E80 Spr '09
MC

Agenda

- Inertial Measurement Unit
  - classic inertial navigation
  - strapdown inertial navigation
- Instrumentation
- Algorithm

- We know that we are interested in maximum altitude of the rocket
  - we're interested in predicting that b4 flight
  - we'll want to measure it during flight
- In First Flight, what did we measure?
  (altitude - actually pressure)
  ↳ altimeter was the instrument we flew
- What did we calculate from these data?
  - accel, velocity
  - were you happy w/ these results?
    Would you do things differently? (this was 1-D; might want 3-D)
    (did rocket go straight up? what affects rocket trajectory? → later)

- You have used Rocksim to predict trajectory.

Next 2 lectures will focus on how we predict trajectory (position) and how we experimentally measure values that allow us to get hold of vehicle position, velocity + acceleration.

- We expect flight trajectory to be 3-D and time-varying. Need x, y, z, t ⇒ How?

(ASK)

accels – how many?

Need to fix 3 accels to our rocket

Will that allow us to get x, y, z, t?

- We need to know the rotation of the rocket "frame" — the local frame — so we can get accel (velocity + position as well) in a global frame.

How to measure rotation? Gyroscope

how many rotations / gyros? : 3 gyros

Inertial Measurement Unit (IMU)

- 3 gyroscopes to measure rotation angles (ϕ_x, ϕ_y, ϕ_z) or rotation rate ($\omega_x, \omega_y, \omega_z$) for orthogonal axes

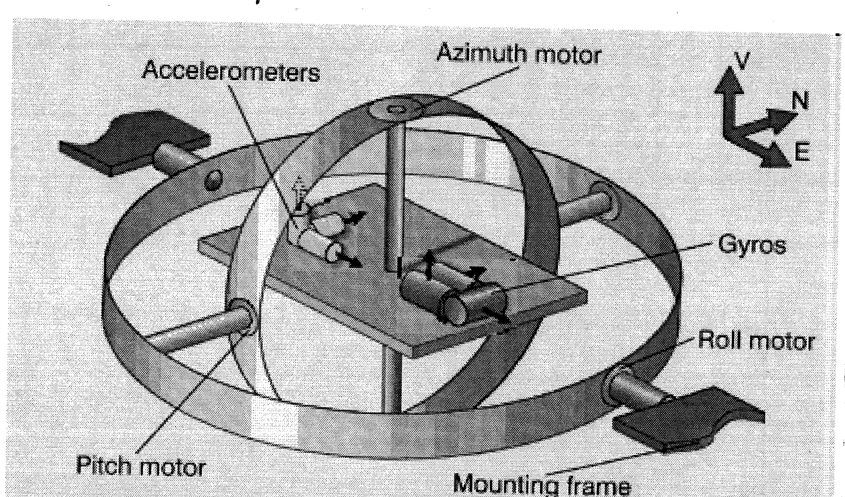
- plus 3 accels to measure linear accel (a_x, a_y, a_z) (again, orthogonal axes)

– couple of different ways to implement this:

Classic Inertial Navigation

– put accels + gyros on a platform

– mount this platform on gimbals that allow the platform to rotate about all 3 axes



– each gimbal frame has a motor that can rotate the frame

- How does it work? If the gyros on the platform sense a rotation, they send a signal to the motors and motors rotate the frames to keep the platform at the same orientation

⇒ classic inertial navigation always has a global frame of reference for the accels.

applications:
surface ships
submarines
rocket engines on large craft
gimbaled
big-game fishing
- gimbal chairs

- gyros used to keep platform in one orientation
- accel data may be integrated to get velocity + position. (remember gravity)

* Do you think we have this type of system in our E80 rockets? — heavy, big, need motors + control system for the motors

NOPE - what do we do instead?

STRAPDOWN INERTIAL NAVIGATION

- Mount accels + gyros to vehicle frame (local frame)

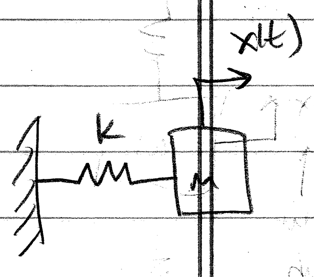
- must do transform to get accel measurements from the local frame to the global frame

- once that's done, integrate to get velocity, and integrate again to get position (remember gravity)

- We'll come back to the transform + integrate in a bit. 1st let's talk about accels + gyros, how they work.

Accelerometers

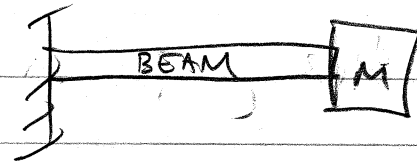
- basic principle is mass-spring system



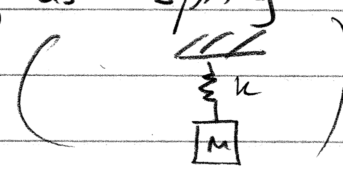
- system undergoing accel has force on the mass $F = Ma$

- the mass undergoes displacement x

$$\Rightarrow \quad a = kx/M$$

As you know from E39, , is a

mass-spring system, w/ the beam acting as a spring



Also could do piezoelectric accels

(vaguely, the crystal lattice deformation leads to an interaction between mechanical + electrical states of the crystal)

— relation between mechanical stress + surface charge on a crystal. Acceleration causes a force → stress in crystal → charge on crystal → convert charge to voltage using circuitry ⇒ output is a voltage proportional to accel

Other types as well — see last year's notes

What do we use? MEMS accels — micro-electro-mechanical systems

· uses principle of mass-spring system, but done really really small

- basically we have 2 microstructures (little beams of polysilicon; 1-10 μm width; 50-100 μm length)
- microstructures next to each other have a certain <u>capacitance</u> between them

Aside: capacitor
- stores energy
- 2 electrical conductors separated by an insulator (dielectric)
- when powered, charge builds up on the conductors, one pos, one neg

<u>capacitance</u> = amount of charge Q stored between the 2 plates for a potential difference existing across the plates

- physically related to the dielectric material area of the plate, and <u>distance between the plates</u>

$$C = \frac{\epsilon A}{d}$$

(ϵ = dielectric constant
A = plate area
d = distance)

OK, we're trying to measure accel. How does having tiny beams + capacitance help us?

Recall: beam can be thought of as a spring-mass system ⇒ accel will give a displacement

displacement in the MEMS system causes what?
— capacitance change

So if we have circuitry to measure capacitance and change it to voltage, ⇒ we have an accel.

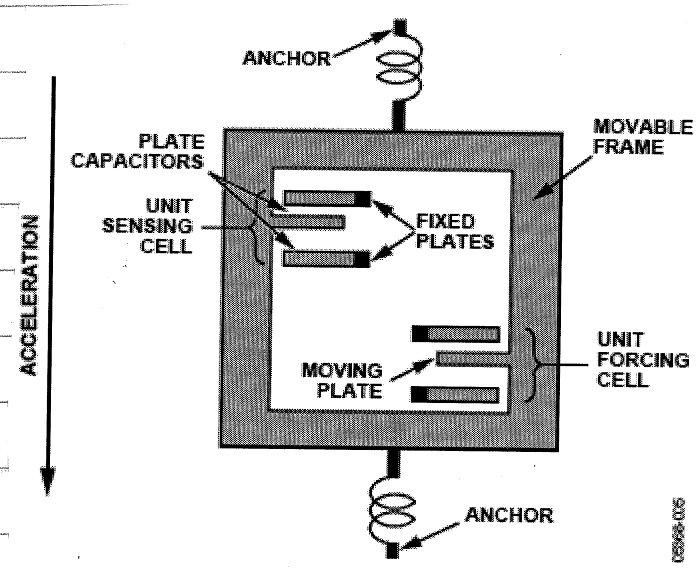


Figure 5. Simplified View of Sensor Under Acceleration

beam deflection ~ femtometers (10^{-15} m)

(will wave my hands on the "on-board circuitry" that measures differential capacitance and outputs voltage)

* Basically, we get a voltage output that is proportional to acceleration.

NOTE: our accels measure static accel forces

(What does this mean? gravity - so remember to take that into account when you use these data)

We have 2 chips (on the IMU) - one one-axis accel (ADXL78) (±35g) along the long axis of the rocket and one dual-axis accel (ADXL320) (±5g) measuring accel in two orthogonal radial directions.

GYROS - see last year's notes if you're interested in gimbaled gyros; we'll go directly to MEMS Gyros

→ rate gyros - measure angular velocities rather than rotation angle

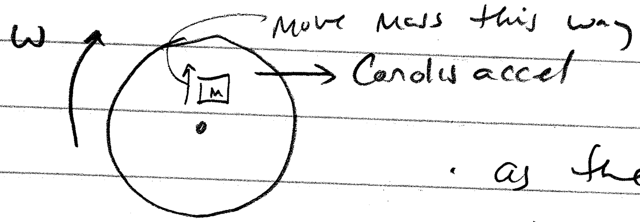
applications
- air bags sensors
- phones + ipods
- gaming
- pedometers
- vehicle skid detection
- drop detection for computers

Errors
- Bias - can measure by mounting on turntable, where orientation is known accurately
- NOISE

How do MEMS Gyros Work?

Coriolis effect

- Let's be an observer in a rotating system. Imagine a mass moving in this system (not parallel to axis of rotation)



- as the mass moves farther from the center, its speed relative to the ground increases.

The rate of increase of the tangential speed, caused by the radial velocity of the rotating system, is the Coriolis accel. ⊥ to relative motion and to rotational axis

$$(- 2\,\omega \times \overline{V})$$

↗ angular velocity of rotating ref. frame

↑ radial velocity of the mass relative to the center of the rotating ref frame

So. If we move a mass in a rotating frame, we get Coriolis accel (and force). What do we want from this gyro? Rotation rate

How?

Coriolis force $= -2m\,\omega \times \bar{V}$

↑ here is rotation rate

or

Coriolis accel $= -2\,\omega \times \bar{V}$

Any ideas?

Put an oscillating mass-spring system in a rotating frame ⇒ cause Coriolis accel

measure accel as 54 (fingers + capacitance)

cap change ~ zeptofarads (10^{-21} farads)

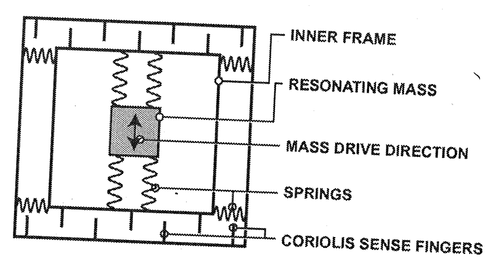


Figure 4. Schematic of the gyro's mechanical structure.

capacitance sensing between microstructures

Analog Dialogue 37-03 (2003)

femtometers (10^{-15} m)

Again we measure a displacement to get hold of accel, and in this case, angular velocity (proportional to voltage)

Applications

Wii

halteres of insects — not instrument application, but shows principle in insect world

aircraft control

robotics

virtual reality

ship stabilization

(Bias + bias drift — output when angular velocity = 0 is called the bias. This bias can drift over time due to factors such as temperature and stress

· how to get bias measurement?

Noise

Our instrument is the IDG300 — Integrated Dual-Axis Gyro. So we have 2 of these so we can get our 3 axes. We get a redundant measurement along long axis of rocket and measurements in the two orthogonal radial directions (same as the linear accel axes)

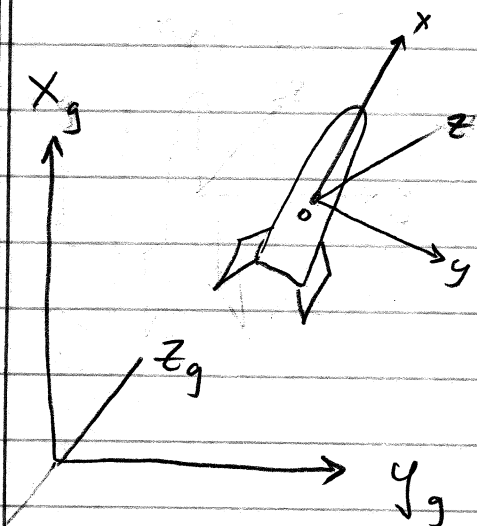
• Let's get back to the Inertial Navigation itself.

Recall we have 3 linear accel measurements.

These accels are fixed to a local (vehicle) frame.

We are after accel (vel, position) in a global frame

Typical axes



Ox: local vertical (up)
Oy: local east
Oz: local north
↖ local, fixed to rocket

← global frame

We need 3 rotation angles (or rates) to relate local frame to global frame

definitions

roll - rotation about long axis

pitch - imagine an axis from left to right (forward is along long axis)
rotation about that axis moves nose (or tail) up and down

yaw - imagine top-to-bottom axis. Rotation moves nose right or left

- Any arbitrary rotation can be decomposed into 3 separate rotations of angle ϕ_x, ϕ_y, ϕ_z

(skipping steps - see last year's notes)

For small angles,

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -\phi_z & \phi_y \\ \phi_z & 1 & -\phi_x \\ -\phi_y & \phi_x & 1 \end{bmatrix}}_{\text{rotation matrix } R} \begin{bmatrix} v_{cx} \\ v_{cy} \\ v_{cz} \end{bmatrix}$$

↑ global variable ↑ local variable

How do we get ϕ's? Also, the rocket continually changes position/orientation. How do we handle that?

We need to get R matrix from measurements we have made.

Recall: $u(t) = R(t)\, v(t)$

$\uparrow$ global $\quad\quad \uparrow$ local

To track the attitude change we must also track the change in R with time

$$\dot{R}(t) = \lim_{\delta t \to 0} \frac{R(t+\delta t) - R(t)}{\delta t}$$

where $R(t+\delta t) = R(t) S(t)$

where $S(t)$ is the rotation matrix which relates the body frame at time t to the body frame at time $t+\delta t$

$$S(t) = \begin{bmatrix} 1 & -\delta\phi_z & \delta\phi_y \\ \delta\phi_z & 1 & -\delta\phi_x \\ -\delta\phi_y & \delta\phi_x & 1 \end{bmatrix} = I + \delta\Phi$$

($\delta\phi_x$, $\delta\phi_y$, $\delta\phi_z$ are small rotations that occur during δt)

So: $\dot{R}(t) = \lim_{\delta t \to 0} \left[\frac{R(t+\delta t) - R(t)}{\delta t} \right]$

$$= \lim_{\delta t \to 0} \left[\frac{R(t)\,S(t) - R(t)}{\delta t} \right]$$

$$= \lim_{\delta t \to 0} \left[\frac{R(t)\overbrace{(I + \delta\bar{\Phi})}^{S(t)} - R(t)}{\delta t} \right] = R(t) \lim_{\delta t \to 0} \frac{\delta \bar{\Phi}}{\delta t}$$

$$\dot{R}(t) = R(t)\,\dot{\bar{\Phi}}$$

Recall that $\dot{\phi}_x = \omega_x$; $\dot{\phi}_y = \omega_y$; $\dot{\phi}_z = \omega_z$

⇒ we will have ω from rate gyros

(remember, we're trying to get R from our measurements)

Let's define

$$\Omega = \dot{\bar{\Phi}} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

so $\dot{R}(t) = R(t)\,\Omega(t)$

$$\left[\frac{dR}{dt} = R\Omega \;; \quad \frac{dR}{R} = \Omega\, dt \right]$$

D.E. soln is $R(t) = R(0) \cdot \exp\left(\int_0^t \Omega(t)\, dt \right)$

where $R(0)$ is the initial attitude of the device

[note: this is matrix math]

IMU gives us samples of angular velocity, not a continuous signal. I will present an integration scheme to integrate the sampled signal

For small δt

$$\int_t^{t+\delta t} \Omega\, dt \approx \Omega(t)\, dt = B$$

(rectangular rule)

$$B = \begin{bmatrix} 0 & -\omega_z\, dt & \omega_y\, dt \\ \omega_z\, \delta t & 0 & -\omega_x\, \delta t \\ -\omega_y\, \delta t & \omega_x \delta t & 0 \end{bmatrix}$$

So $R(t+\delta t) = R(t) \exp B$

$(\omega_x, \omega_y, \omega_z)$ are the angular

velocity samples corresponding to the update period

Let $\sigma^2 = (\omega_x^2 + \omega_y^2 + \omega_z^2)\delta t^2$, do Taylor series

expansion of the exponential (see Wang's

lecture '08) $\Rightarrow$

$$R(t+\delta t) = R(t)\left(I + \frac{\sin\sigma}{\sigma} B + \frac{1-\cos\sigma}{\sigma^2} B^2 \right)$$

every time we get new ω's, we update the

rotation matrix; this allows us to get the variables

in the <u>global frame</u>. $[a_{global} = R\, a_{local}]$

Then integrate accel to get velocity; integrate again to

get position

<u>ERRORS</u> — accel + gyro measurements have noise +
bias <u>AND</u> we're integrating these data
over + over.
Can lead to large errors ↑ in position, vel over time

ACCEL + GYRO (IMU) LAB

- need to calibrate accels + gyros
  (accels → on IMU and on R-DAS; gyros → on IMU)

- need to trouble shoot IMU's (some IMUs have broken accels and/or gyros)

- Calculate the change in global position and orientation using IMU data ⇒ errors? explain

You are given a controllable turntable, IMU, R-DAS

Up to team to come up w/ Design of Experiments and a plan to do the lab.