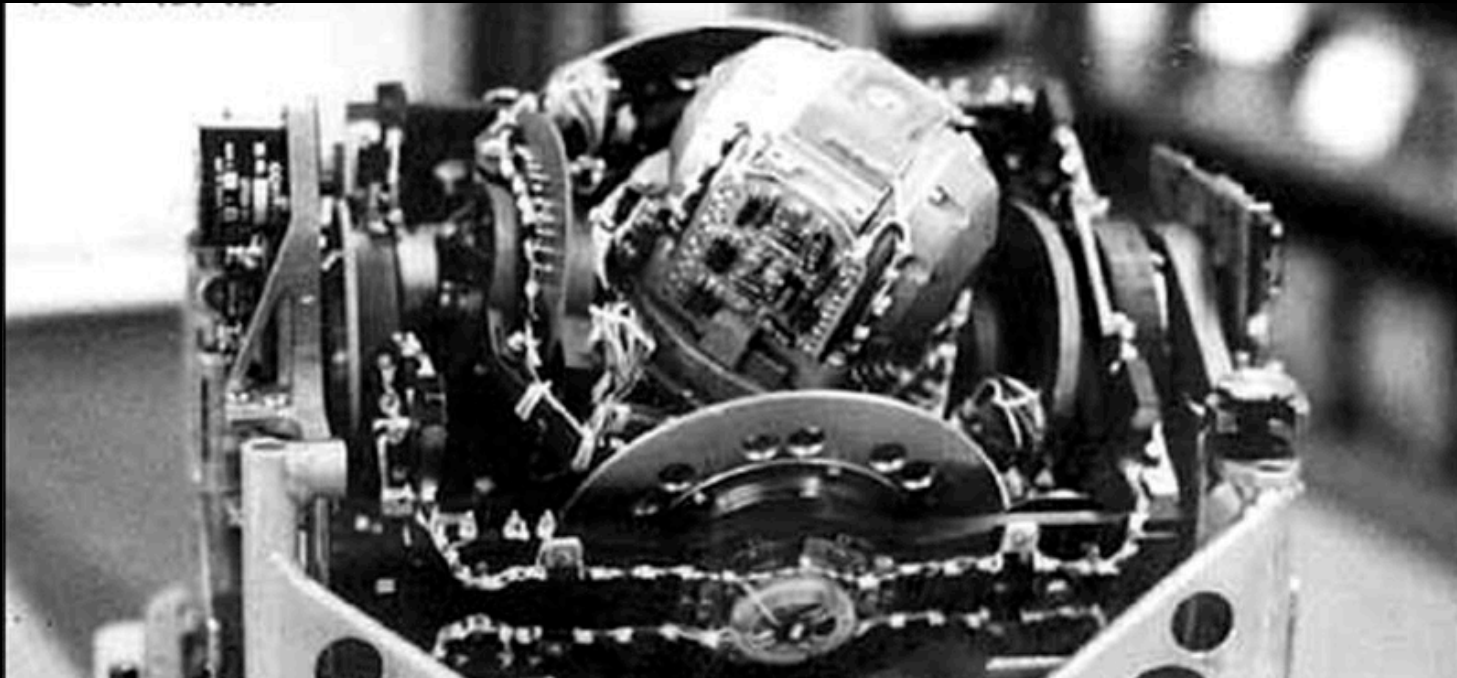
Where is the rocket?

Outline

- Sensors
  - People
  - Accelerometers
  - Gyroscopes
- Representations
- State Prediction
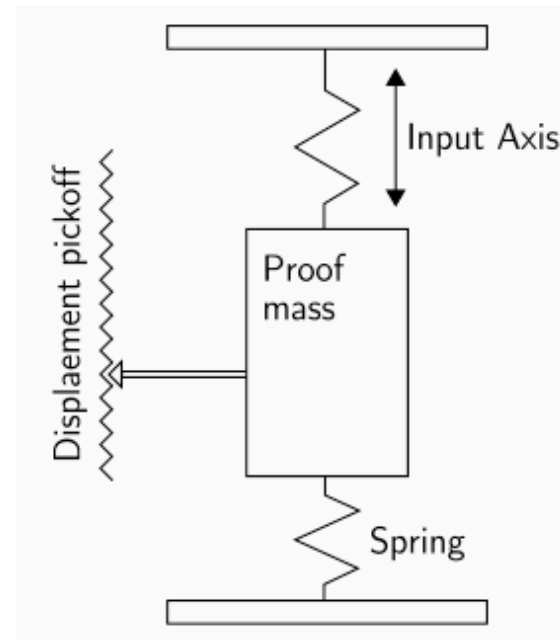- Example Systems
- Bounding with KF

Sensors

- People

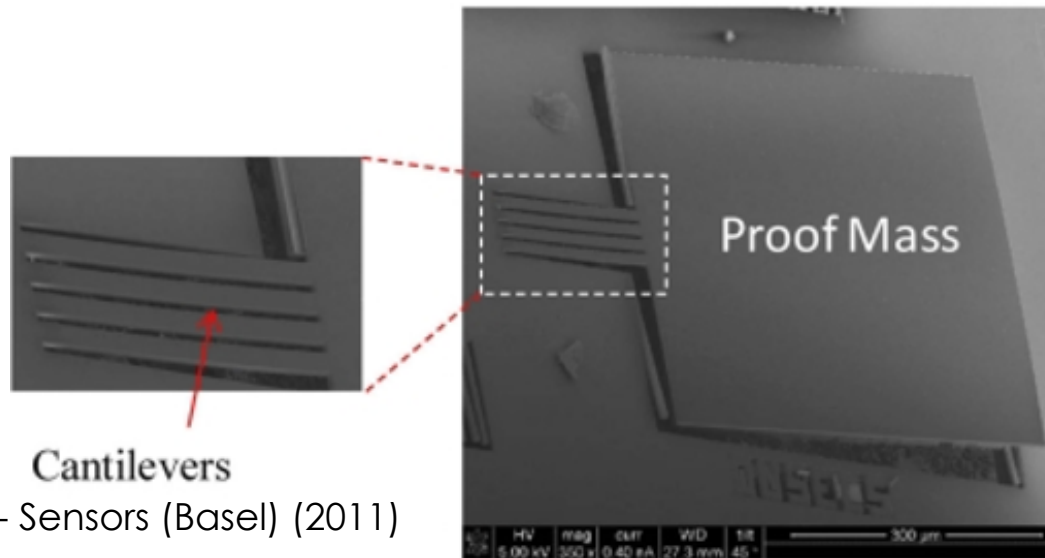http://en.wikipedia.org/wiki/File:Bigotolith.jpg

Sensors

- Accelerometers

$$a = k\,x\,/\,m$$

Sensors

- Accelerometers
  - The accelerometers are typically MEMS based
  - They are small cantilever beams (~100 µm)
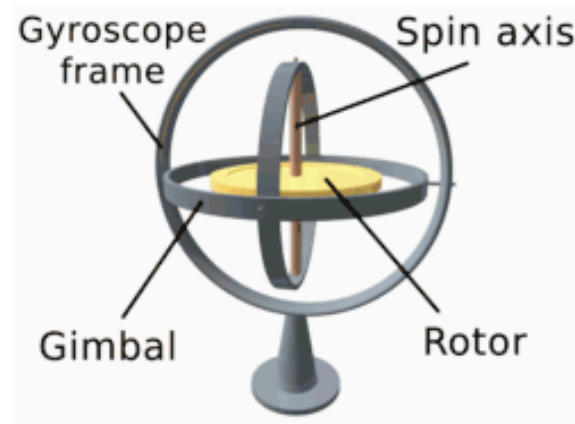

Cantilevers

Proof Mass

Khir MH, Qu P, Qu H - Sensors (Basel) (2011)
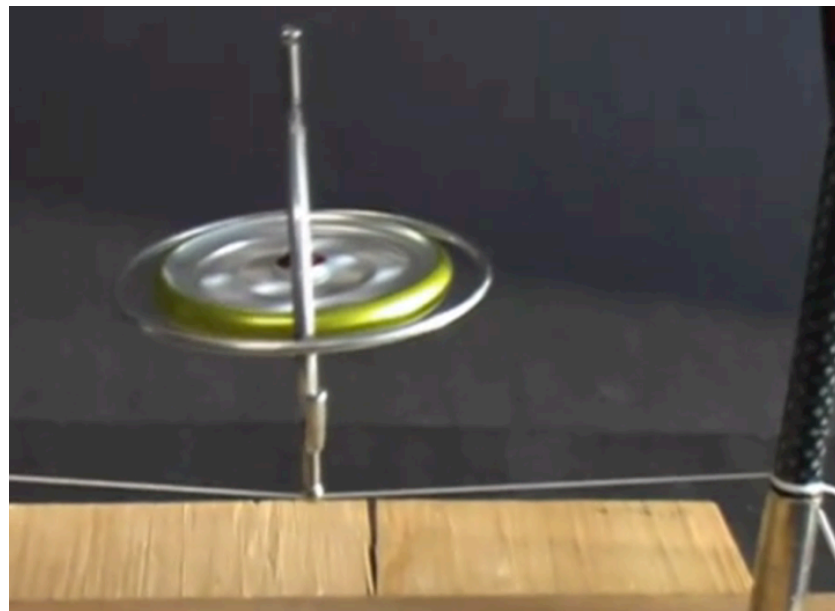
Sensors

- ■ Gyroscopes (original)
    - ❑ Mounted on two nested gimbals, the spinning wheel of the gyroscope is free to take any orientation.

    - ❑ High spin rate leads to high angular momentum

Sensors

http://www.youtube.com/watch?v=cquvA_IpEsA

Sensors

- ## Gyroscopes (original)
  - The gyroscope resists any change of orientation caused by external torques, due to the principle of conservation of angular momentum.
  - The orientation of the gyroscope remains nearly fixed, regardless of any motion of the platform.
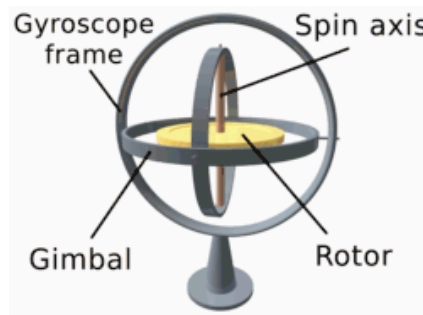
Sensors

- Gyroscopes (original)
    - Gyroscope orientation can be obtained by measuring the angles between adjacent gimbals.
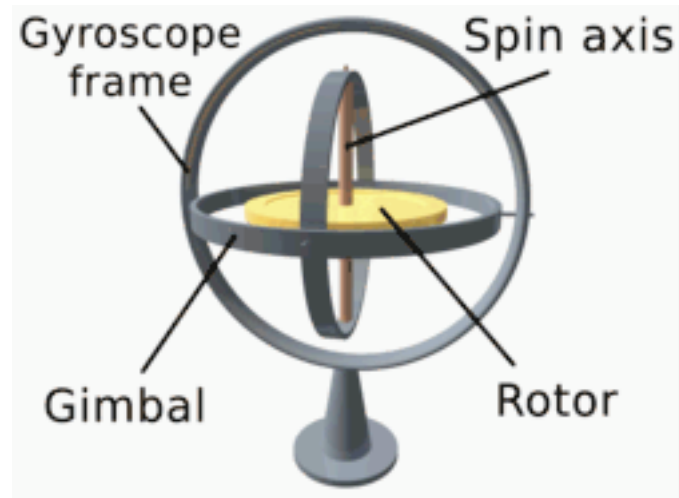
Sensors

- MEMS (Microelectromechanical Systems) Gyroscopes
  - Two masses oscillate back and forth from the center of rotation with velocity v.
  - A rotation will cause a Coriolis force in this coordinate frame.
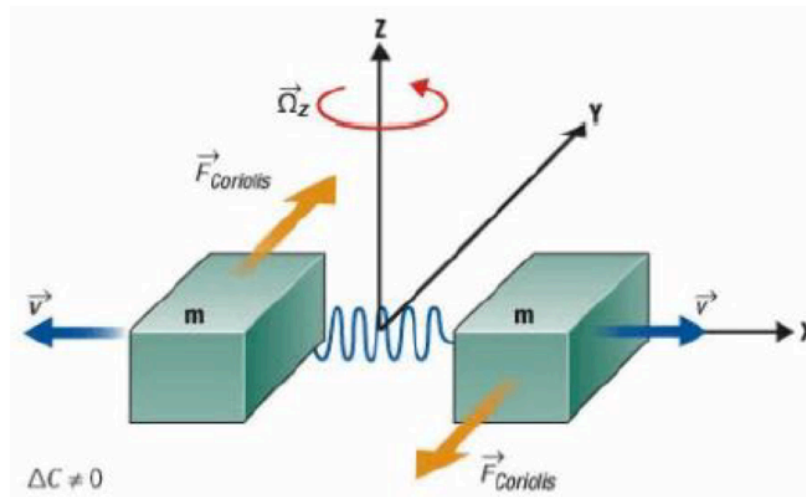
Sensors

- **MEMS Gyroscopes**
  - Their deflection y is measured, to establish a force

  $$F_{Coriolis} = ky$$

  - The acceleration is obtained since the mass is known

  $$-2m\ |\boldsymbol{\Omega} \times \boldsymbol{v}| = F_{Coriolis}$$

Sensors

- Inertial Measurement Units
    - 3 Accelerometers
    - 3 Gyroscopes
    - 3 Magnetometers(?)
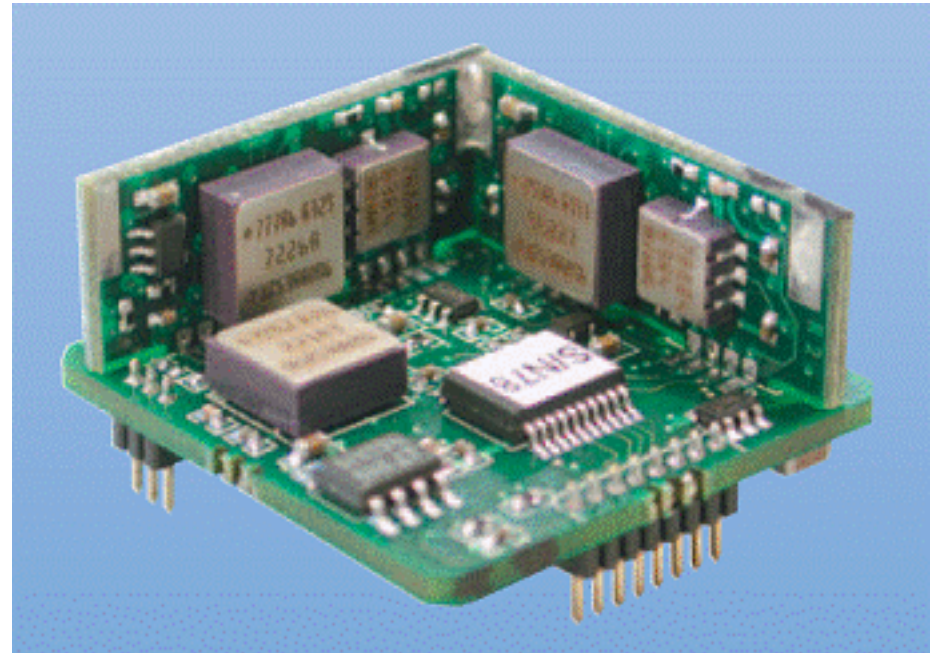
Sensors

- Inertial Measurement Units
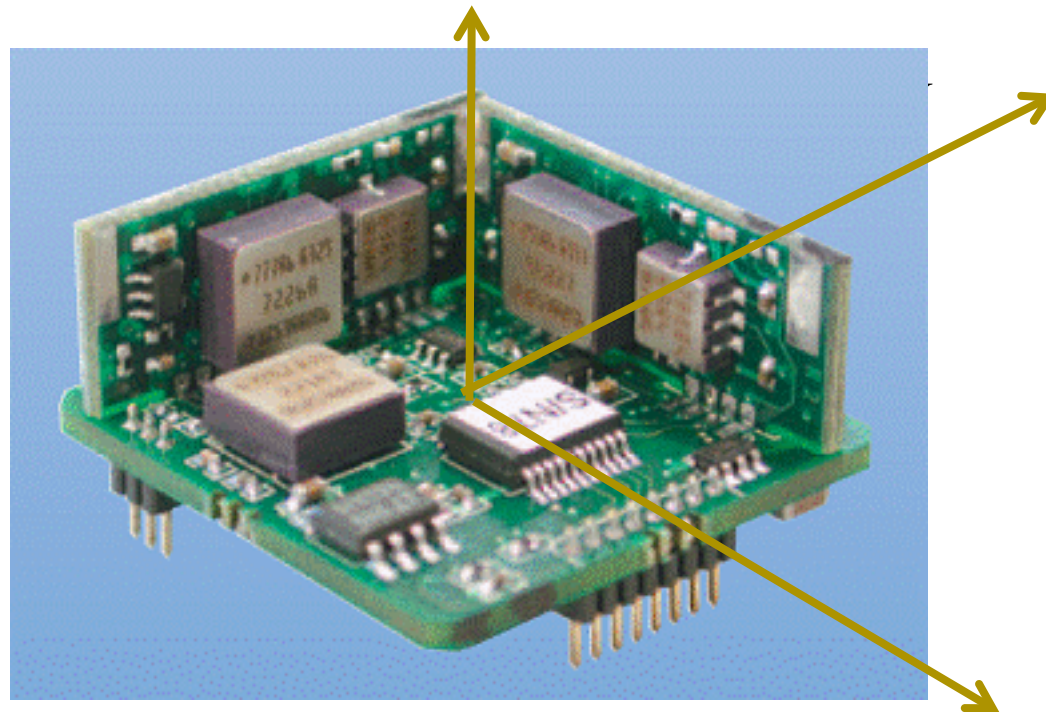
Outline

- Sensors
- Representations
  - Cartesian Coordinate Frames
  - Transformations
- State Prediction
- Example Systems
- Bounding with KF

Representations

- Cartesian Coordinate Frames
  - We can represent the 3D position of a vehicle with the vector

    $$\mathbf{r} = [\, x\, y\, z\,]$$
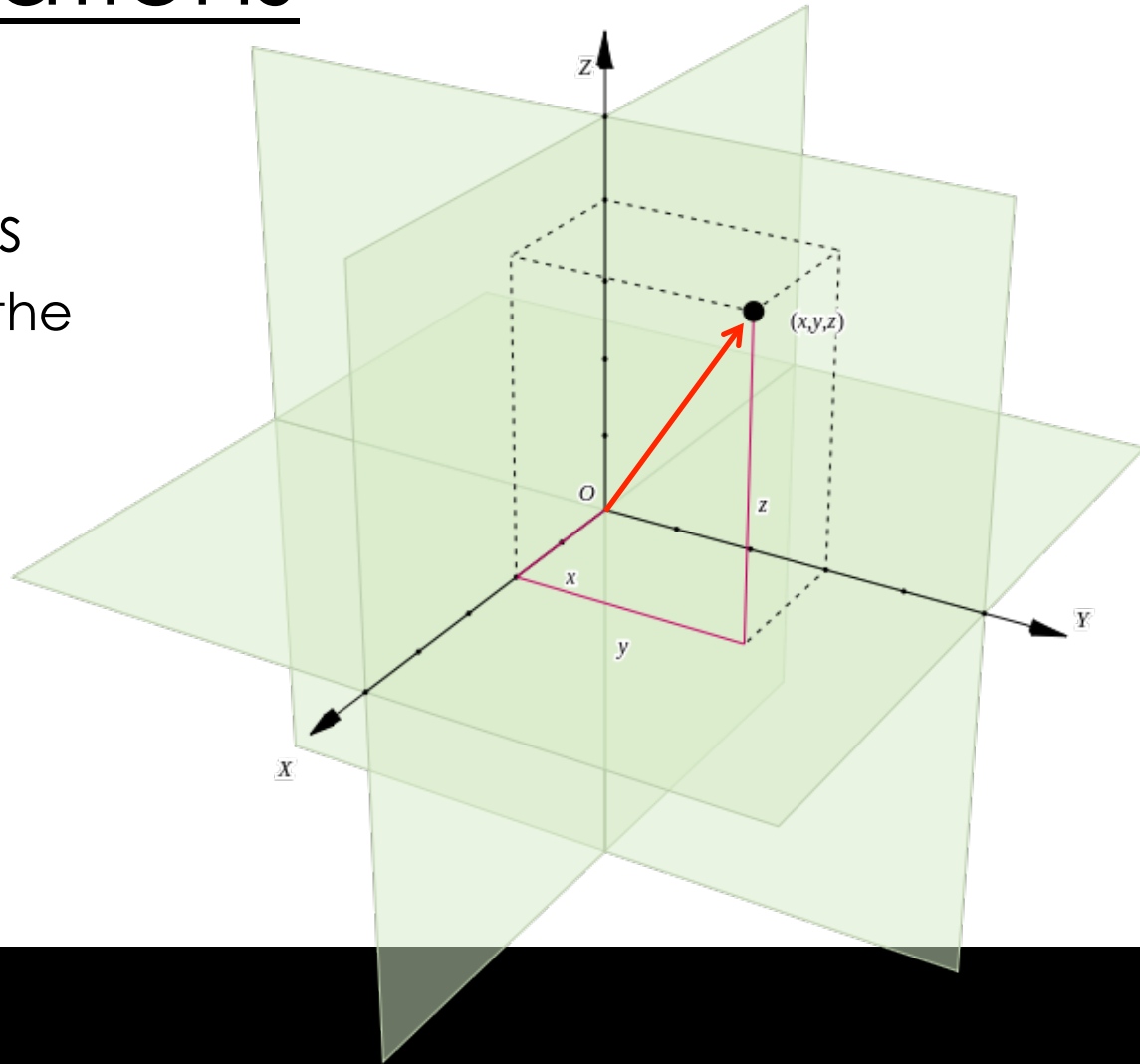
Representations

- Euler Angles
  - We can represent the 3D orientation of a vehicle with the vector

    $$\phi = [\ \alpha\ \beta\ \gamma\ ]$$

    Roll - α
    Pitch - β
    Yaw - γ
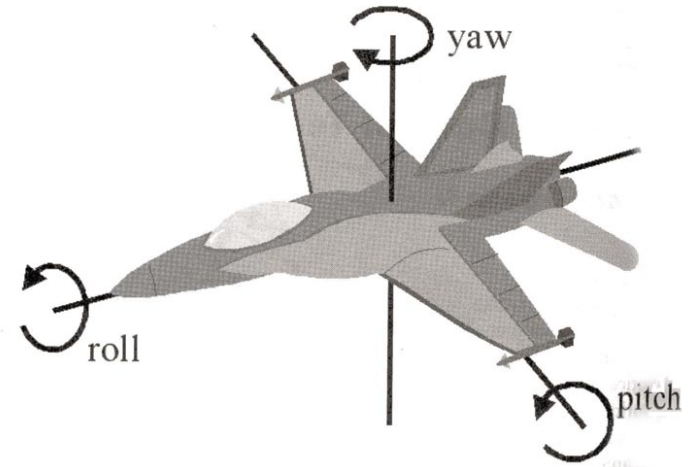
Representations

- Where do we place the origin?

    - We can fix the origin at a specific location on earth, e.g. a rocket's launch pad.

    - This is called the **global** or inertial coordinate frame

Z

$\mathbf{r} = [x\ y\ z]$

Y

X
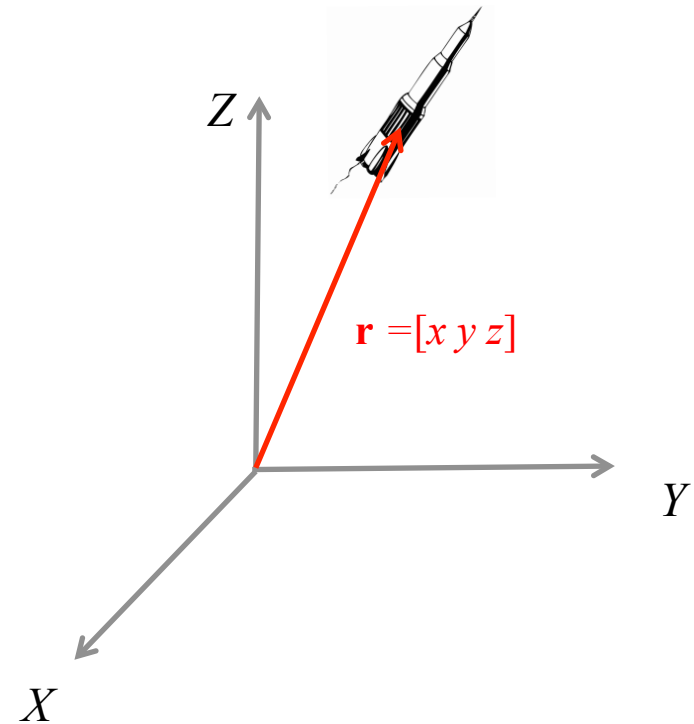
Representations

- Where do we place the origin?
    - We can ALSO fix the origin on a vehicle.
    - This is called the **local** coordinate frame

$\mathbf{r} = [x\ y\ z]$
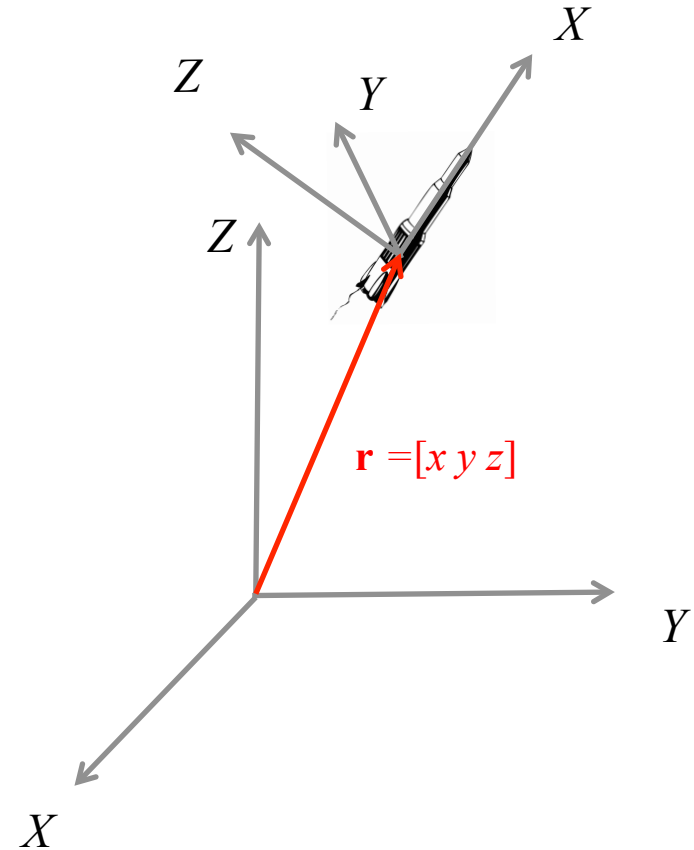
Representations

- Where do we place the origin?
    - We must differentiate between these two frames.
    - What is the real difference between these two frames?
    - A Transformation consisting of a **rotation** and **translation**

Z_L Y_L X_L

Z_G

$\mathbf{r} = [x\,y\,z]$

X_G Y_G
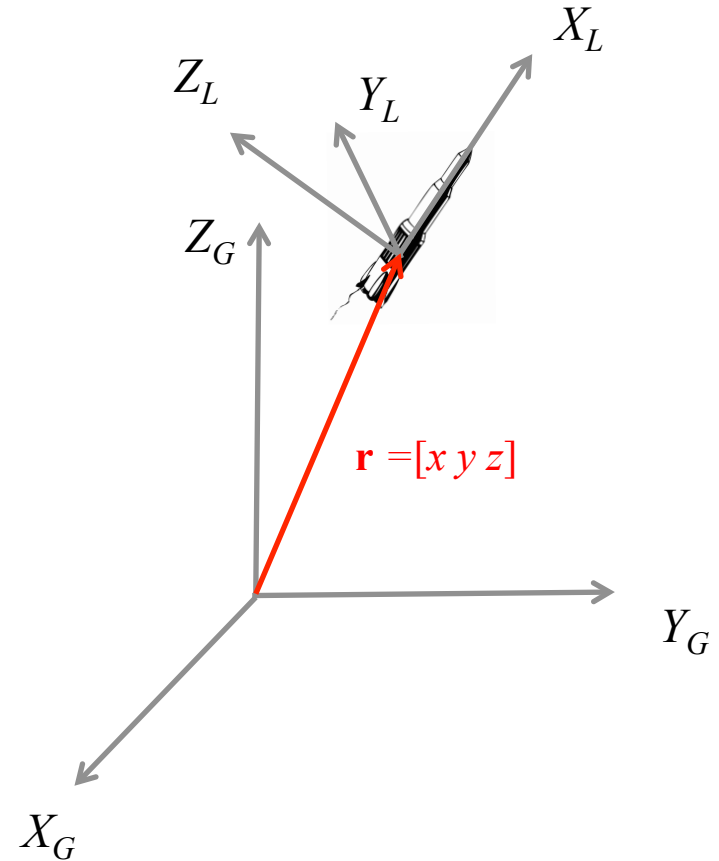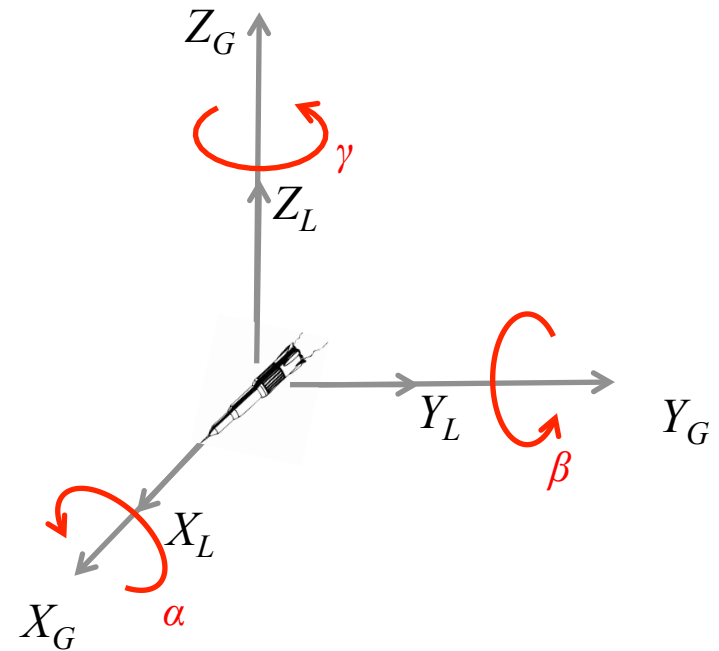
Representations

- Transformations
    - The **rotation** can be about 3 axes (i.e. the roll, pitch, yaw)

Representations

- Transformations
  - The **rotation** can be about 3 axes (i.e. the roll, pitch, yaw)

Z_G

X_L

Z_L

Y_L

Y_G

X_G
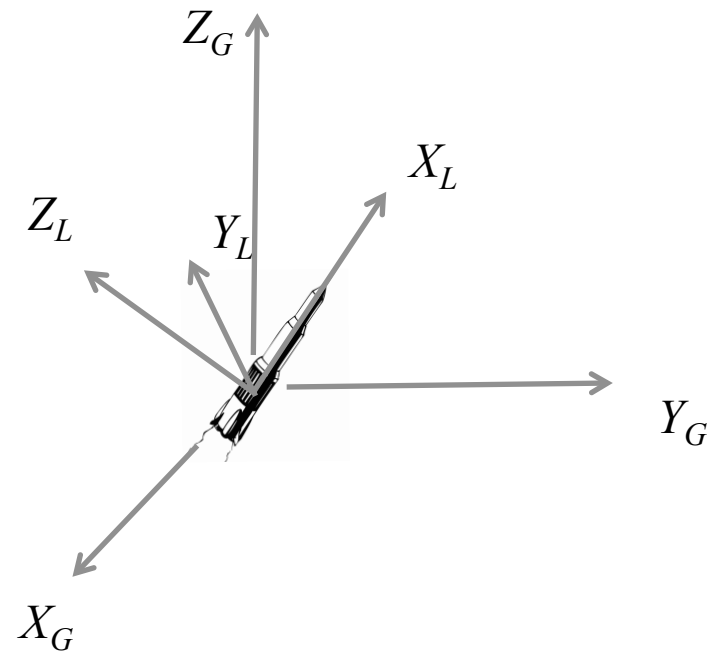
Representations

- Transformations
  - The **translation** can be in three directions

Z_L

Y_L

X_L

Z_G

$\mathbf{r} = [x\ y\ z]$

Y_G

X_G
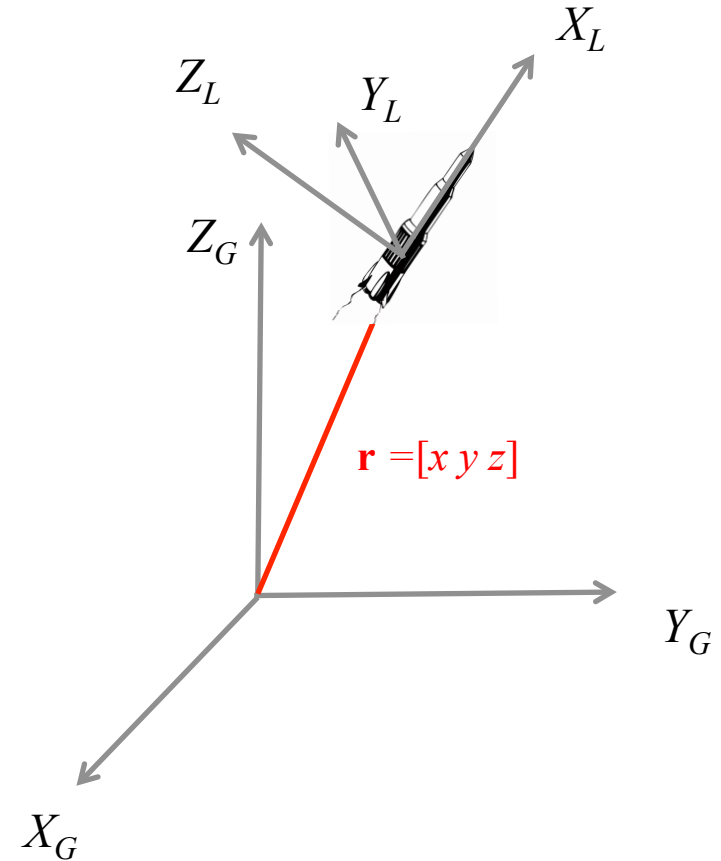
Representations

- **Rotations**
  - In 2D, it is easy to determine the effects of rotation on a vector

$$\mathbf{q}_2 = \begin{pmatrix} cos(\alpha) & -sin(\alpha) \\ sin(\alpha) & cos(\alpha) \end{pmatrix} \mathbf{q}_1$$

$$= \mathbf{R}(\alpha) \, \mathbf{q}_1$$
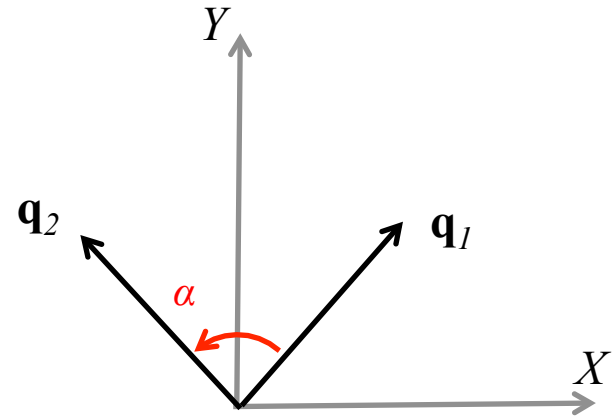
Representations

- Rotations
  - In 3D, we can use similar rotation matrices

$$\mathbf{q}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos(\alpha) & -sin(\alpha) \\ 0 & sin(\alpha) & cos(\alpha) \end{pmatrix} \mathbf{q}_1$$

$$= \mathbf{R_x}(\alpha)\ \mathbf{q}_1$$
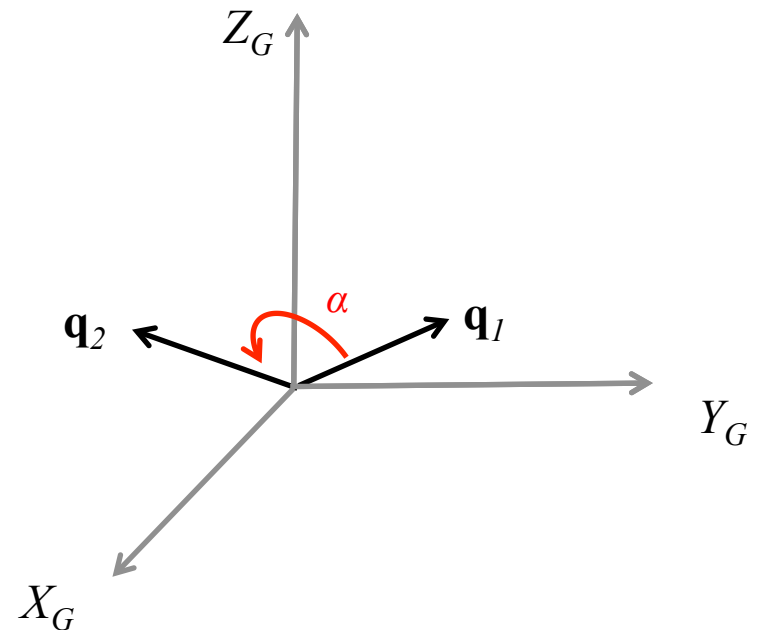
Representations

$$\mathbf{R_x}(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos(\alpha) & -sin(\alpha) \\ 0 & sin(\alpha) & cos(\alpha) \end{pmatrix}$$

$$\mathbf{R_y}(\beta) = \begin{pmatrix} cos(\beta) & 0 & sin(\beta) \\ 0 & 1 & 0 \\ -sin(\beta) & 0 & cos(\beta) \end{pmatrix}$$

$$\mathbf{R_z}(\gamma) = \begin{pmatrix} cos(\gamma) & -sin(\gamma) & 0 \\ sin(\gamma) & cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Representations

- Rotations
  - For 3 rotations, we can write a general Rotation Matrix

$$\mathbf{R}(\alpha,\,\beta,\,\gamma) = \mathbf{R_x}(\alpha)\,\mathbf{R_x}(\beta)\,\mathbf{R_x}(\gamma)$$

  - Hence, we can rotate any vector with the general Rotation Matrix

$$\mathbf{q}_2 = \mathbf{R}(\alpha,\,\beta,\,\gamma)\,\mathbf{q}_1$$

Outline

- Sensors
- Representations
- State Prediction
  - Updating $\mathbf{R}(t)$
  - Updating $\mathbf{r}(t)$
- Example Systems
- Bounding Errors with KF

State Prediction

- **Strapdown Inertial Navigation**
  - Our IMU is fixed to the **local** frame
  - We care about the state of the vehicle in the **global** frame

X_L

Z_L

Y_L

Z_G

IMU

$\mathbf{r} = [x\, y\, z]$

Y_G

X_G
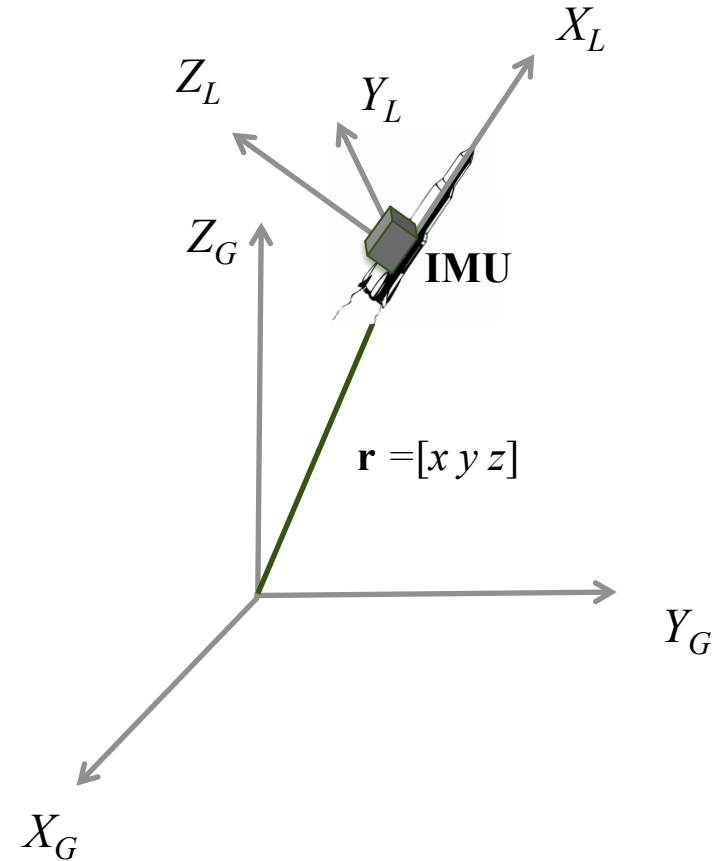
Updating $\mathbf{R}(t)$

Given: $\qquad \boldsymbol{\omega}_L(t) = [\ \omega_{x,L}(t)\ \omega_{y,L}(t)\ \omega_{z,L}(t)\ ]$

Find: $\qquad \mathbf{R}(t)$

Updating $\mathbf{R}(t)$

- Lets define the rotational velocity matrix based on our gyroscope measurements $\boldsymbol{\omega_L} = [\omega_{x,L}(t)\ \omega_{y,L}(t)\ \omega_{z,L}(t)]$

$$\Omega(t) = \begin{pmatrix} 0 & -\omega_z(t) & \omega_y(t) \\ \omega_z(t) & 0 & -\omega_x(t) \\ -\omega_y(t) & \omega_x(t) & 0 \end{pmatrix}$$

Updating $\mathbf{R}(t)$

- It can be shown that the vehicle rotating with velocity $\Omega(t)$ for δt **seconds** will (approximately) yield the resulting rotation Matrix $\mathbf{R}(t+\delta t)$:

$$\mathbf{R}(t+\delta t) = \mathbf{R}(t) \, [\, \mathbf{I} + \Omega(t)\delta t \,]$$

Updating $\mathbf{r}(t)$
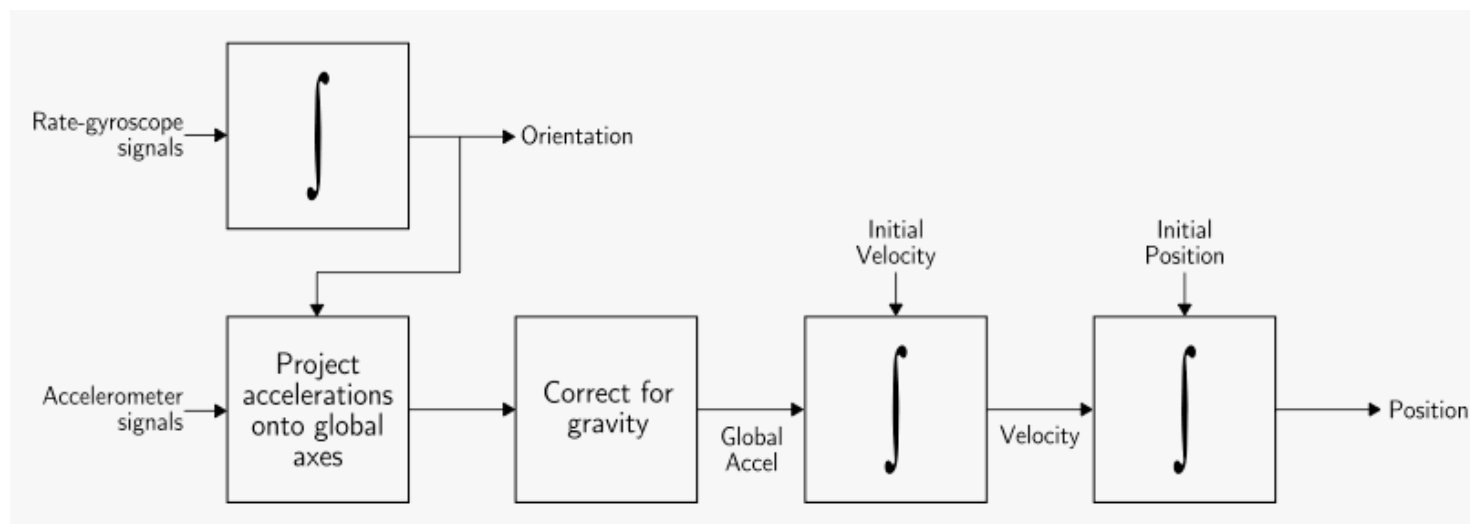
Given: $\mathbf{a}_L = [\ a_{x,L}\ a_{y,L}\ a_{z,L}\ ]$
$\mathbf{R}(t)$

Find: $\mathbf{r}_G = [\ a_{x,L}\ a_{y,L}\ a_{z,L}\ ]$

Updating $\mathbf{r}(t)$

Updating **r**(t)

- First, convert to global reference frame

$$\mathbf{a}_G(t) = \mathbf{R}(t)\ \mathbf{a}_L(t)$$

- Second, remove gravity term

$$\mathbf{a}_G(t) = [\ a_{x,G}(t) \quad a_{y,G}(t) \quad a_{z,G}(t)\text{-}g\ ]$$

Updating $\mathbf{r}(t)$

- Third, integrate to obtain velocity

$$\mathbf{v}_G(t) = \mathbf{v}_G(0) + \int_0^t \mathbf{a}_G(\tau)\, d\tau$$

- Fourth, integrate to obtain position

$$\mathbf{r}_G(t) = \mathbf{r}_G(0) + \int_0^t \mathbf{v}_G(\tau)\, d\tau$$

Updating $\mathbf{r}$*(t)*

- Third, integrate to obtain (approximate) velocity

$$\mathbf{v}_G(t+\delta t) = \mathbf{v}_G(t) + \mathbf{a}_G(t+\delta t)\ \delta t$$

- Fourth, integrate to obtain (approximate) position

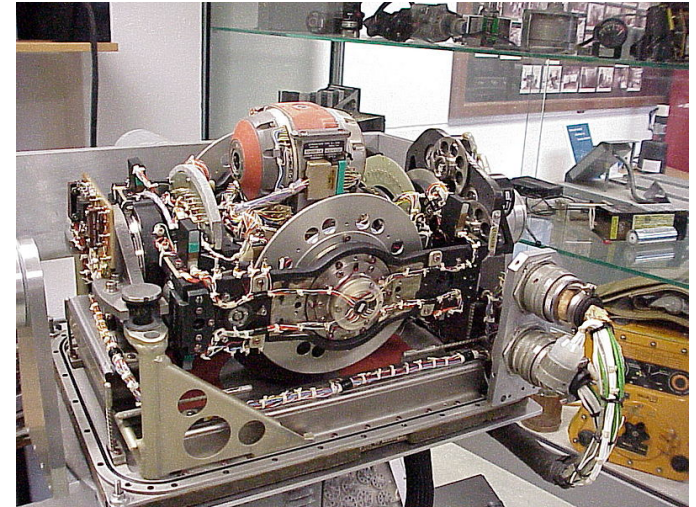$$\mathbf{r}_G(t+\delta t) = \mathbf{r}_G(t) + \mathbf{v}_G(t+\delta t)\ \delta t$$

Outline

- Sensors
- Representations
- State Prediction
- Example Systems
    - LN-3
    - The Jaguar
- Bounding Errors with KF

Example Systems

- LN-3 Inertial Navigation System
  - Developed in 1960's
  - Used gyros to help steady the platform
  - Accelerometers on the platform were used to obtain accelerations in global coordinate frame
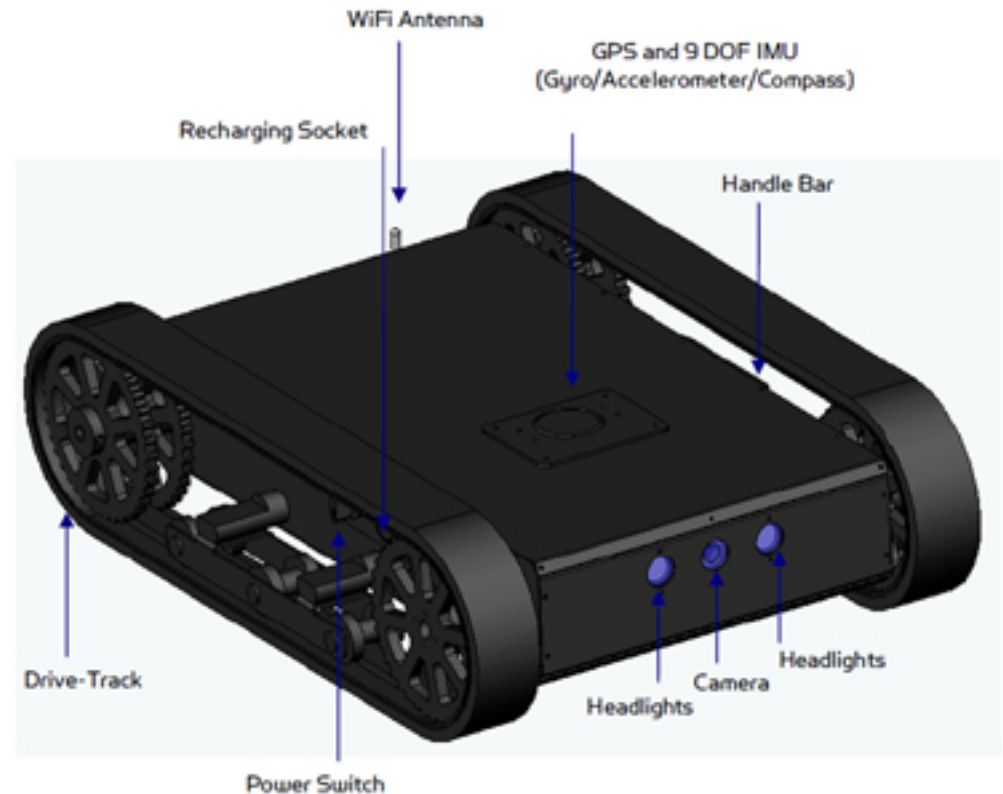 - Accelerations (double) integrated to obtain position

E80
Experimental Engineering

Example Systems

Litton LN3-2A Inertial Navigator Platform

0:06 / 1:19

- http://www.youtube.com/watch?feature=player_embedded&v=ePbr_4yMehs

Example Systems

- The Jaguar Lite
  - Equipped with an **IMU**, camera, laser scanner, encoders, GPS

WiFi Antenna

GPS and 9 DOF IMU
(Gyro/Accelerometer/Compass)

Recharging Socket

Handle Bar

Drive-Track

Headlights

Camera

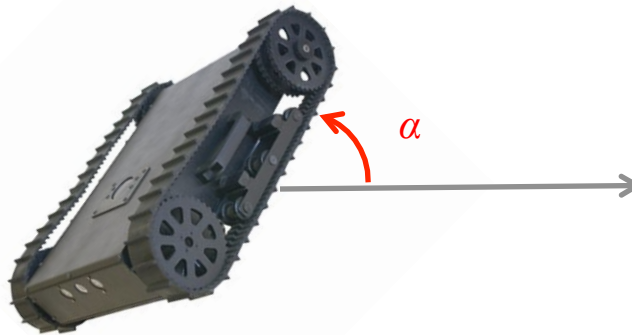Headlights

Power Switch

Example Systems

- ## The Jaguar Lite
  - ### GUI provides acceleration measurements
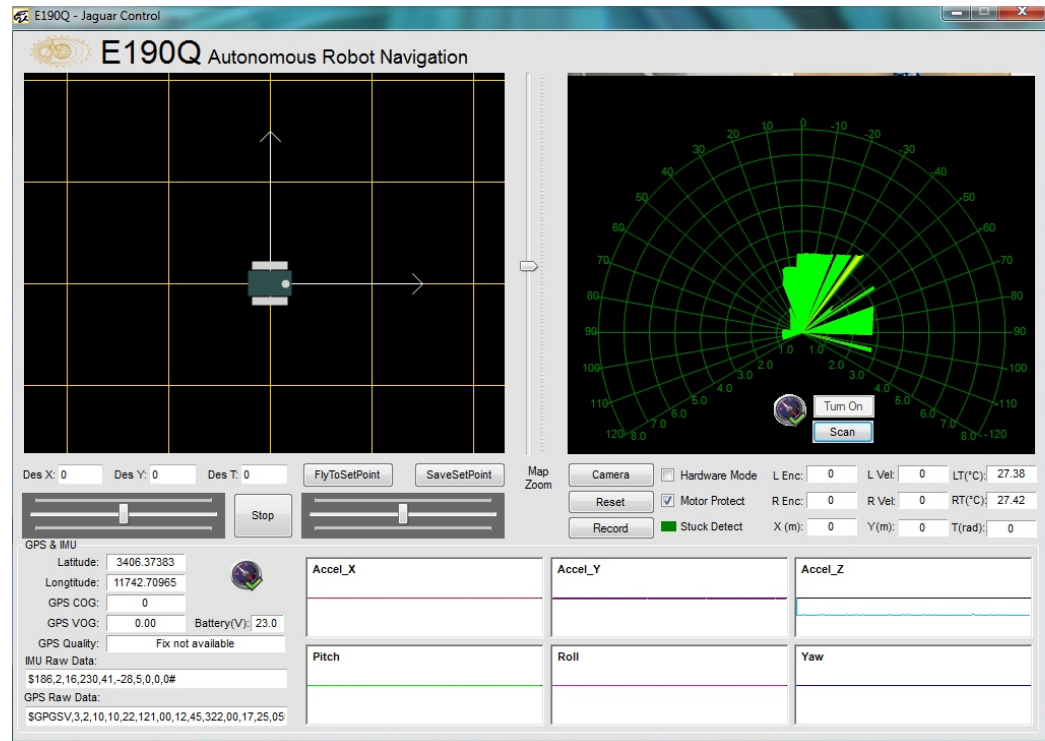
Example Systems

- Question:
  - Can we use the accelerometers alone to measure orientation?

α

Outline

- Sensors
- Representations
- State Prediction
- Example Systems
- Bounding Errors with KF
    - Exteroceptive Sensing
    - Fusing measurements

Bounding our Errors

- Exteroceptive sensors
    - Drift in inertial navigation is a problem
    - We often use exteroceptive sensors – which measure outside the robot – to bound our errors
    - Examples include vision systems, GPS, range finders
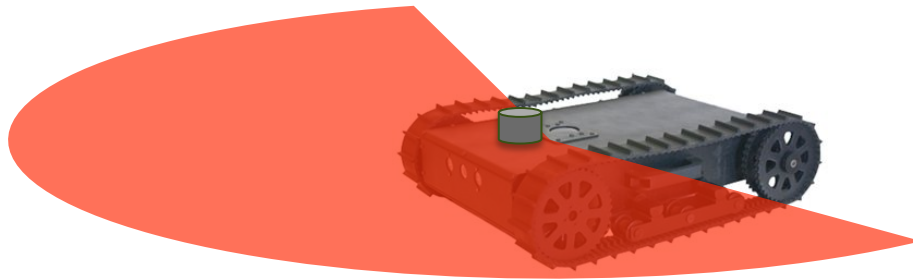
Bounding our Errors

- Exteroceptive sensors
  - We can fuse measurements, e.g. integrated accelerometer measurements and range measurements, by averaging.
  - For example, consider the 1D position estimate of the jaguar.

$$x = 0.5\ (\ x_{IMU} + (x_{wall} - x_{laser})\ )$$

x_{IMU} is the double integrated IMU measurement
x_{wall} is the distance from the origin to the wall
x_{laser} is the central range measurement

Bounding our Errors

- Exteroceptive sensors
  - Lets weight the average, where weights reflect measurement confidence

$$x = \frac{w_{IMU}x_{IMU} + w_{laser}(x_{wall} - x_{laser})}{w_{IMU} + w_{laser}}$$

Bounding our Errors

- Exteroceptive sensors
  - This leads us to a 1D Kalman Filter

$$x_t = x_{IMU,t} + K_t [(x_{wall} - x_{laser,t}) - x_{IMU,t}]$$

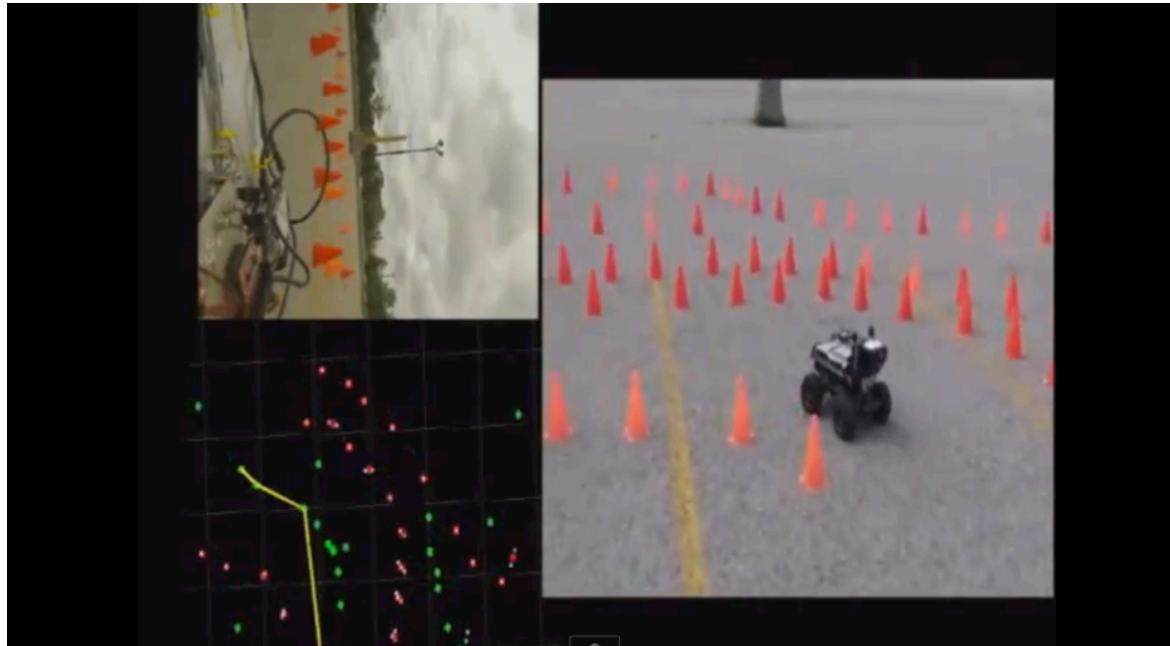$$K_t = \frac{\sigma^2_{IMU}}{\sigma^2_{IMU} + \sigma^2_{laser}}$$

$$\sigma^2_x = (1 - K_t) \sigma^2_{IMU}$$

Bounding our Errors

- IMU
  - Higher sampling rate
  - Small errors between time steps (maybe centimeters)
  - Uncertainty increases
  - Large build up over time (unbounded)
- GPS
  - Lower sampling rate
  - Larger errors(maybe meters)
  - Uncertainty decreases
  - No build up (bounded)

E80
Experimental Engineering

Bounding our Errors

http://www.youtube.com/watch?v=I_cCeGm4x4c