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# Pressure and Temperature Sensors

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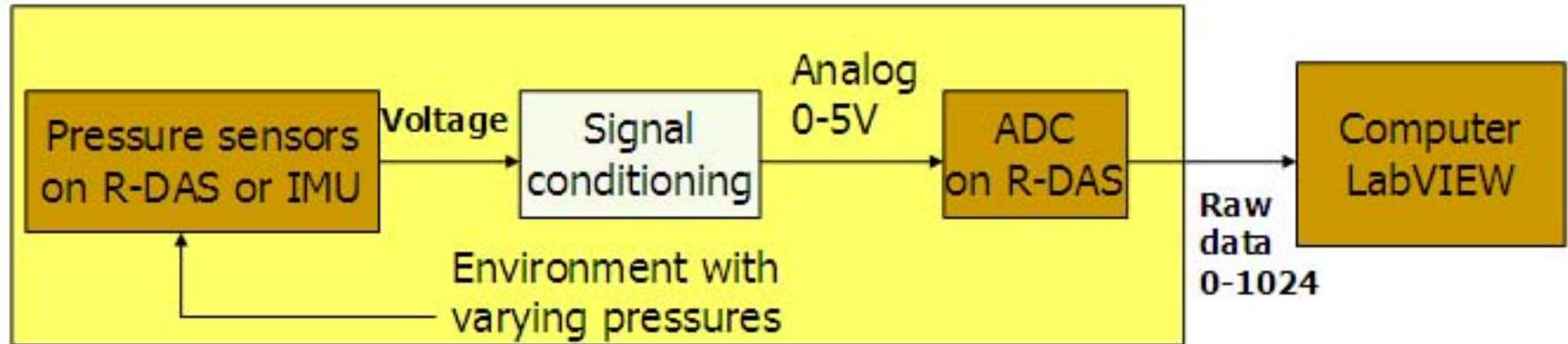
E80

Feb 17, 2009

# Agenda

- Pressure sensors and calibration
- Relating pressure to altitude
- Temperature sensors and calibration (Steinhart-Hart constants)

# Pressure Sensors

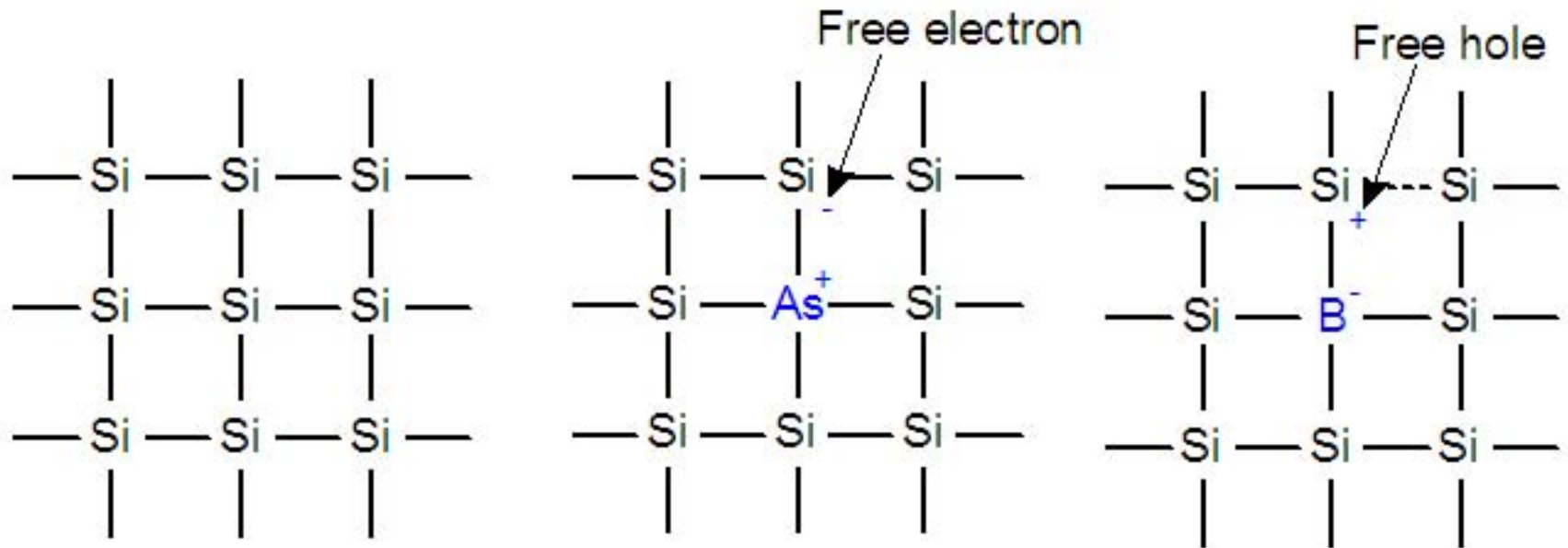


## ■ Why use pressure sensors?

→ Can figure out altitude from pressure

# How Pressure Sensors Work

Made from semiconductors



Silicon Lattice

n-doped Silicon

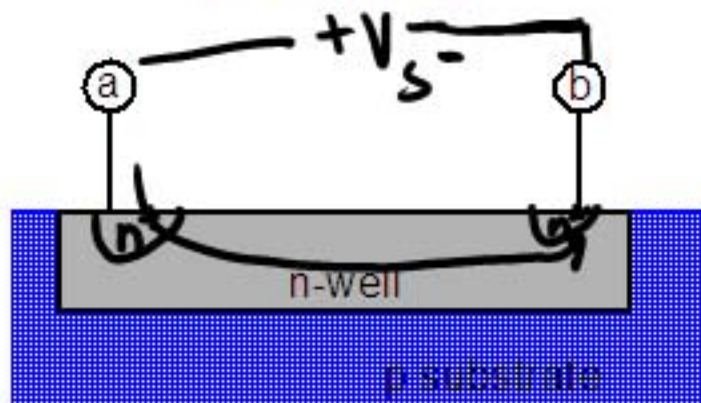
*negative charges  
floating around*

p-doped Silicon

*positive  
charges*

# How Pressure Sensors Work

Device

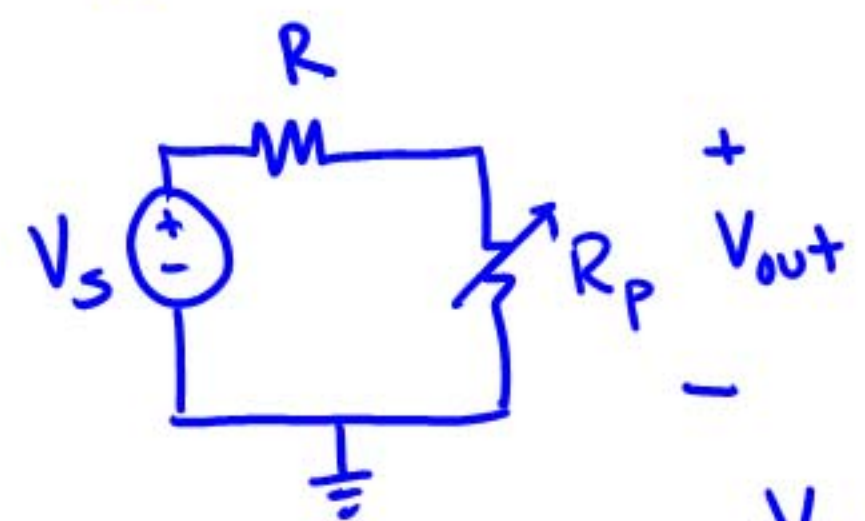


Symbol



# How do we measure resistance?

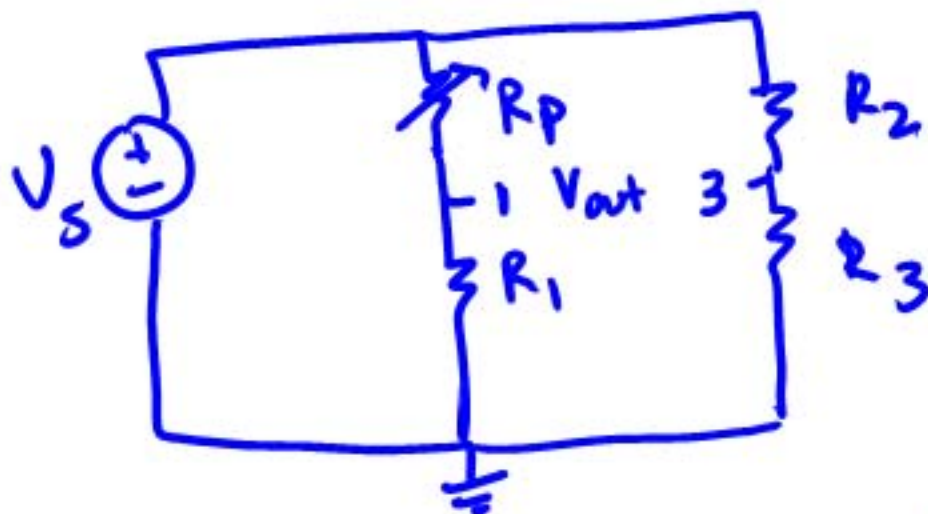
Voltage Divider:



$$V_{out} = \left[ \frac{R_p}{R_p + R} \right] V_s$$

# How do we measure resistance?

Wheatstone Bridge ckt



If all  $R$ 's equal,  
 $V_{out} = 0$

If  $R_2 = R_3$

$$V_{out} = \left[ \frac{R_1}{R_1 + R_P} - \frac{R_3}{2R_3} \right] V_S$$
$$\rightarrow R_P = R_1 \left[ \frac{V_S - 2V_{out}}{V_S + 2V_{out}} \right]$$

$$V_{out} = V_1 - V_3$$

$$= \left( \frac{R_1}{R_1 + R_P} \right) V_S - \left( \frac{R_3}{R_3 + R_2} \right) V_S$$

$$= \left[ \left( \frac{R_1}{R_1 + R_P} \right) - \left( \frac{R_3}{R_3 + R_2} \right) \right] V_S$$

# Pressure sensors

## MPX4115A(IMU) / MPXA6115A (R-DAS)

SMALL OUTLINE PACKAGE



MPXA4115A6U  
CASE 482



MPXA4115AC6U  
CASE 482A

### Features

- 1.5% Maximum Error over 0° to 85°C
- Ideally suited for Microprocessor or Microcontroller-Based Systems
- Temperature Compensated from -40° to +125°C
- Durable Epoxy Unibody Element or Thermoplastic (PPS) Surface Mount Package

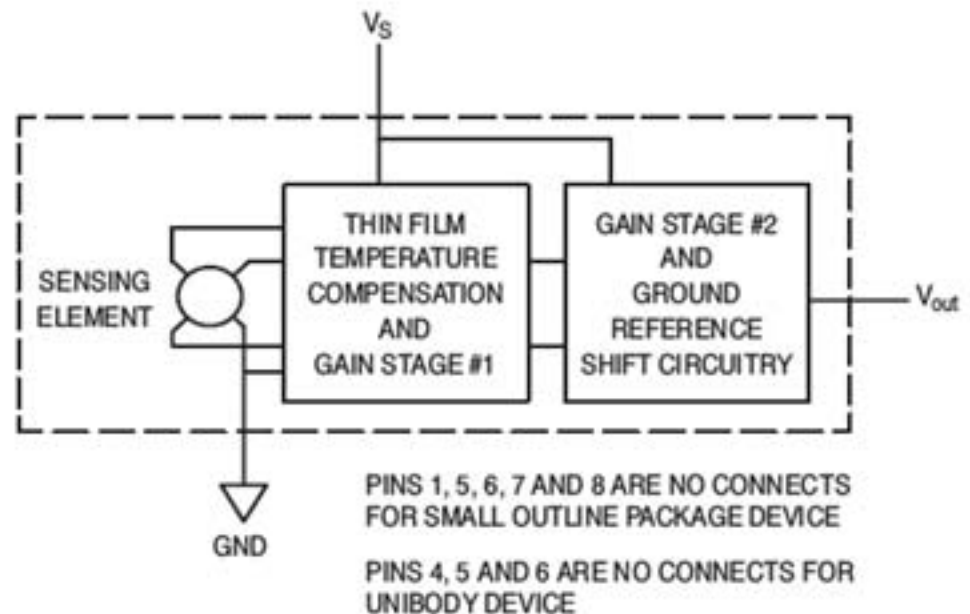


Figure 1. Fully Integrated Pressure Sensor Schematic

[http://www.freescale.com/files/sensors/doc/data\\_sheet/MPX4115A.pdf?pspll=1](http://www.freescale.com/files/sensors/doc/data_sheet/MPX4115A.pdf?pspll=1)

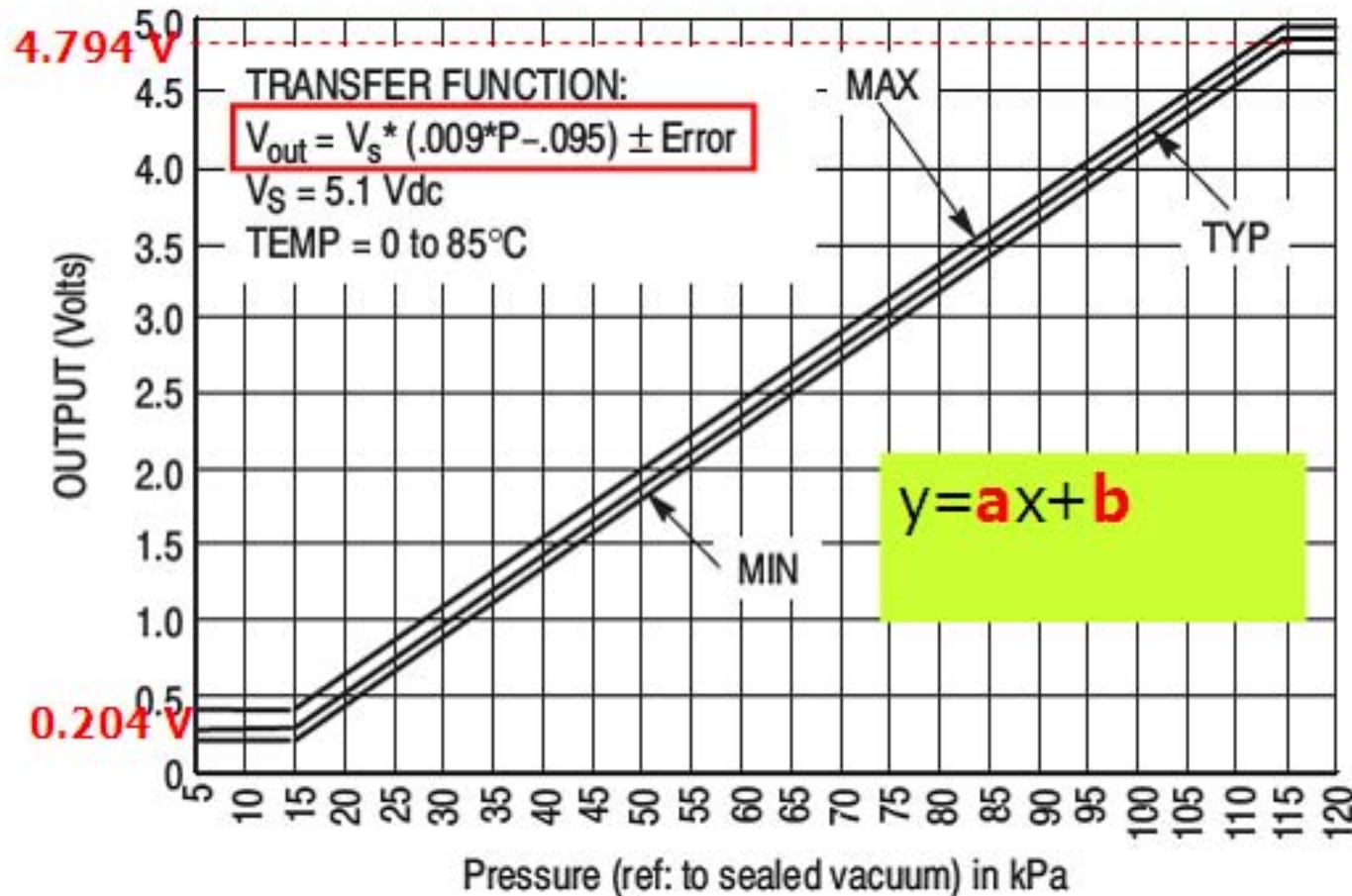
<http://www.eng.hmc.edu/NewE80/PDFs/MPXA6115A.pdf>



# Pressure sensors-MPX4115A

- **Pressure range:** 15-115 kPa
- **Sensitivity:** 45.9mV/kPa
- **Supply voltage:** 5V
- **Output analog voltage:**
  - Offset voltage ( $V_{off}$ ): output voltage at minimum rated pressure (Typical@ 0.204V)
  - Full scale output ( $V_{fso}$ ): output voltage at maximum rated pressure (Typical@ 4.794 V)
- **Pressure units**
  - Pascal (Pa)=N/m<sup>2</sup>: standard atmosphere  
 $P_0=101325=101.325\text{kPa}$
  - Psi= (Force) pound per square inch: 1 Psi=6.89465 KPa

# How does voltage correlate to pressure?

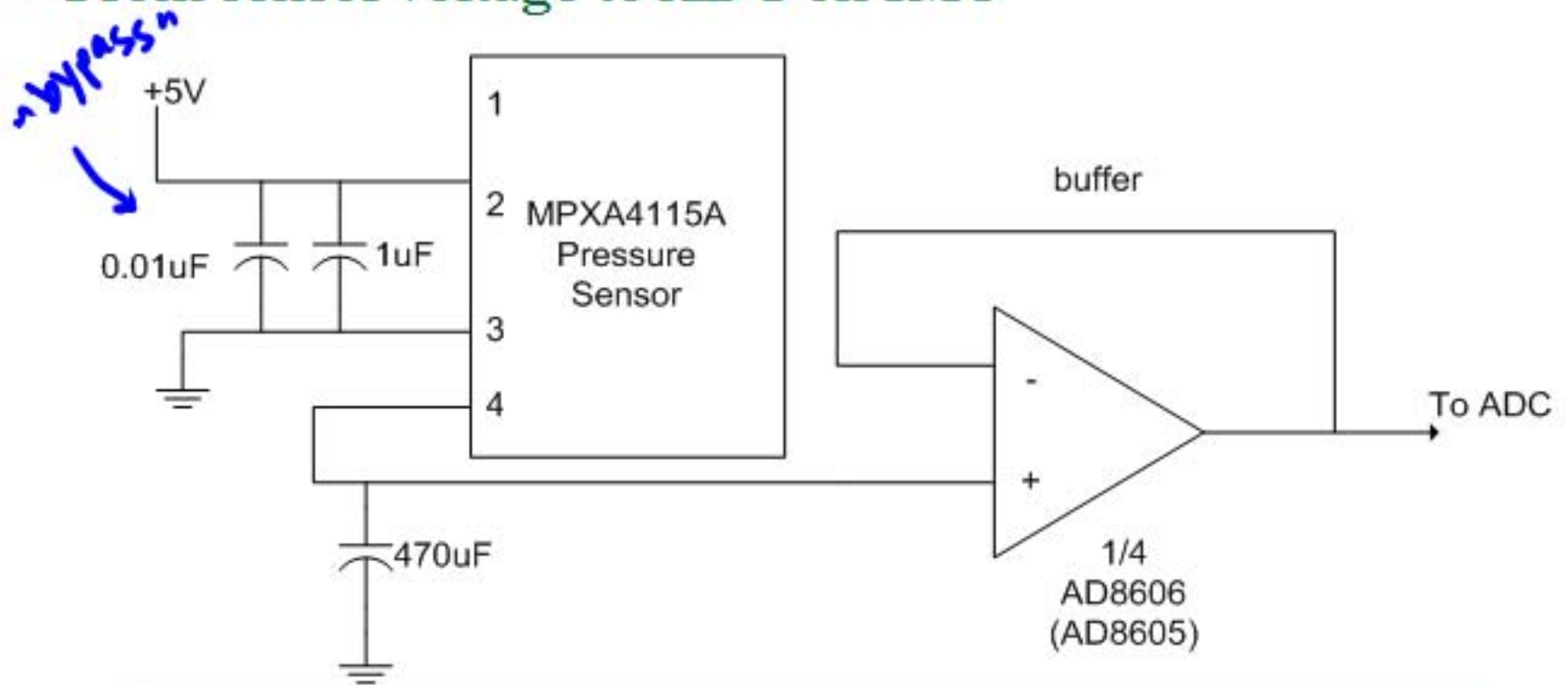


Manufacturer  
Specs

Figure 4. Output versus Absolute Pressure

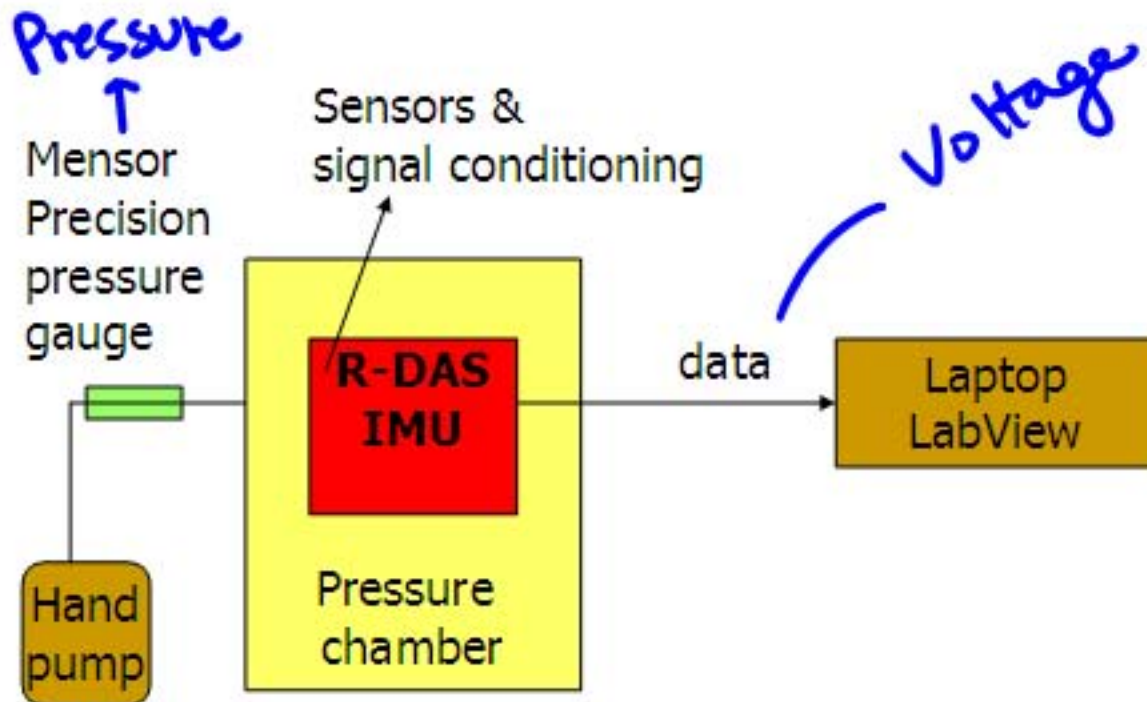
# Signal Conditioning Circuitry

- From sensor voltage to ADC on IMU



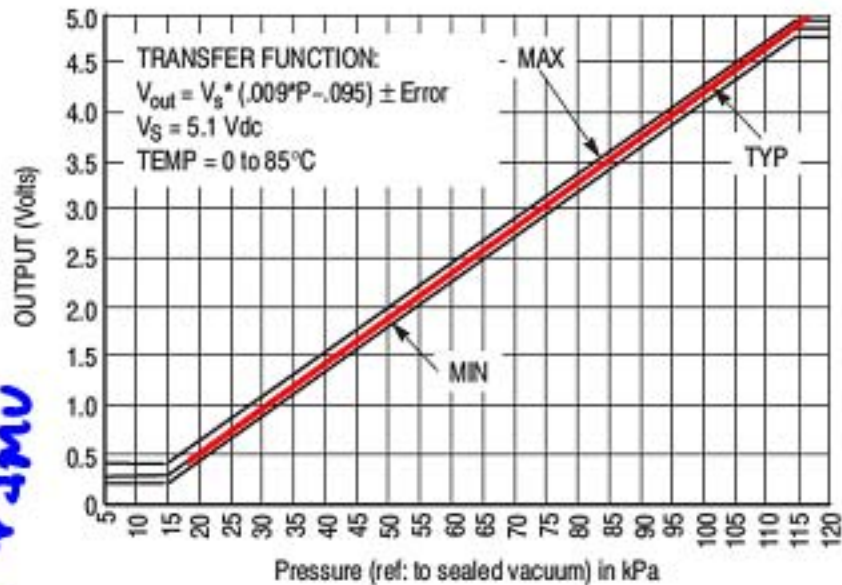
- 0.2-4.8V
- IMU  $R_{in} = 1k\Omega$ , so unity gain buffer for loading
- Low pass filter before ADC
- All power supplies bypassed to reduce noises

# Calibrate in the lab



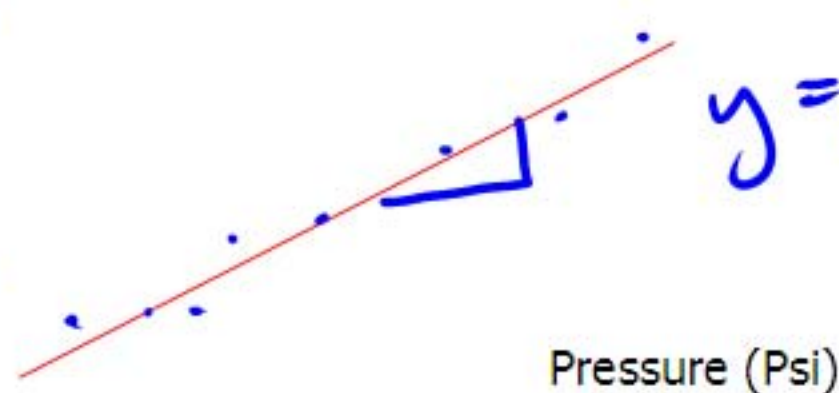
- Data (0-1024) → (0-5V)
- Pressure reading of Mensor is in units of psi

# Calibration curve options



R-DA1 v SMU

Digital reading



$y = ax + b$   
↑                    ↑  
Figure out these variables

from Mensor

# Error

Manufacturer

- Voltage Error = Pressure Error x Temperature Error Factor x 0.009 x Vs
- Temperature Error Factor=1 (0°C - 85°C)
- Pressure Error: +/- 1.5kPa

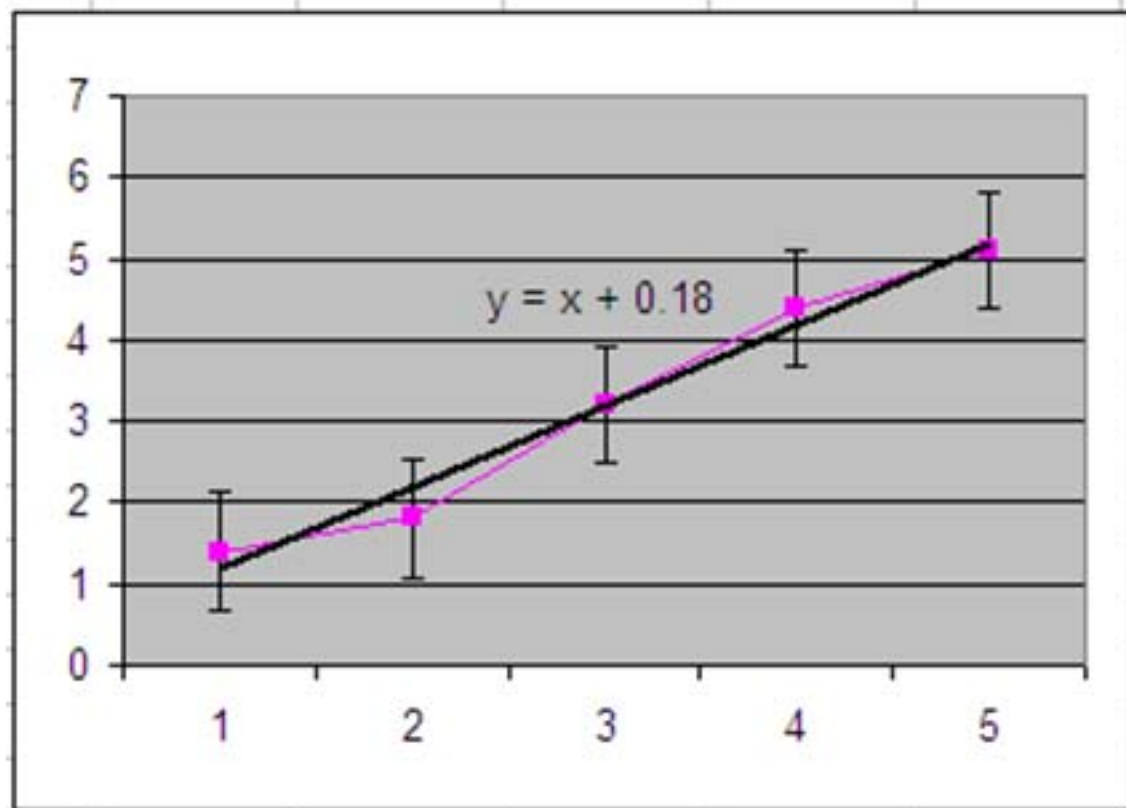
# Find $a$ and $b$ in calibration curve

$$y = ax + b$$

- Collect data sets  $(x_1, y_1)$   $(x_2, y_2)$ ..... $(x_n, y_n)$ ,  $n > 2$
- Best fit (regression or least square) line
- Excel, Matlab, KlaidaGraph, LabView

## Excel Example

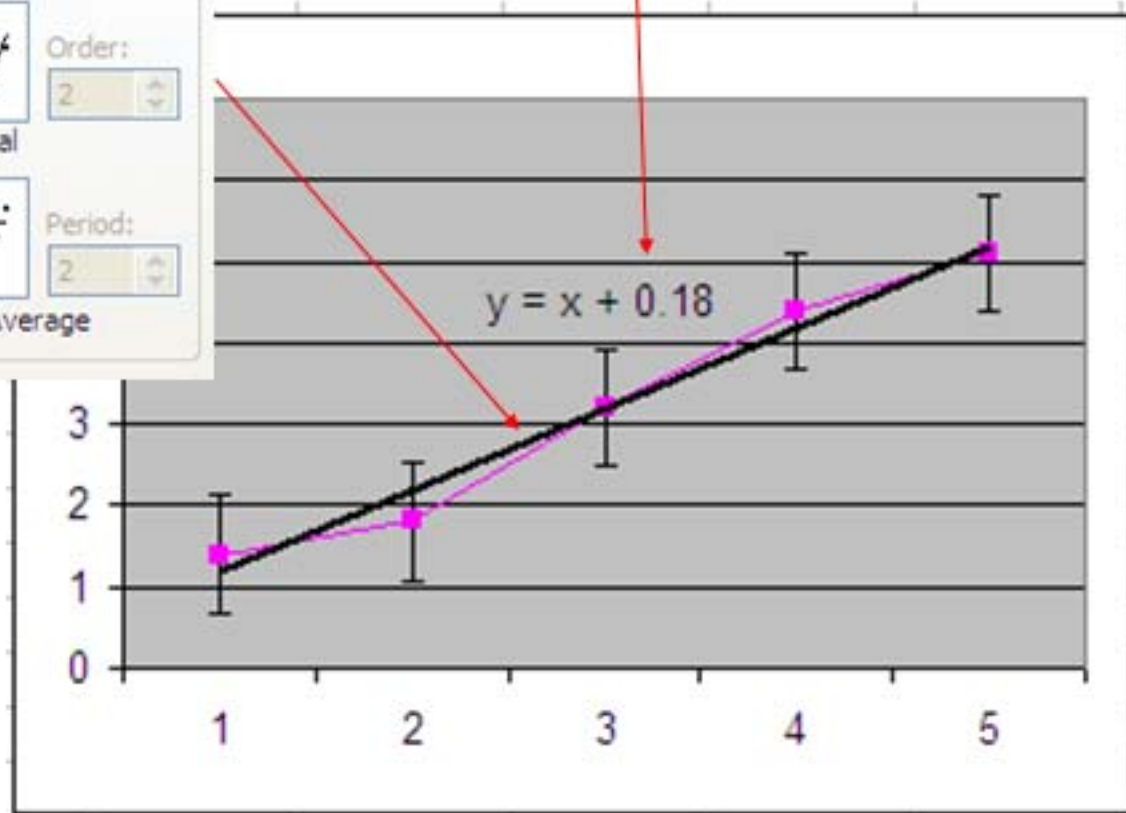
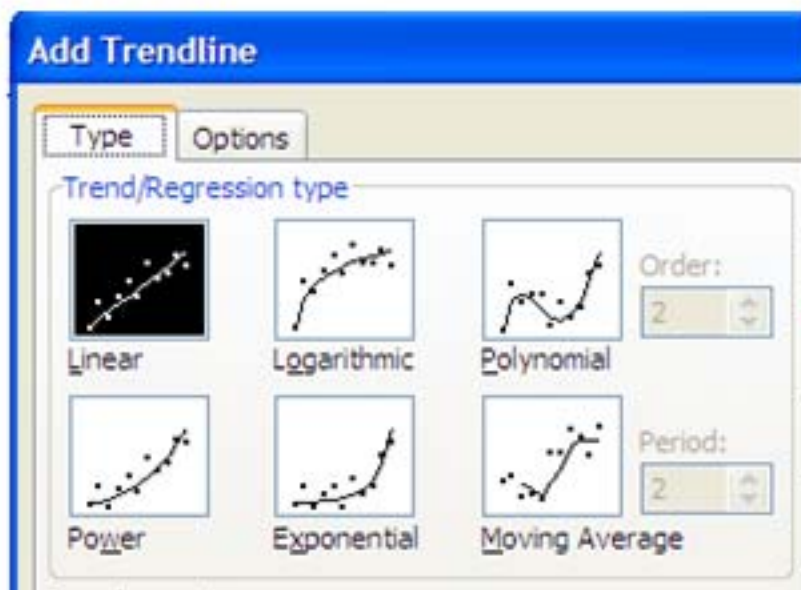
	A	B
1		
2		
3		
4	1	1.4
5	2	1.8
6	3	3.2
7	4	4.4
8	5	5.1
9		



# Adding Trendlines

Chart -> Add Trendline

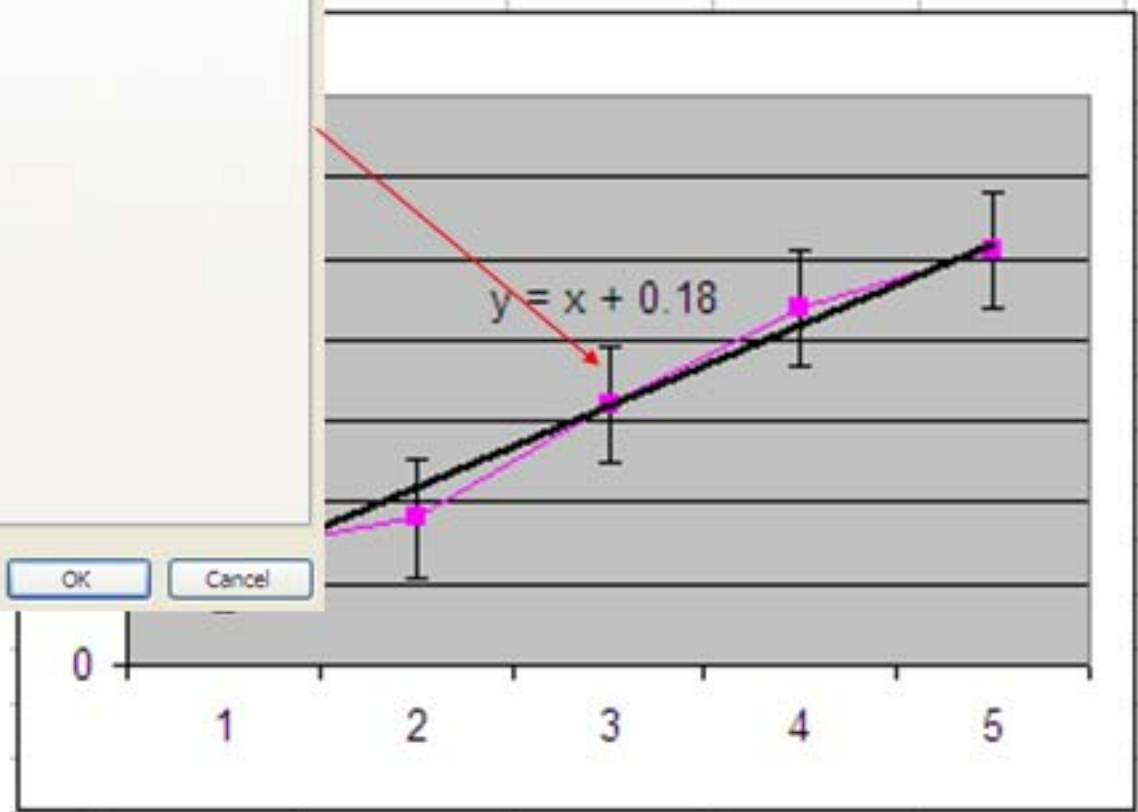
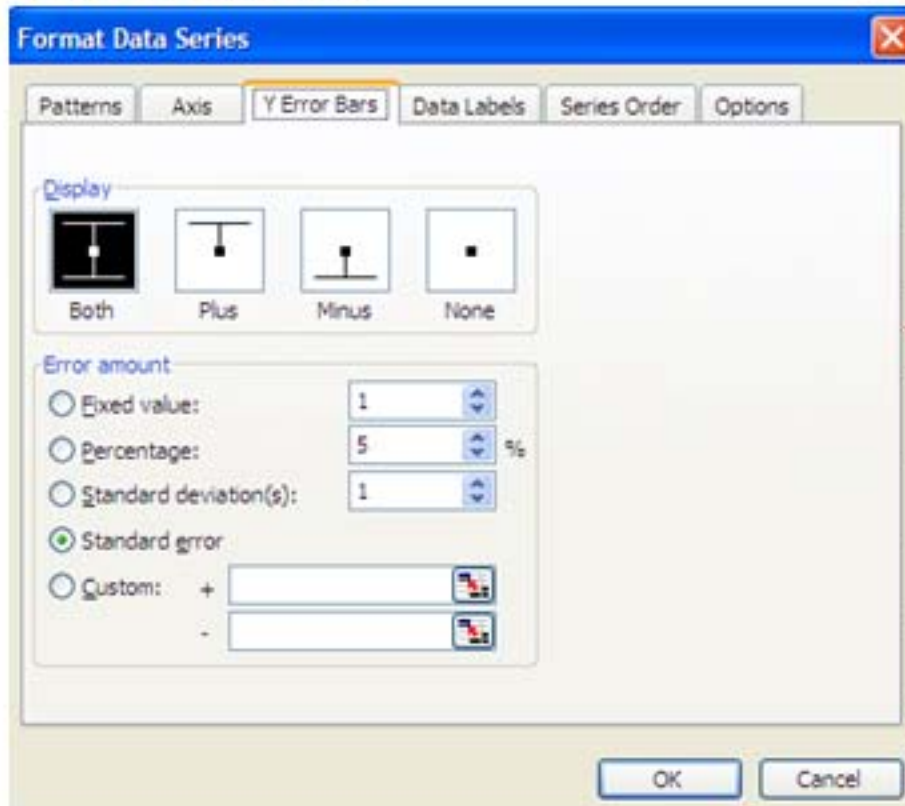
Options -> Display Equation on Chart





# Adding Standard Error

**Double-click on data series**



Can also find **a** and **b** by hand

$$\text{Slope : } a = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$
$$\text{Intercept : } b = \bar{y} - a\bar{x}$$

Can also find error bars by hand

$$S_{\varepsilon} = \sqrt{\frac{\sum_{i=1}^n \varepsilon_i^2}{N-2}}$$

$$S_b = S_{\varepsilon} \sqrt{\frac{1}{N} + \frac{\bar{x}^{-2}}{\sum_{i=1}^N (x_i - \bar{x})^2}}$$

$$S_a = S_{\varepsilon} \sqrt{\frac{1}{\sum_{i=1}^N (x_i - \bar{x})^2}}$$

$$\lambda_b = tS_b$$

$$\lambda_a = tS_a$$

# Example

	A	B
1		
2		
3		
4	1	1.4
5	2	1.8
6	3	3.2
7	4	4.4
8	5	5.1
9		

$$a = 1 \quad \checkmark$$

$$b = 0.18 \quad \checkmark$$

$$S_\varepsilon = 0.29$$

$$S_a = 0.09$$

$$S_b = 0.3$$

$$\lambda_a = 0.3 \quad (95\% \text{ confidence})$$

$$\lambda_b = 0.97 \quad (95\% \text{ confidence})$$

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# Example using Matlab

```
% matrix of x values
```

```
x = [1 2 3 4 5]';
```

```
% matrix of y (observed outputs)
```

```
Y = [1.4 1.8 3.2 4.4 5.1]';
```

```
X = [ones(size(x)) x]; % the first column of this matrix must be 1's
```

```
% Perform linear regression using least squares
```

```
% B: a matrix of the coefficients, B0 and B1 (b and a in previous example)
```

```
% BINT: a matrix of the confidence intervals for each coefficient
```

```
% R: a matrix of the residuals
```

```
% RINT: a matrix of the confidence intervals of the residuals, centered
```

```
% around each residual
```

```
[B, BINT, R, RINT] = regress(Y,X);
```

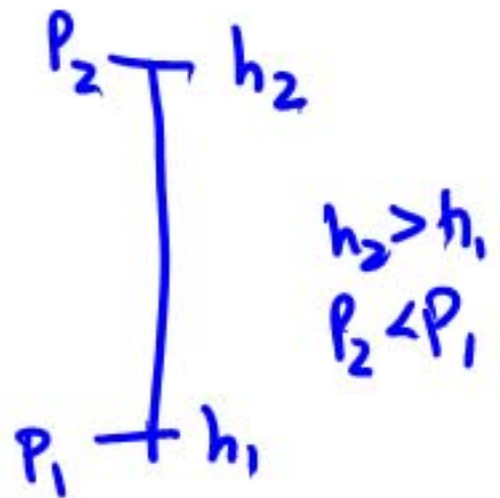
How does pressure (P) relate to altitude (h)?

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{m}{V/h} \cdot g = \underline{\rho gh}$$

Assume  $\rho$  is constant.

$$\Delta P = -\rho g \Delta h$$

$$P_2 - P_1 = -\rho g (h_2 - h_1)$$



How does pressure (P) relate to altitude (h)?

$\rho$  isn't constant. ✓

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{nM}{nRT/P}$$

$n = \# \text{ moles}$   
 $M = \text{molar mass}$   
(kg/mol)

$$= \frac{MP}{RT}$$

$$\Delta P = -\rho g \Delta h$$

$$= -\frac{MP}{RT} g \Delta h$$

$$\frac{\Delta P}{\Delta h} = -\frac{Mg}{\frac{RT}{A}} P(h)$$

$$\frac{dP(h)}{dh} = -A P(h)$$

$$P(h) = C e^{-Ah}$$

$$\text{@ } h=0 \quad P(h) = C = P_0$$

$$P(h) = P_0 e^{-\frac{Mgh}{RT}} \quad \checkmark$$

## How does pressure (P) relate to altitude (h)?

$$h = \frac{T_0}{-\left(\frac{dT}{dh}\right)} \cdot \left[ 1 - \left(\frac{P_0}{P}\right)^{\frac{\left(\frac{dT}{dh}\right)_R}{gM}} \right]$$

where

- $h$  = altitude (above sea level) (in meters)
- $P_0$  = standard atmosphere pressure = 101.325kPa
- $T_0$  = 288.15K (+15°C)
- $dT/dh = -0.0065$  K/m: thermal gradient or standard temperature lapse rate
- $R$  = gas constant (8.31432 N\*m/mol\*K)
- $g = (9.80665 \text{ m/s}^2)$
- $M$  = molar mass of earth's air (0.0289644 kg/mol )

Reference: (1976 US standard atmosphere)



How does pressure (P) relate to altitude (h)?

Plug in all the constants

$$h = 4.43 \times 10^4 \times \left( 1 - \left( \frac{101.325 \text{ kPa}}{P} \right)^{-0.1902} \right) \quad (1)$$

- h is measured **in meters**.
- Equation calibrated up to 36,090 feet (11,000m).
- Reference: [http://en.wikipedia.org/wiki/Atmospheric\\_pressure](http://en.wikipedia.org/wiki/Atmospheric_pressure)
- Different values of dT/dh for different layers of the atmosphere

How does pressure ( $P$ ) relate to altitude ( $h$ )?

Plug in all the constants

$$h = 4.43 \times 10^4 \times \left( 1 - \left( \frac{101.325 \text{ kPa}}{P} \right)^{-0.1902} \right) \quad (1)$$

# Example

Suppose,  $P = 85 \text{ kPa}$  (from Pressure sensor)

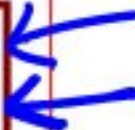
**Method 1:**

$$\Delta h = -\frac{\Delta P}{\rho g} = -\frac{(85 - 101) \text{ kPa}}{\left(1.2 \frac{\text{kg}}{\text{m}^3} * 9.8 \frac{\text{m}}{\text{s}^2}\right)} = 1.36 \text{ km}$$

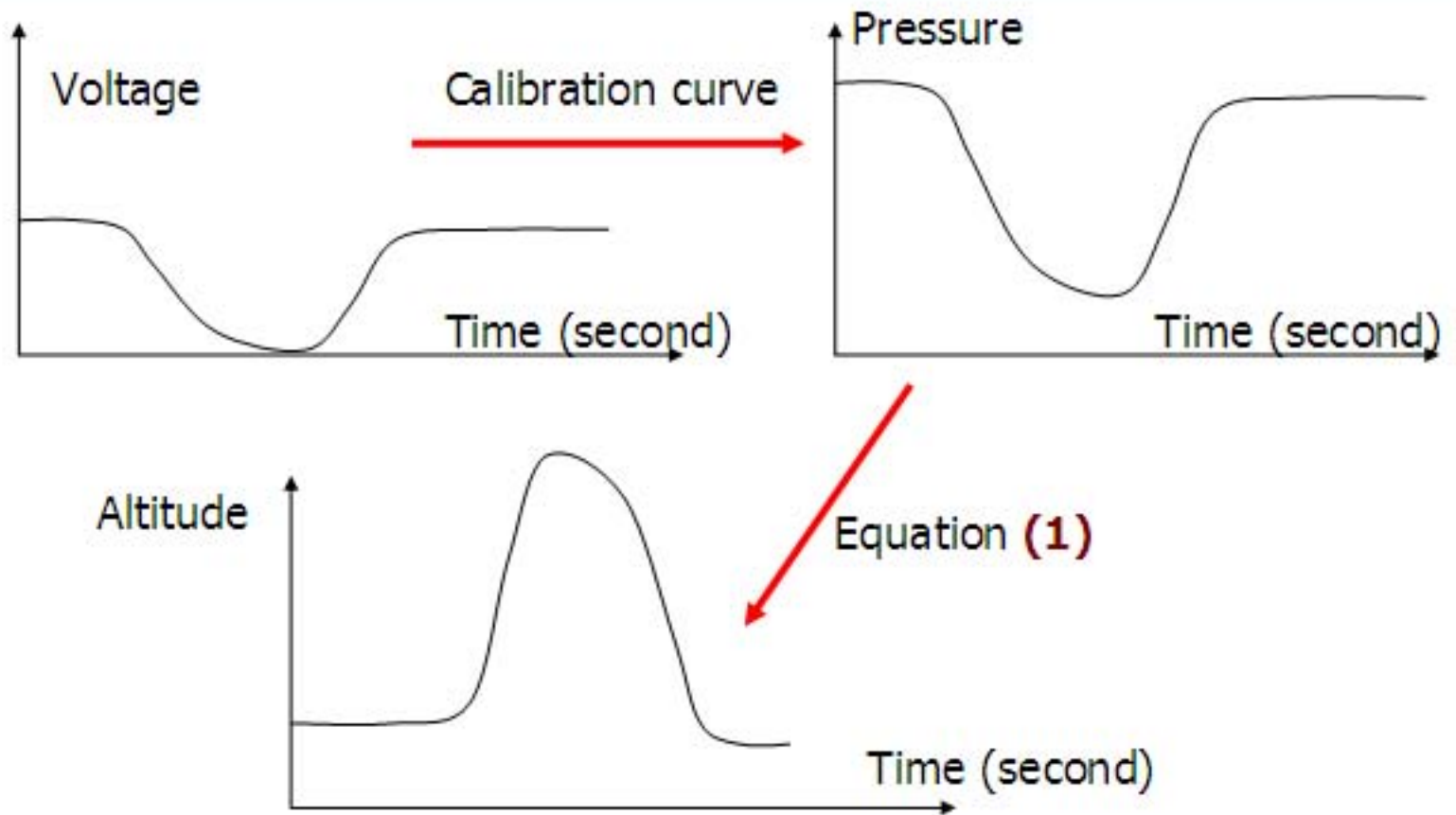
**Method 2:**

$$h = -\frac{RT}{Mg} \ln\left(\frac{P}{P_0}\right) = -8440 \ln\left(\frac{85 \text{ kPa}}{101 \text{ kPa}}\right) = 1.46 \text{ km}$$

**Method 3:**

$$h = 4.43 \times 10^4 \times \left(1 - \left(\frac{101.325 \text{ kPa}}{85 \text{ kPa}}\right)^{-0.1902}\right) = 1.43 \text{ km}$$


# Calibration process



# How do we measure temperature?

- Thermometer

- Thermistor: resistance changes w/ temp.

- Galileo thermometer

# Temperature measuring devices

## ■ Thermocouples

- ❑ Metals generate a small voltage under a temperature difference
- ❑ Measure voltage, know temperature difference
- ❑ Largest temperature range (-200 °C – 1000 °C), low accuracy

## ■ Resistance Temperature Detectors (RTDs)

- ❑ Usually made from platinum
- ❑ Resistance varies linearly with temperature
- ❑ Large temperature range (-200 °C – 500 °C), medium accuracy

## ■ Thermistors

- ❑ Usually made of ceramic or polymer
- ❑ Smaller temperature range (-40 °C – 125 °C), medium-high accuracy

# Thermistors

- Sensitive, accurate, reliable, inexpensive \$3
- Temperature dependent resistors
- Most common: Negative-Temperature Coefficient (NTC) thermistors  $T \uparrow R \downarrow$
- NTC themistors have nonlinear R-T characteristics
- Steinhart-Hart equation models the R-T relationship.



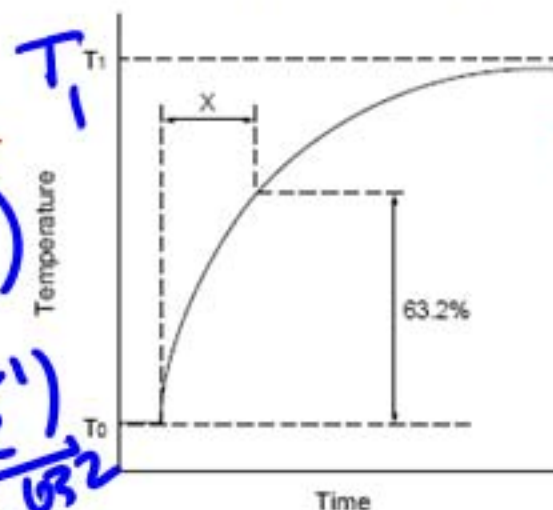
# Examples: thermistors in your car

- **Air conditioning**
- **Seat temperature controls**
- **Electronic fuel injection** air/fuel mixture and cooling water temperatures are monitored to help determine the fuel concentration for optimum injection
- **Warning indicators** oil and fluid temperatures, oil level and turbo-charger switch off
- **Fan motor control** based on cooling water temperature
- **Frost sensors** outside temperature measurement



# Basic characteristics of thermistors

- Temperature range  $-40^{\circ}\text{C}$  to  $125^{\circ}\text{C}$
- Resistance:
  - $R = R_0 e^{B(1/T - 1/T_0)}$
  - $B = \ln(R/R_0) / (1/T - 1/T_0)$
- Power dissipation
  - $P = C(T_2 - T_1)$
  - C is the thermal dissipation constant (mW/°C).
- Thermal time constant



$$T(t) = T_1 (1 - e^{-t/\tau})$$

@  $t = \tau$   $T(\tau) = \frac{T_1 (1 - e^{-1})}{0.632}$

# R-T characteristics of thermistor

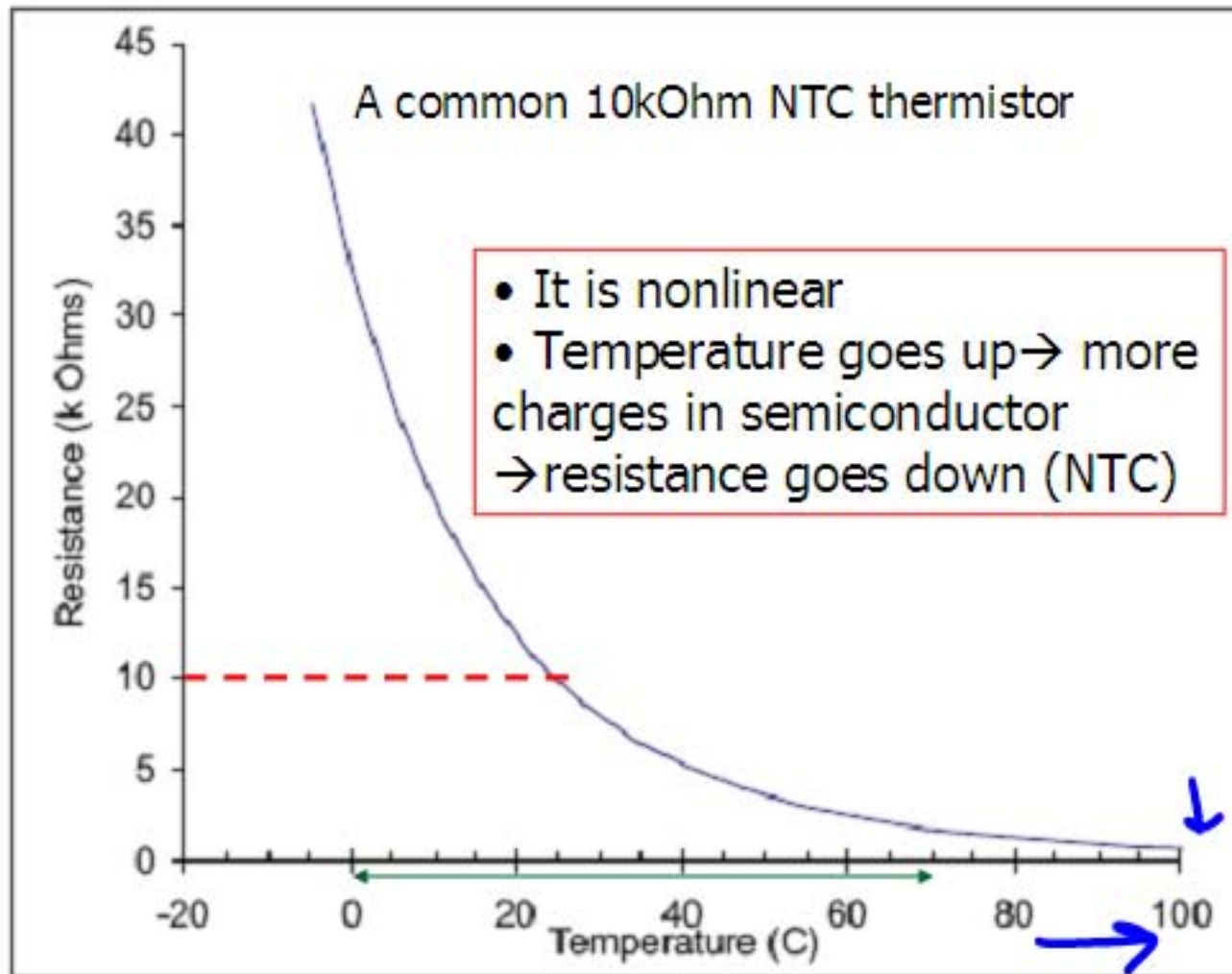
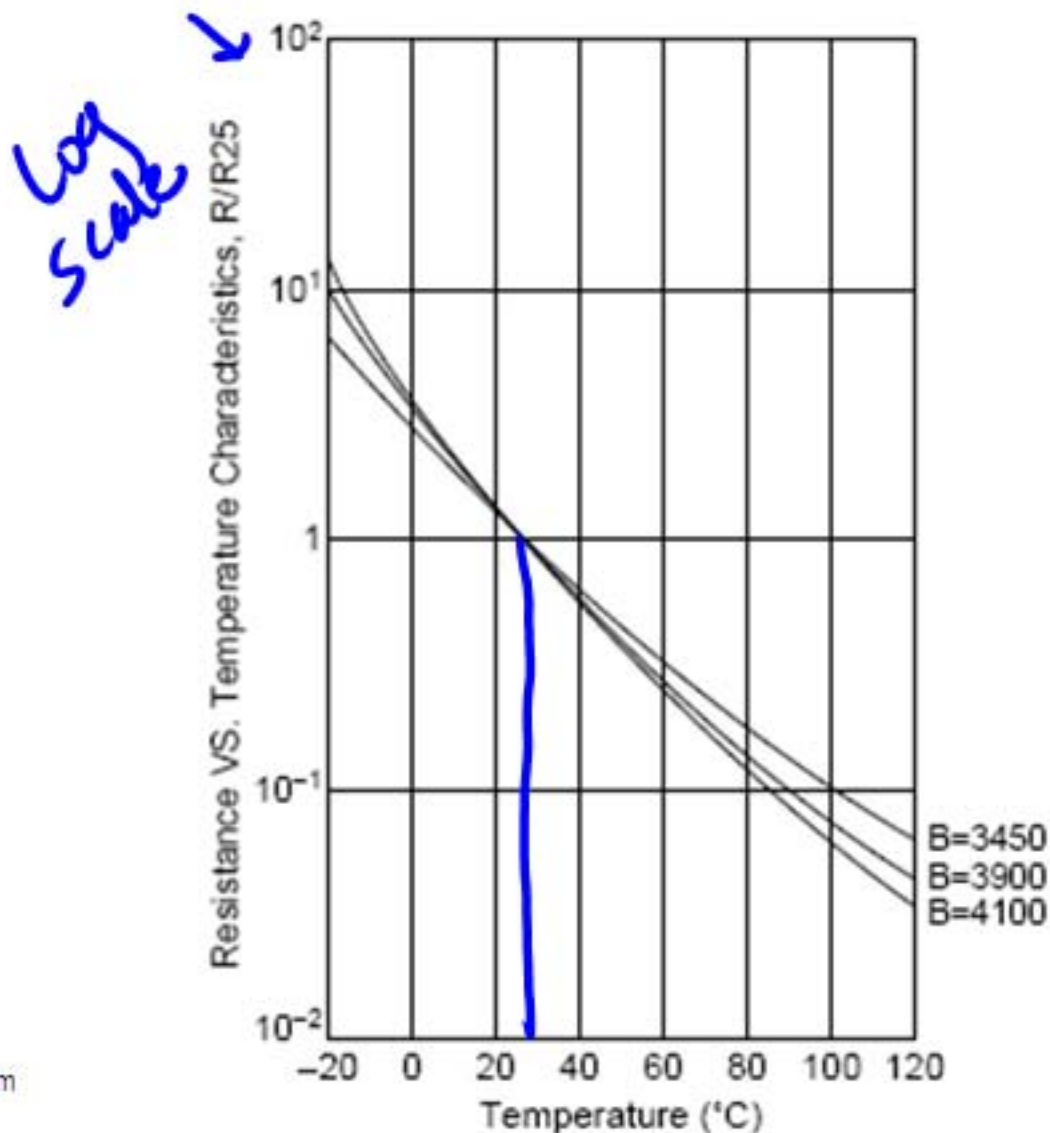


Figure 1. NTC R-T response curve.

# R-T characteristics of thermistor

Resistance vs. Temperature



$$\frac{R}{R_{@25^{\circ}\text{C}}}$$

## Relating $T$ to $R$ : Steinhart-Hart (S-H) Equations

- **3 term form:**

$$\frac{1}{T} = C_1 + C_2 \ln(R) + C_3 (\ln(R))^3$$



- 2 term form:

$$\frac{1}{T} = C_1' + C_2' \cdot \ln(R)$$

Note  $C_1' \neq C_1, C_2' \neq C_2$

- $T$ , in Kelvin.

- How do we measure thermistor resistance?

*Voltage  
Divider*

- How do we figure out  $C_1$ ,  $C_2$  and  $C_3$ ?

How do we measure thermistor resistance?

- Voltage divider

How do we figure out  $C_1$ ,  $C_2$ , and  $C_3$ ?

External  
Sensor

$$\frac{1}{T} = C_1 + C_2 \ln(R) + C_3 \ln(R)^3$$

thermistor

① - Need at least 3  $\Rightarrow$  3 equations,  
3 unknowns

② - Take more samples:

Linear regression fit: Linear Regression  
Wikipedia.org  $\Rightarrow$  Example

## How do we figure out $C_1$ , $C_2$ , and $C_3$ ?

% Measured Resistance values

```
R = [37e3 20e3 10e3 5e3 4e3 2e3]';
```

% Observed temperatures, corresponding to resistance matrix

```
T = [0 10 25 38 50 80]';
```

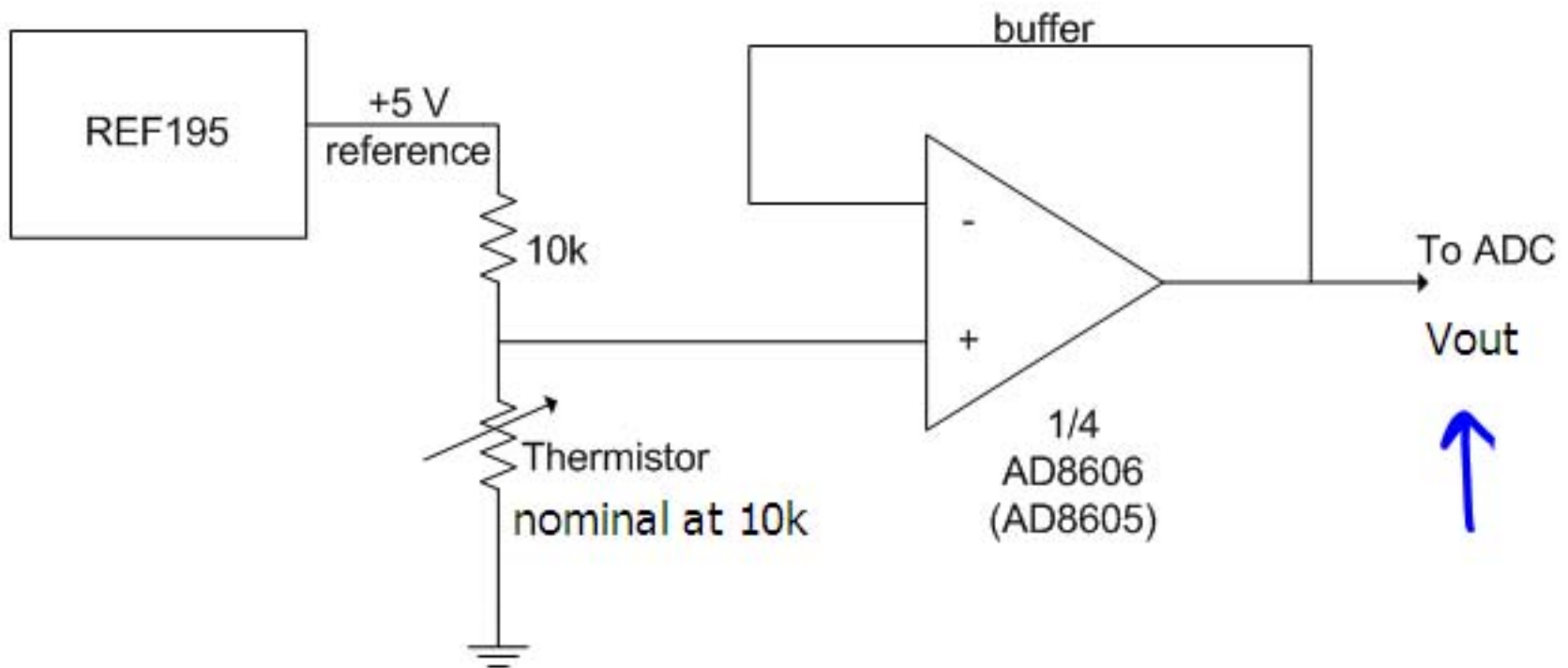
% Convert temperatures to Kelvin and create  $Y = 1/T$

```
Y = 1./(T + 273.15);
```

```
X = [ones(size(R)) log(R) (log(R).^3)];
```

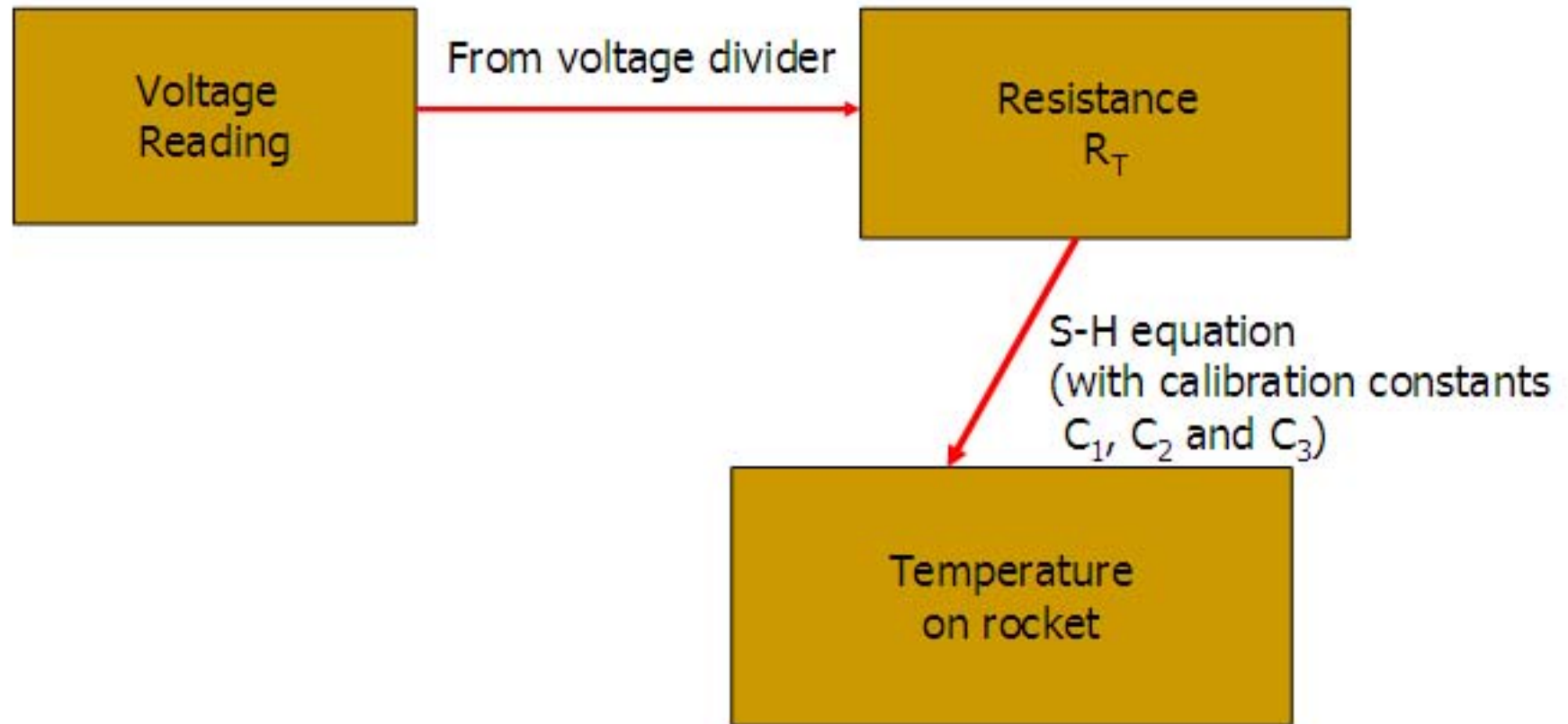
```
[B, BINT, R, RINT] = regress(Y,X);
```

# Thermistor signal conditioning circuits





# Thermistor on the rocket



# Summary

