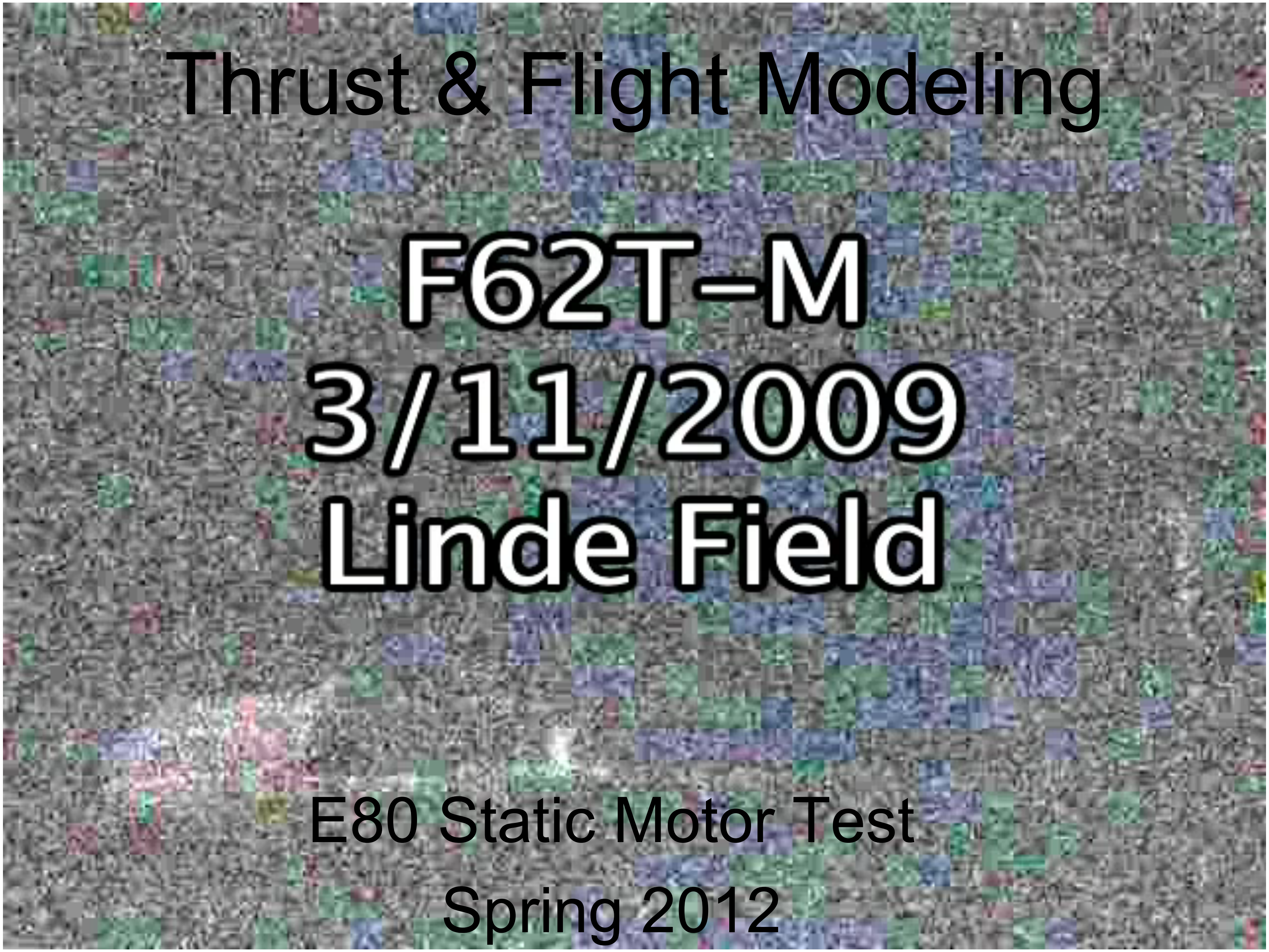
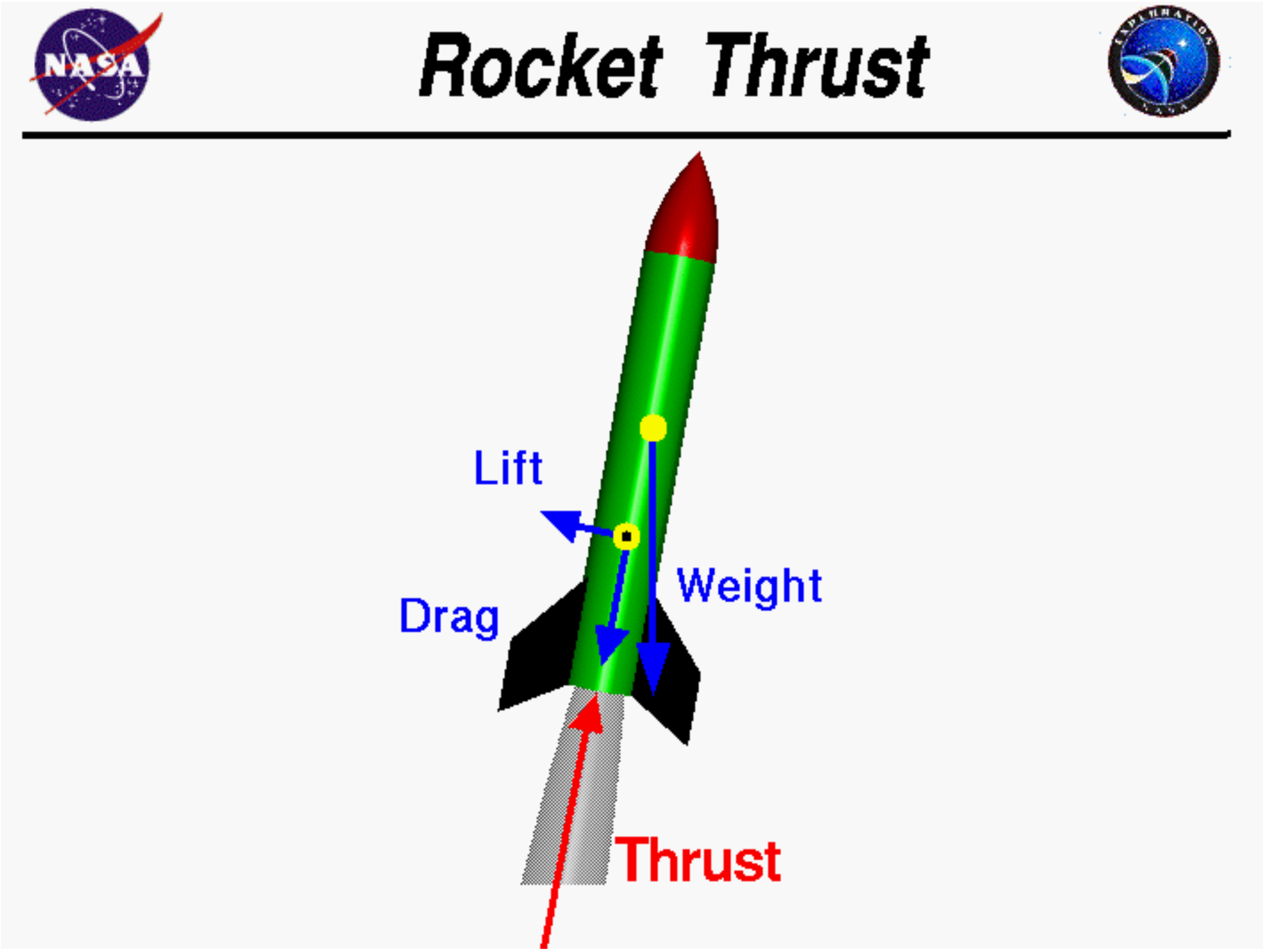
Thrust & Flight Modeling



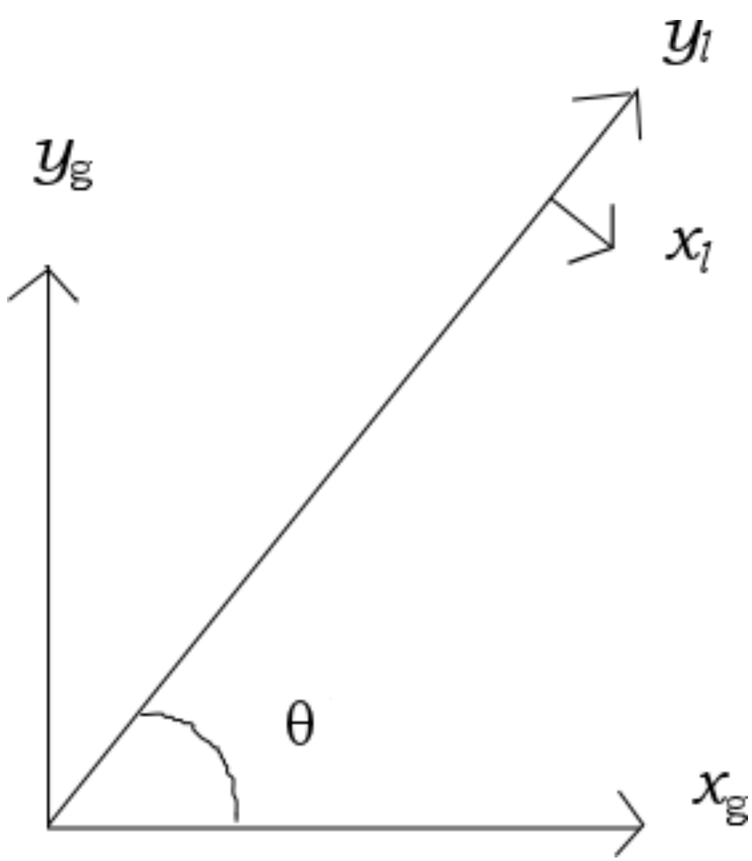
E80 Static Motor Test

Spring 2012



http://exploration.grc.nasa.gov/education/rocket/bgmr.html

Local and Global Coordinate Systems



Thrust is always $+y_l$

Drag is always $-y_l$

Lift is always $+x_l$ or $-x_l$

Gravity is always $-y_g$

Wind is always $+x_g$ or $-x_g$

See for example http://www.kwon3d.com/theory/transform/rot.html ,
and http://www.kwon3d.com/theory/transform/transform.html

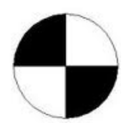
3 DOF Equations of Motion (global coordinates)

- x – horizontal

$$m\ddot{x} = F_{Tx} - F_{Dx} + F_{Lx}$$

- y – vertical

$$m\ddot{y} = F_{Ty} - F_{Dy} + F_{Ly} - mg$$

- θ – Angle (about CM )

$$J\ddot{\theta} = T_L$$ You may need to include damping.

CM symbol from http://naturalginger.blogspot.com/2011/04/locus-of-effect.html

Drag & Lift

- Drag (acting at CP) $\odot$

$$\vec{F}_D = -C_D A_P \rho \frac{V^2}{2} \hat{y}_l$$

- Lift (acting at CP) $\odot$

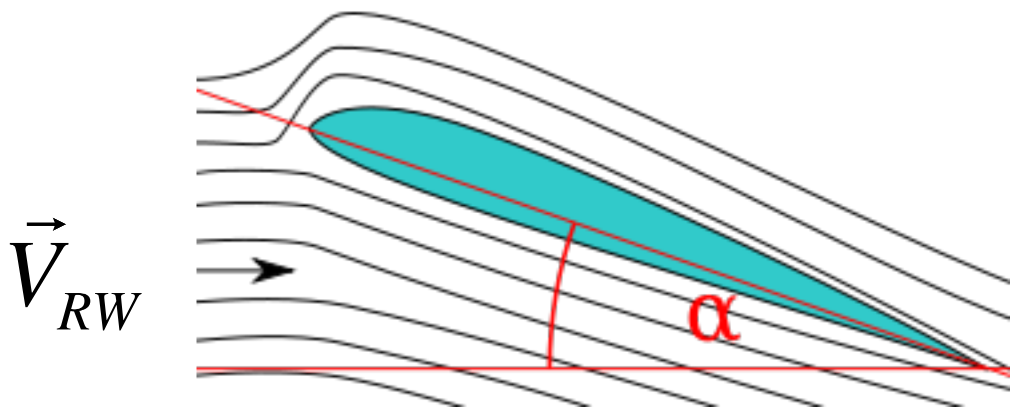
$$\vec{F}_L = \pm C_L S \rho \frac{V^2}{2} \hat{x}_l$$

Angle of Attack

- Relative Wind

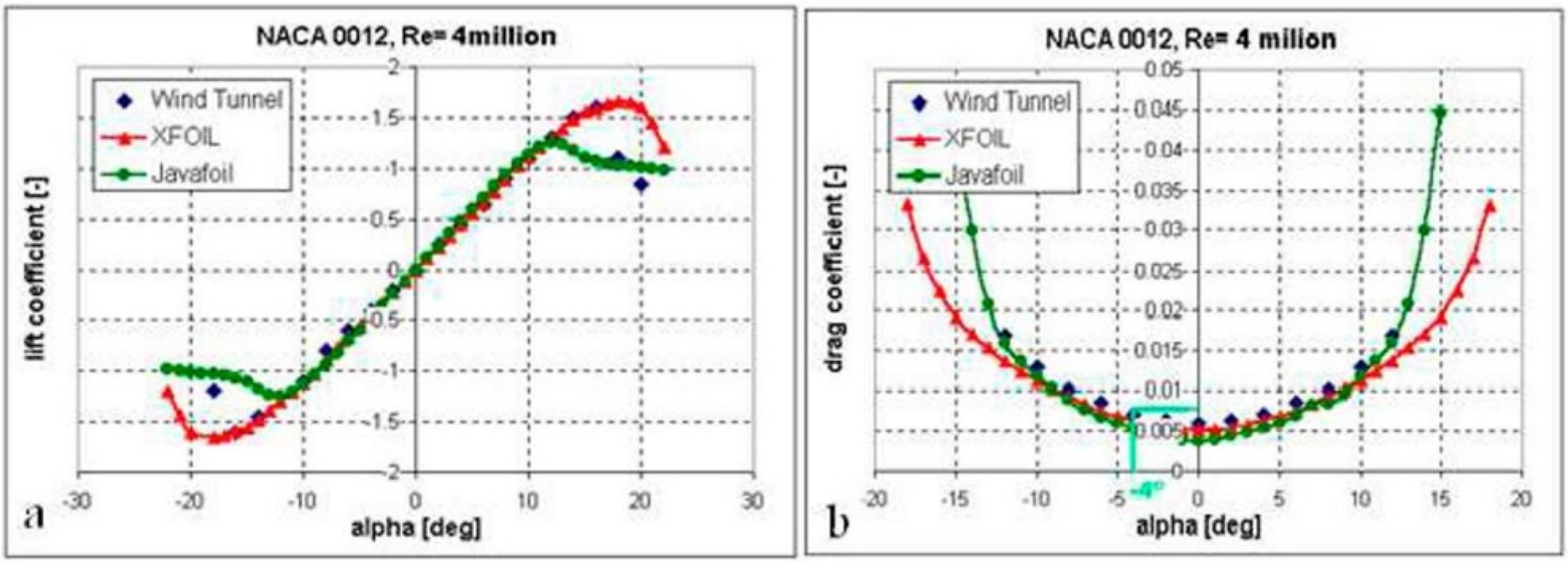
$$\vec{V}_{RW} = -\vec{V}_{Rocket} + \vec{V}_{Wind}$$

- Angle of Attack



http://en.wikipedia.org/wiki/File:Angle_of_attack.svg

C_D and C_L versus α



http://what-when-how.com/experimental-and-applied-mechanics/parametric-study-to-optimize-aluminum-shell-structure-under-various-conditions-experimental-and-applied-mechanics-part-1/

Numerical Solution of ODEs

- Express as system of 1st-order ODEs

$$\frac{dV_x}{dt} = \frac{F_{Tx}(t) - F_{Dx}(t)}{m}$$

$$\frac{dx}{dt} = V_x$$

Numerical Solution of ODEs

- Determine initial conditions, e.g.,

$$V_{x0}, x_0, V_{y0}, y_0, \omega_0, \theta_0$$

- Choose a solution method
  - Explicit Euler
  - Implicit Euler
  - Runge-Kutta

Explicit Euler

- The set of equations to solve is

$$\dot{\mathbf{y}}(t) = \mathbf{F}\left(\mathbf{y}(t), t\right)$$

- For each time step the solution is

$$\mathbf{y}_{n+1} = \mathbf{y}_n + h\mathbf{F}\left(\mathbf{y}_n, t_n\right)$$

$$t_{n+1} = t_n + h$$

Explicit Euler

- Stability of solution depends on time step
  - Small h good for stability, bad for solution size
- For example

$$\dot{y} = -\alpha y$$

$$y_{n+1} = y_n + h\left(-\alpha y_n\right) = \left(1 - h\alpha\right) y_n$$

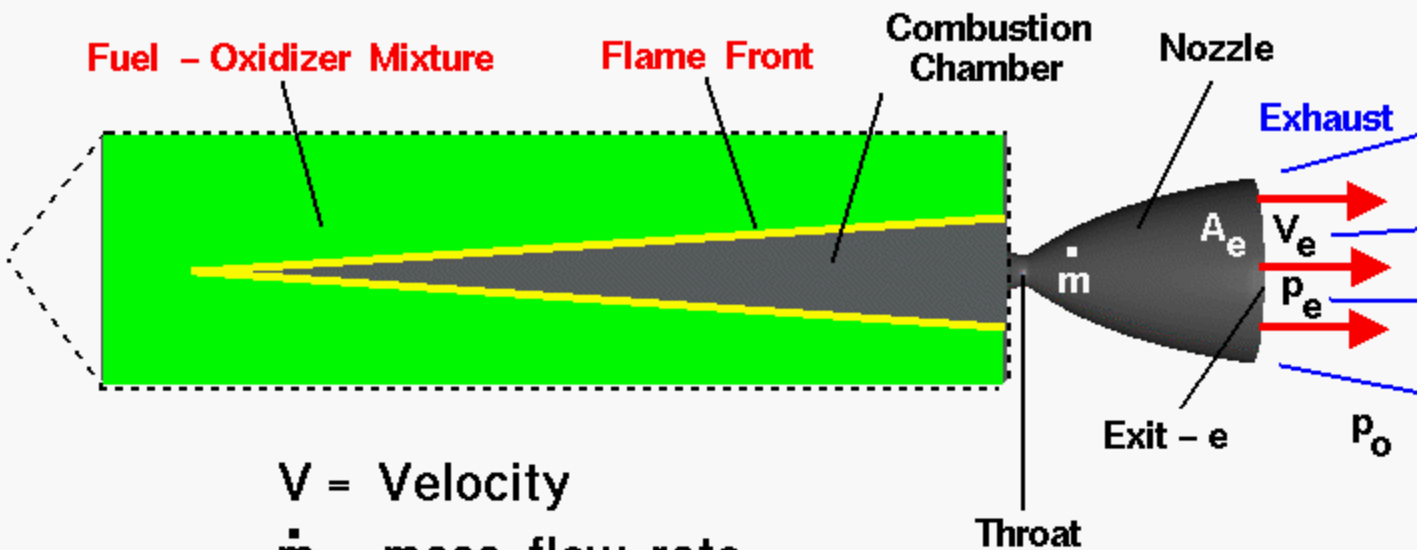
What Do You Need?

- 1-DOF
  - Mass
  - Thrust Curve
  - Drag Coefficient
  - Projected Area

What Do You Need?

- 3-DOF add
  - Lift Coefficient
  - Planform Area (or do you?)
  - Center of Mass
  - Center of Pressure
  - Moment of Inertia

Solid Rocket Engine



V = Velocity

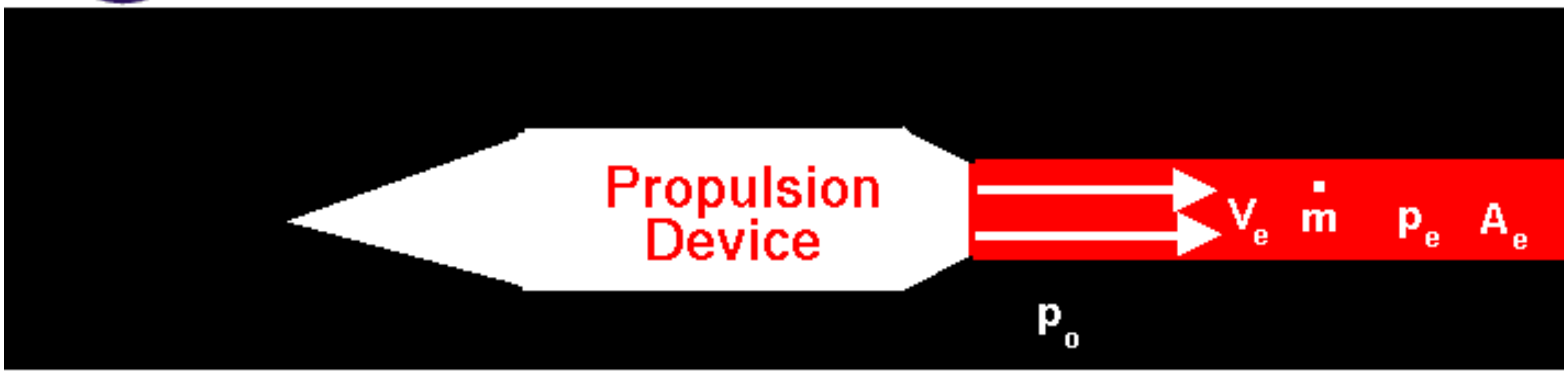
$\dot{m}$ = mass flow rate

p = pressure

Thrust = $F = \dot{m}\, V_e + (p_e - p_0)\, A_e$

http://exploration.grc.nasa.gov/education/rocket/srockth.html

Specific Impulse



Rocket Thrust Equation $F = \dot{m}\, V_e + (p_e - p_o)\, A_e$

where p = pressure, V = velocity, A = area, $\dot{m}$ = mass flow rate , F = thrust

Define: Equivalent Velocity: $V_{eq} = V_e + \dfrac{(p_e - p_o)\, A_e}{\dot{m}}$ $\quad F = \dot{m}\, V_{eq}$

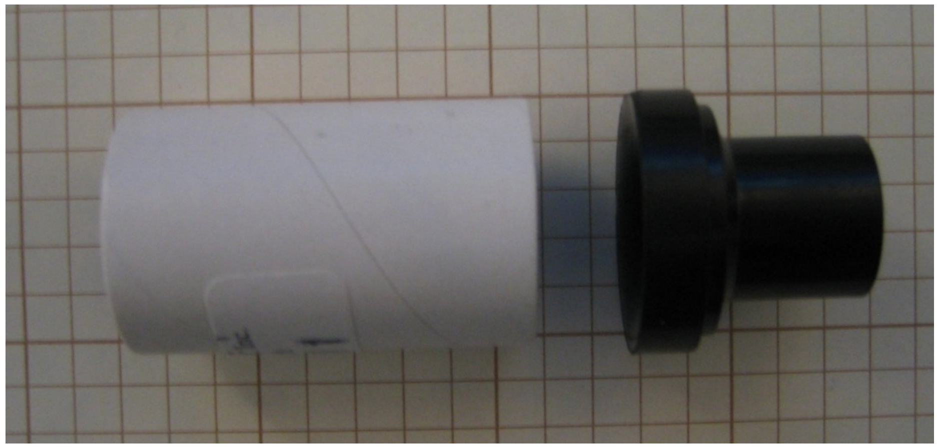
Define: Total Impulse: $I = F \Delta t = \int F\, dt = \int \dot{m}\, V_{eq}\, dt = m\, V_{eq}$

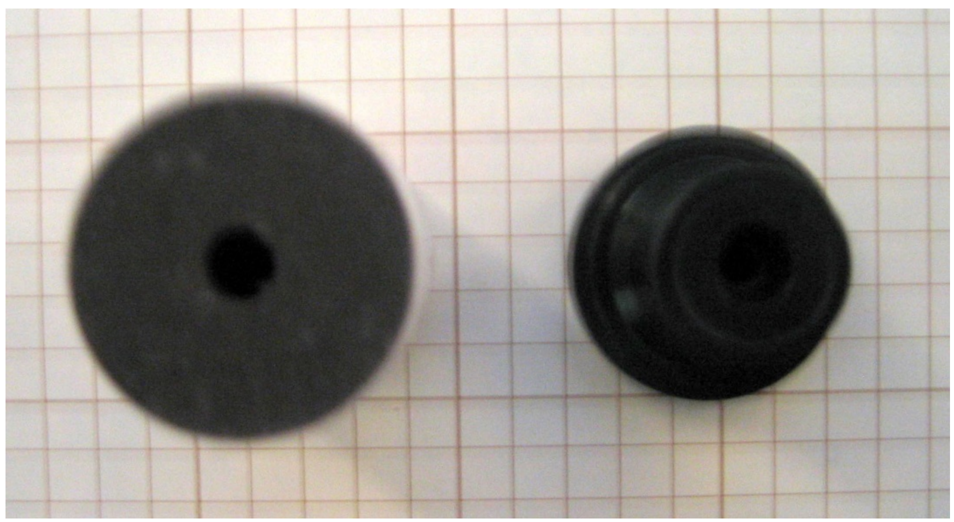
Define: Specific Impulse: $\dfrac{\text{Total Impulse}}{\text{Weight}}$ $\quad Isp = \dfrac{I}{m\, g_o} = \dfrac{V_{eq}}{g_o}$ $\quad$ units = sec

$$\boxed{Isp = \frac{F}{\dot{m}\, g_o}}$$

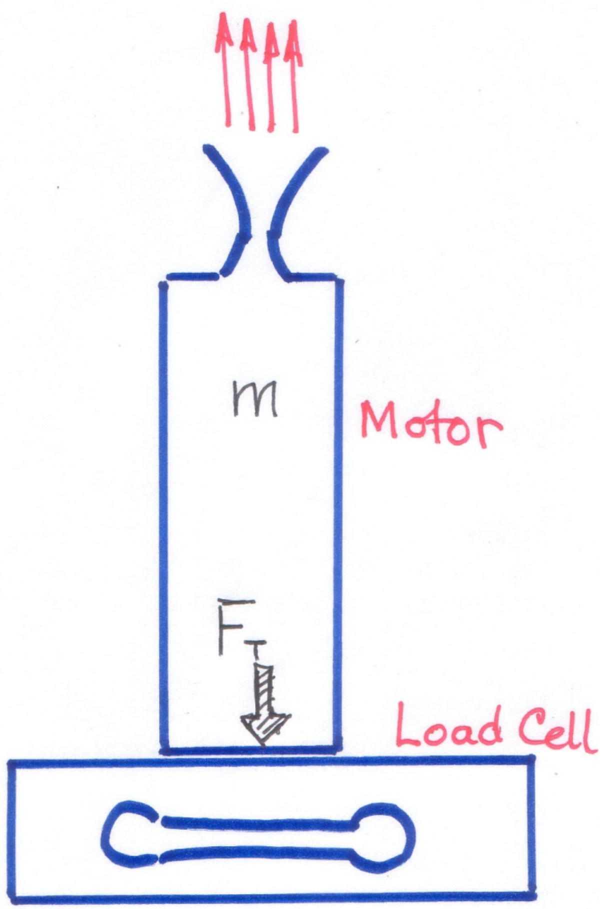
http://exploration.grc.nasa.gov/education/rocket/specimp.html

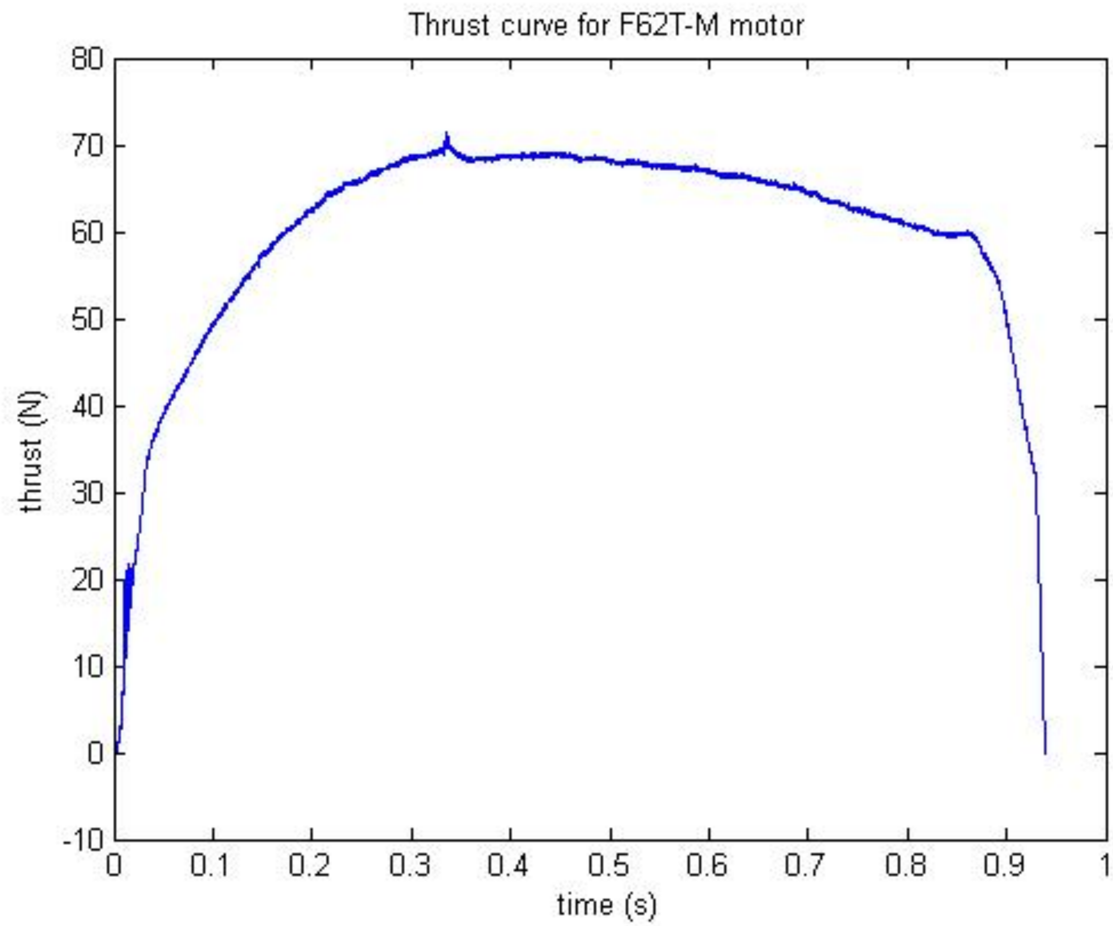
Static Motor Test





Static Motor Test





Static Motor Test

$$F_T = \dot{m}V_e + \left(P_e - P_0\right)A_e = \dot{m}V_{eq}$$

$$\dot{m} = \frac{m_{final} - m_{initial}}{t_{burn}} = \frac{\Delta m}{t_{burn}}$$

$$V_{eq} \cong \frac{F_{T,average}}{\dot{m}}$$

Thrust Analysis

KINETICS
(Non equilibrium processes)

Fuel Burn Rate

$$\dot{m}$$

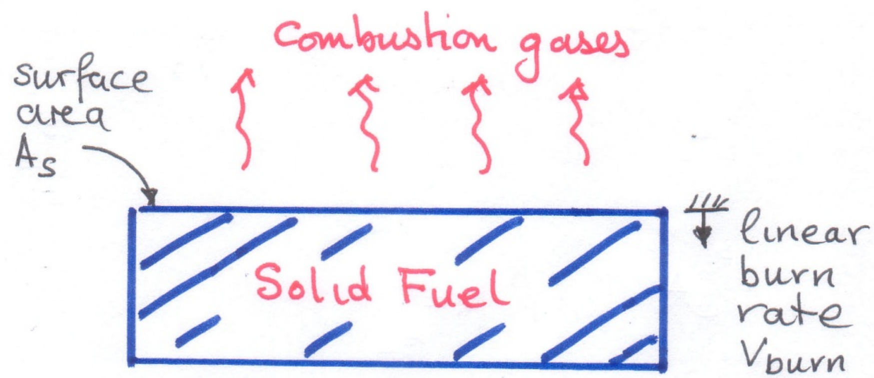
THERMODYNAMICS
(Equilibrium processes)

Exit conditions from nozzle

$$V_e \qquad P_e$$

Kinetics of Solid Fuel Combustion

$$6NH_4ClO_4(s) + 10Al(s) \rightarrow 4Al_2O_3(s) + 2AlCl_3(g) + 3N_2(g) + 12H_2O(g)$$
$$+\,ENERGY!!!!!!$$



$$\dot{m} = \rho_{fuel} A_s V_{burn}$$

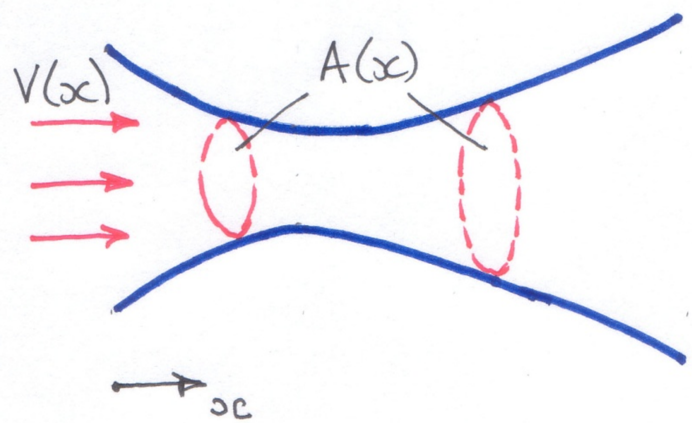
$$V_{burn} = kP_t^n$$

Thermodynamics of Flow Processes

$$\rho A V = \text{constant} \quad (\textit{mass balance})$$

$$h + \tfrac{1}{2} V^2 = \text{constant} \quad (\textit{energy balance})$$

Isentropic Flow through Nozzles



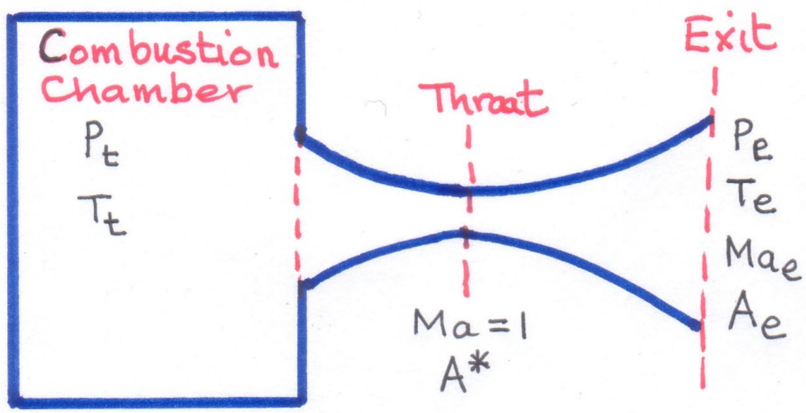
$$\frac{dA}{dV} = -\frac{A}{V}\left(1 - Ma^2\right)$$

For subsonic flow (Ma<1) $\quad \frac{dA}{dV} < 0$

For supersonic flow (Ma>1) $\quad \frac{dA}{dV} > 0$

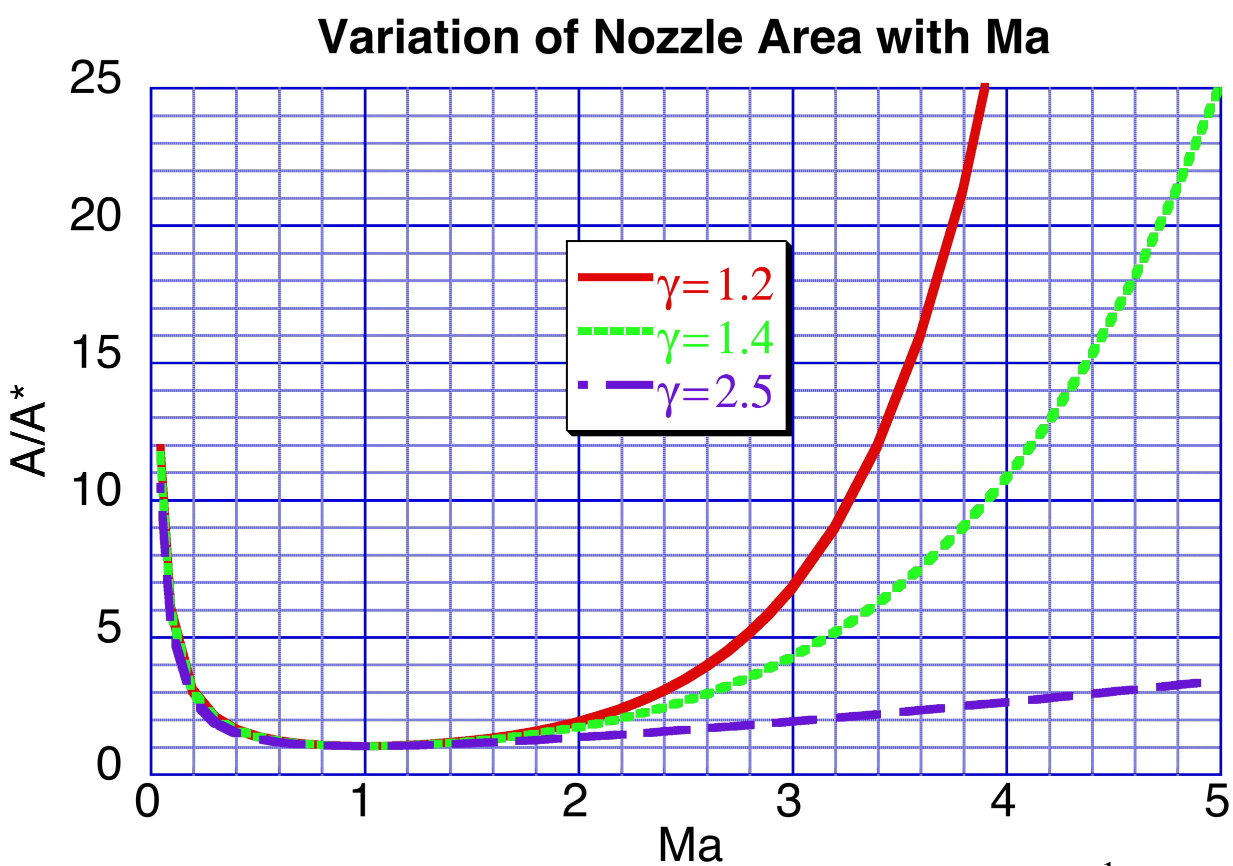
For sonic flow (Ma=1) $\quad \frac{dA}{dV} = 0$

Isentropic Flow of Perfect Gas Through Converging-Diverging Nozzle



$$\frac{A_e}{A^*} = \frac{1}{Ma_e}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}Ma_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Isentropic Flow of Perfect Gas Through Converging-Diverging Nozzle



$$\frac{A}{A^*} = \frac{1}{Ma}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}Ma^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Rocket Thrust Summary

Known:

p_t = Total Pressure

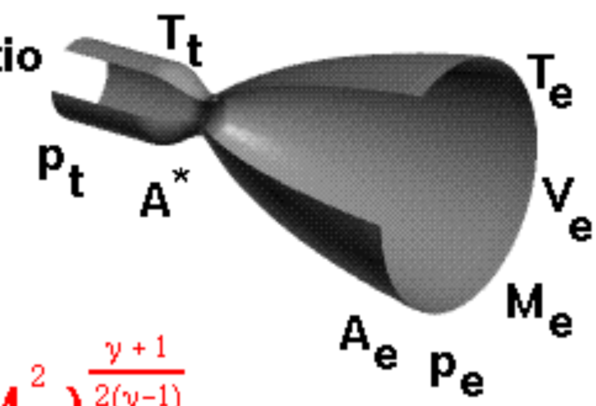
T_t = Total Temperature

p_o = Free Stream Pressure

γ = Specific Heat Ratio

R = Gas Constant

A = Area



Mass Flow Rate: $$\dot{m} = \frac{A^* p_t}{\sqrt{T_t}} \sqrt{\frac{\gamma}{R}} \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

Exit Mach: $$\frac{A_e}{A^*} = \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \frac{\left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M_e}$$

Exit Temperature: $$\frac{T_e}{T_t} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{-1}$$

Exit Pressure: $$\frac{p_e}{p_t} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{-\frac{\gamma}{\gamma-1}}$$

Exit Velocity: $$V_e = M_e \sqrt{\gamma R T_e}$$

Thrust: $$\boxed{F = \dot{m}\, V_e + (p_e - p_0)\, A_e}$$

http://exploration.grc.nasa.gov/education/rocket/rktthsum.html

E80 Static Motor Test Lab

1. Calibrate Load Cell
2. Measure Thrust Curve of Rocket Motor
3. Measure Average Mass Flow Rate
4. Model Flight 1-DOF & 3-DOF
5. Compare with Rocksim.