Dynamic Beam Analytical Solution

The general solution for displacement of a single-span, free-free beam shown in figure 1 is:

$$y(x,t) = \sum_{n=1}^{\infty} \left(A_n \sin \omega_n t + B_n \cos \omega_n t\right) \sin \frac{n\pi x}{L}$$

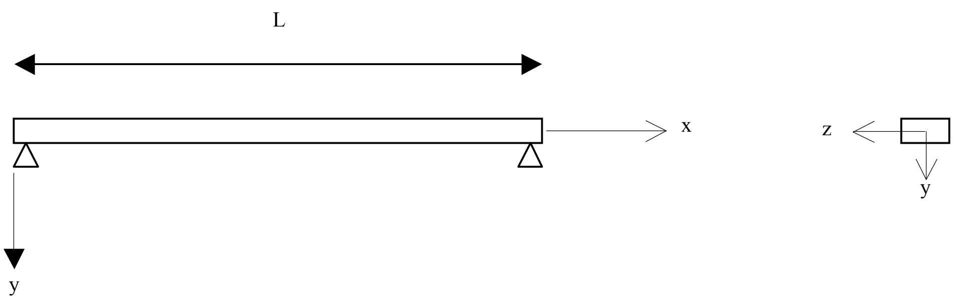


Figure 1: Schematic of a free-free beam

For this system, natural frequencies are

$$\omega_n = \beta_n^{\,2} \sqrt{\frac{EI_z}{\rho A}} = \left(\beta_n L\right)^2 \sqrt{\frac{EI_z}{\rho A L^4}}$$

in which E is Young's modulus, I_z is second moment of the beam, A is cross sectional area of the beam and ρ is density. See Appendix 1 for basic beam calculations.

β_n is dependent upon the beam's boundary conditions. Values for β_n for some typical boundary conditions are presented in Table 1.

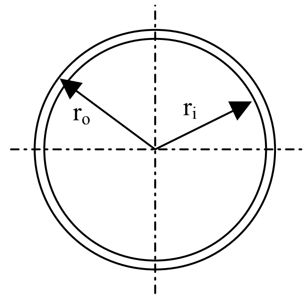
Boundary Conditions	**Frequency Equations**	$\beta_1 L$	$\beta_2 L$	$\beta_2 L$
Pinned-pinned	$\sin \beta L = 0$	3.141	6.282	9.423
Fixed-free	$\cos \beta L \cosh \beta L + 1 = 0$	1.875	4.694	7.855
Fixed-pinned (and pinned-free)	$\tan \beta L = \tanh \beta L$	3.927	7.069	10.210
Fixed-fixed (and free-free)	$\cos \beta L \cosh \beta L = 1$	4.730	7.853	10.996
Fixed-sliding (and free-sliding)	$\tan \beta L + \tanh \beta L = 1$	2.365	5.498	8.639

Table 1: Natural Frequencies for Single-Span Beams

For the experiments of dynamic beam, use Polycarbonate as the material used and measure cross sectional dimensions of the beam. Use proper equation from Table 1 to obtain the values for β_n.

Reference: "Structural Dynamics, The theory and applications," Joseph W. Tedesco, Addison Wesley Longman Inc., 1999

Appendix 1: Calculation of beam's cross sectional properties



Cross section of a hollow cylinder

$$A = \pi(r_o^2 - r_i^2)$$

$$I = \frac{1}{4}\pi(r_o^4 - r_i^4)$$

Reference: "Structural Dynamics, The theory and applications," Joseph W. Tedesco, Addison Wesley Longman Inc., 1999