Basic Electrical Measurements

E80 – 1/27/09 - Professor Sarah Harris

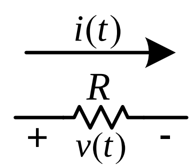
Overview

- **Electrical Building Blocks: R, L, C**
  - Impedance: Z
  - Voltage Division
  - Experimental Plots
    - Time Domain
    - V_{out} vs. V_{in}
    - Bode Plots
- **Instrumentation**
  - Signal generators
  - Measurement instruments
  - Instruments affect the measurement!

Electrical Building Blocks

- Electrical building blocks are characterized by their current-voltage (I-V) relationship.

- **Resistor:** $v(t) = i(t)R$



Example:

Given $R = 5\Omega$ and $i(t) = \sin(4\pi t)$, then $v(t) = i(t)R = 5\sin(4\pi t)$

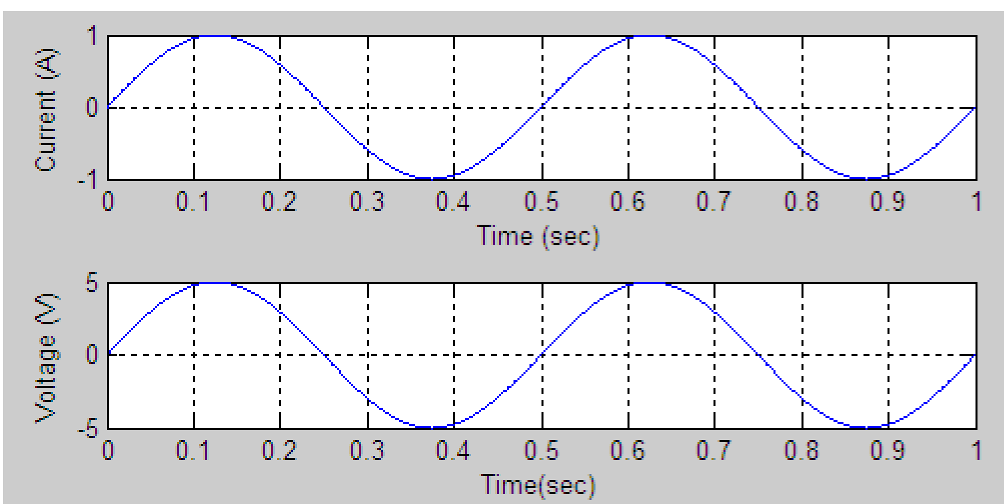
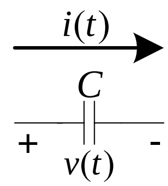


Figure 1. Resistor: Current and voltage are **in phase**.

- **Capacitor:** $$i(t) = C\frac{\partial v(t)}{\partial t}$$



Example:

Given $C = 1\mu F$ and $v(t) = \sin(4\pi t)$, then

$$i(t) = C\frac{\partial v(t)}{\partial t} = 1\mu F[4\pi\cos(4\pi t)] \approx 1.25\times10^{-5}\cos(4\pi t) = 1.25\times10^{-5}\sin(4\pi t + \frac{\pi}{2})$$

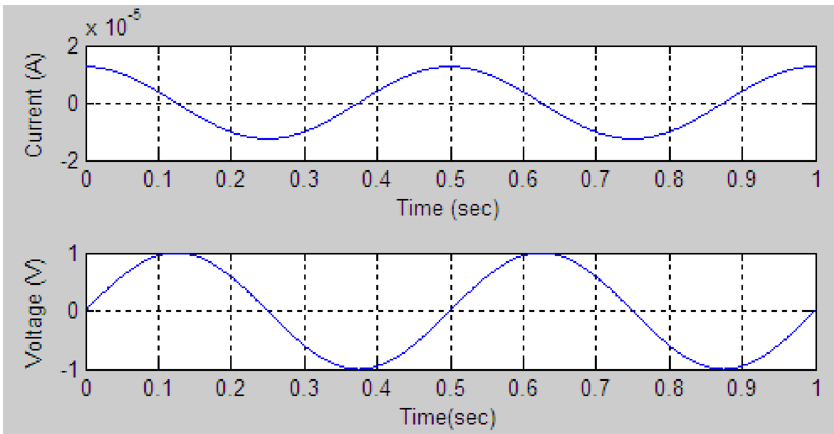
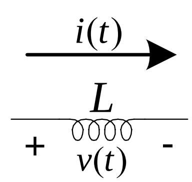


Figure 2. Capacitor: **Current is 90˚ ahead of voltage**.

- **Inductor:** $$v(t) = L\frac{\partial i(t)}{\partial t}$$



Example:

Given $L = 1\mu H$ and $i(t) = \sin(4\pi t)$, then

$$v(t) = L\frac{\partial i(t)}{\partial t} = 1\mu H[4\pi\cos(4\pi t)] \approx 1.25\times10^{-5}\cos(4\pi t) = 1.25\times10^{-5}\sin(4\pi t + \frac{\pi}{2})$$

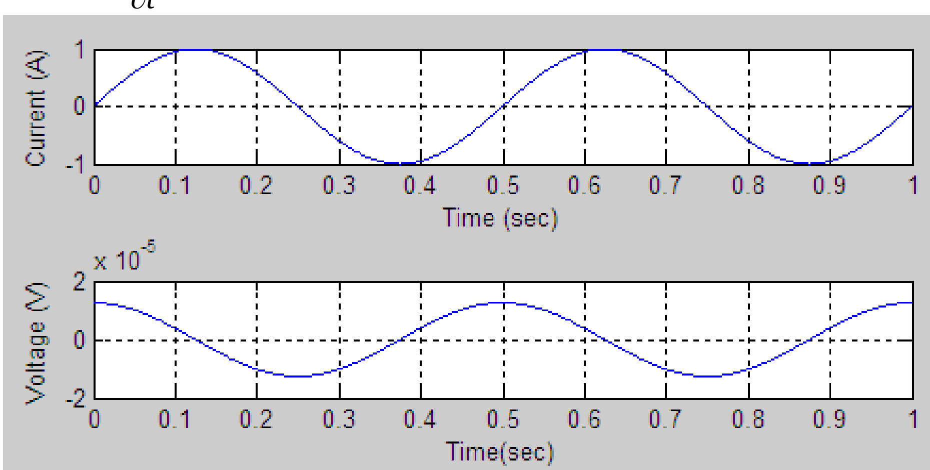


Figure 3. Inductor: **Voltage is 90˚ ahead of current.**

Impedance (Z)

- Impedance is the ratio of the voltage to current: $Z = \dfrac{V}{I}$

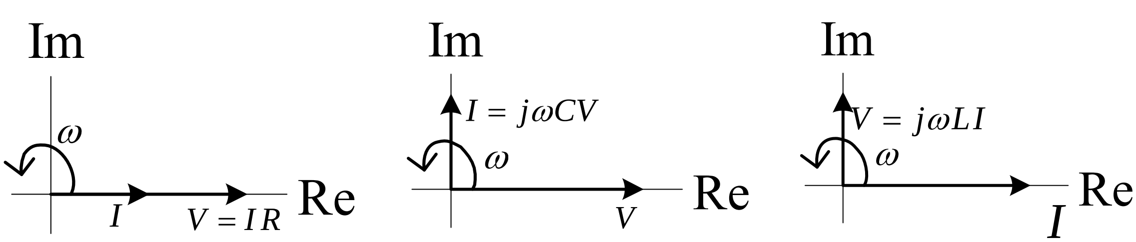
- Suppose current and voltage are represented as complex exponentials: $v(t) = Ve^{j\omega t}$, $i(t) = Ie^{j\omega t}$, where V and I are phasors, e.g. of the form $V = Ae^{j\theta}$.

- **Resistor:**
$$v(t) = i(t)R$$
$$Ve^{j\omega t} = Ie^{j\omega t}R$$
$$Z_R = \frac{V}{I} = R$$

- **Capacitor:**
$$i(t) = C\frac{\partial v(t)}{\partial t}$$
$$Ie^{j\omega t} = C\frac{\partial Ve^{j\omega t}}{\partial t}$$
$$Ie^{j\omega t} = C[\,j\omega Ve^{j\omega t}\,]$$
$$Z_C = \frac{V}{I} = \frac{1}{j\omega C}$$

- **Inductor:**
$$v(t) = L\frac{\partial i(t)}{\partial t}$$
$$Ve^{j\omega t} = L\frac{\partial Ie^{j\omega t}}{\partial t}$$
$$Ve^{j\omega t} = L[\,j\omega Ie^{j\omega t}\,]$$
$$Z_L = \frac{V}{I} = j\omega L$$

- **Phasor plots**



Resistor: Current and voltage are in phase.

Capacitor: Current is 90˚ ahead of voltage.

Inductor: Voltage is 90˚ ahead of current.

Figure 4. Phasor representation of current and voltage.

The impedance and phasor analysis gives the same results as the earlier time domain plots.

Voltage Division

We show two methods for solving for the output voltage, $v_{out}(t)$, of Circuit 1 in terms of the input voltage $v_{in}(t)$.

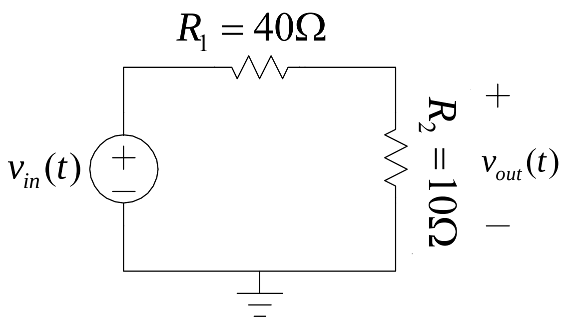


Figure 5. Circuit 1

Method 1:
Since the circuit is purely resistive, we can use simple voltage division:

$$v_{out}(t) = \frac{R_2}{R_2 + R_1} v_{in}(t) = \frac{10}{10+40} v_{in}(t) = \frac{1}{5} v_{in}(t)$$

Method 2:
For any circuit, we can find the frequency response function, *FRF*, using impedances. The $FRF = \frac{V_{out}}{V_{in}}$, where V_{out} and V_{in} are the output and input phasors of the circuit. The *FRF* gives the gain and phase of the output relative to the input. For the given circuit:

$$FRF = \frac{V_{out}}{V_{in}} = \frac{Z_{R2}}{Z_{R2} + Z_{R1}} = \frac{10}{10+40} = \frac{1}{5}$$

Since the *FRF* is completely real in this case, there is no phase shift of the output relative to the input and the output is $1/5^{th}$ the magnitude of the input.

We now use Method 2 to solve for the output voltage of a more complex system, Circuit 2 shown below.

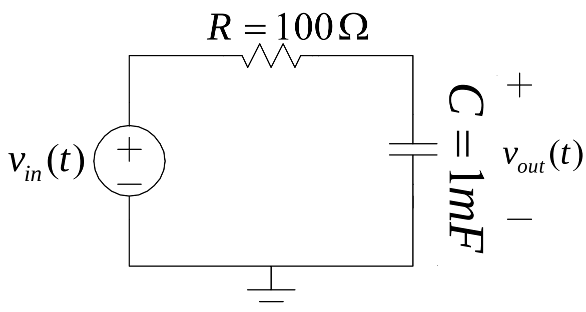


Figure 6. Circuit 2

$$FRF = \frac{V_{out}}{V_{in}} = \frac{Z_C}{Z_C + Z_R} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega RC + 1} = \frac{1}{j\omega(0.1) + 1}$$

The gain and phase depend on the frequency of the input.

Case 1: Suppose $v_{in}(t) = \sin(2\pi(50)t)$, then

$$\frac{V_{out}}{V_{in}} = \frac{1}{j\omega(0.1)+1} = \frac{1}{j2\pi(50)(0.1)+1} = \frac{1}{j31+1} \approx \frac{1e^{j0}}{31e^{j\frac{\pi}{2}}} = \frac{1}{31}e^{-j\frac{\pi}{2}} = 0.03e^{-j\frac{\pi}{2}}$$

Thus, the magnitude of $v_{out}(t)$ is 0.03 smaller than $v_{in}(t)$ and its phase is shifted by about $-\frac{\pi}{2}$.

$$v_{out}(t) \approx 0.03\sin(2\pi(50)t - \frac{\pi}{2})$$

Case 2: Suppose $v_{in}(t) = \sin(2\pi(0.05)t)$, then

$$\frac{V_{out}}{V_{in}} = \frac{1}{j\omega(0.1)+1} = \frac{1}{j2\pi(0.05)(0.1)+1} = \frac{1}{j0.031+1} \approx \frac{1e^{j0}}{1e^{j0}} = 1e^{j0} = 1$$

Thus, the magnitude and phase of $v_{out}(t)$ are the same as $v_{in}(t)$.

$$v_{out}(t) = v_{in}(t) = \sin(2\pi(0.05)t)$$

Experimental Plots
Plotted experimental results give insights into the characteristics of a system. Here we discuss several useful experimental plots: time domain plots, v_{out} vs. v_{in} and Bode plots.

- **Time Domain**

For Circuit 2 in Figure 6, we plot $v_{in}(t)$ and $v_{out}(t)$. If $v_{in}(t) = \sin(2\pi(50)t)$, $v_{out}(t) = 0.03\sin(2\pi(50)t - \frac{\pi}{2})$. The experimental results can be obtained using a signal generator as the input and an oscilloscope to measure the output.

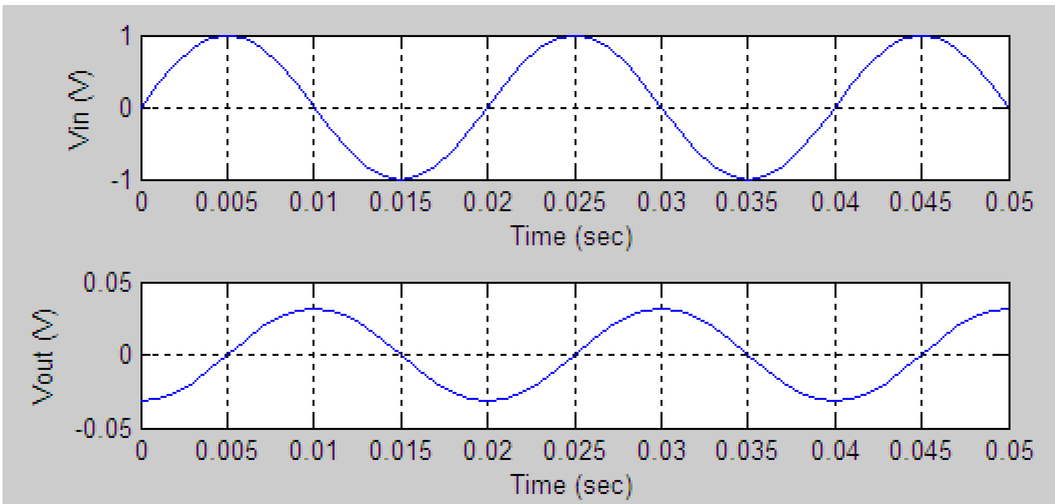


Figure 7. Time domain plots

- v_{out} vs v_{in}

For Circuit 2 in Figure 6, we can plot v_{out} vs v_{in} at various frequencies. Below we show v_{out} vs v_{in} for $\omega = 2\pi(50)t$. The magnitude of the output is measured for various input magnitudes, as shown for 0 to 20 V. The slope of the line (0.3/10 = 0.03) gives the gain of the system at the set frequency, in this case 50 Hz.

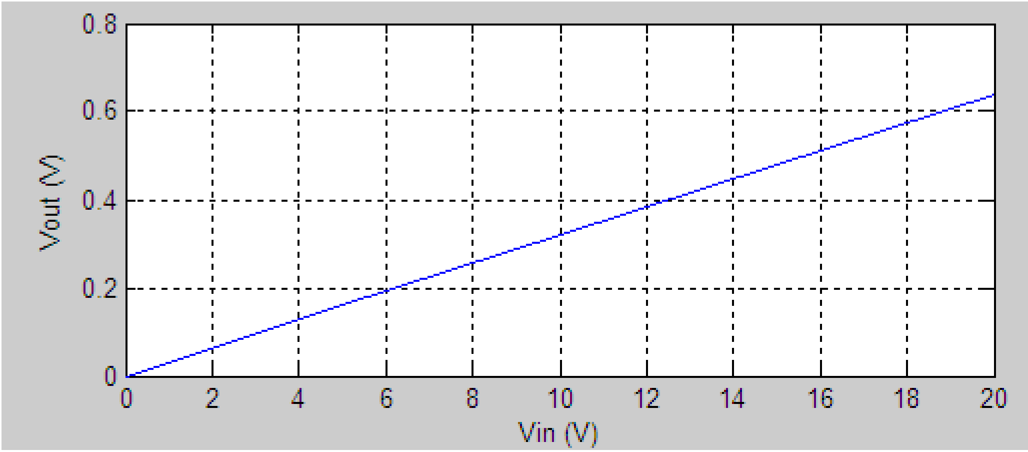


Figure 8. v_{out} vs v_{in}

- **Bode Plots**

A system's response at different frequencies is clearly displayed by its Bode plot. Below is the Bode plot of Circuit 2. A Bode plot can be experimentally determined by measuring the output voltage and phase on an oscilloscope while varying the input frequency. Recall that the magnitude plot gives the Log Magnitude,

$$Lm = 20\log_{10}\left|\frac{V_{out}}{V_{in}}\right|.$$

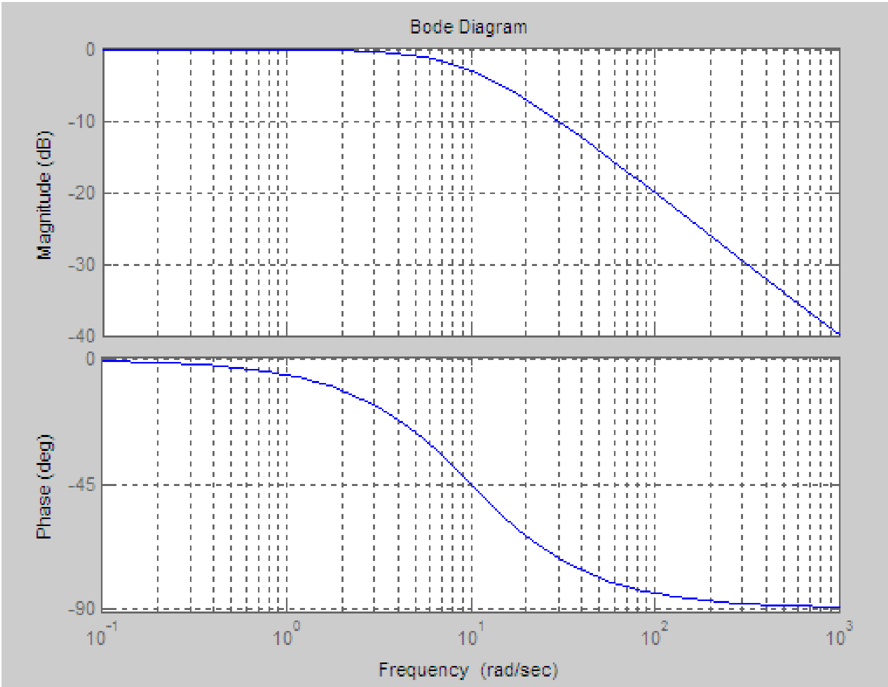


Figure 9. Bode plots. Note that in this plot the x-axis is in rad/s.

Remember that to make an experimental Bode plot you must measure **both** the output and the input.

Instrumentation

Instrumentation is used to generate and measure signals and devices. Some useful instruments are: signal generators, power supplies, multimeters (ohmmeters, ammeters, voltmeters), and oscilloscopes. Specifications and operation of a specific instrument are found in its manual.

- **Signal generators** (e.g., HP/Agilent 33120A)

Signals: Signal generators (also called function generators) generate sine waves, square waves, triangle waves, etc. Because the signals change over time, the signal generator is as an AC (alternating current) supply.
Parameters: The interface allows the user to select the signal amplitude (Ampl), frequency (Freq), etc.
Output Impedance: The signal generator has an internal resistance, sometimes referred to as the "output resistance or impedance" or "source resistance or impedance". A signal generator can be modeled as shown below, with the output/source resistance labeled R_S.
Vpp: The Agilent 33120A allows you to set the peak-to-peak voltage Vpp. This measures the signal from its maximum to its minimum. So, the peak voltage is twice the signal's magnitude: Vpp = 2Vm. For example, Vpp for $v_s(t) = 3\sin(2\pi(50)t)$ is 6 V.
Out Term: The signal generator allows you to select among two loads: High Z (large impedance) or 50 Ω. A "load" is the circuit to which you connect the signal generator. Notice that in the figure below, the signal source $v_s(t) = 5\sin(2\pi t)$ is actually the same, only the signal generator settings have changed. The signal generator can't tell what kind of load you actually use, so if you set the Out Term to 50 Ω and then use a very large (relative to 50 Ω) load, the measured amplitude won't match the displayed amplitude. If you are driving a load that is neither very large nor 50 Ω, you'll need to choose one of the settings and then model the entire circuit to find out what voltage will be delivered to the load.

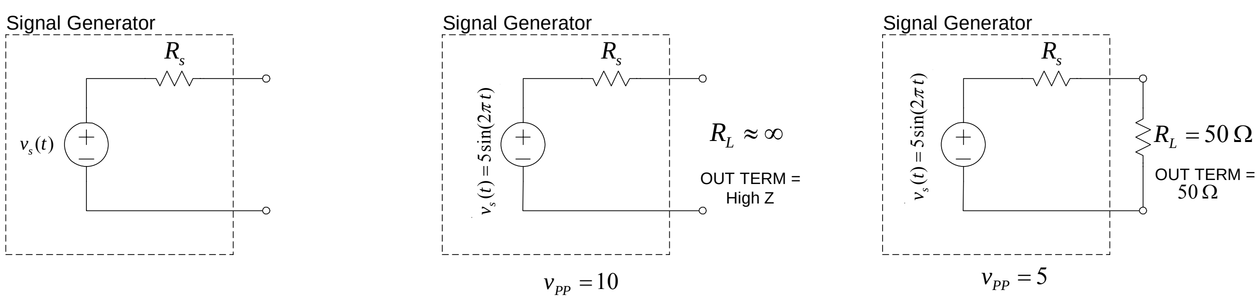


Figure 10. Signal generator with various loads

- **Power supplies** (e.g., HP/Agilent 6236)

Power supplies generate a DC (direct current or unchanging) signal.
Note: The "COM" port is the common reference (usually referred to as ⏚ in your circuit) for all of the generated power supplies. The "GND" port is the ground reference coming in from the wall. "COM" and "GND" are **not** connected internally.

- **Multimeters** (e.g., Simpson 260 (analog), Elenco (digital))

Using different settings, multimeters can measure voltage, resistance, or current.

Ammeter: measures current. Place multimeter leads in series with circuit. Internal impedance is low to avoid changing the behavior of the measured circuit. For example, the Simpson 260 can measure current as low as 50μA. Digital multimeters can usually measure lower currents.
Voltmeter: measures voltage. Place multimeter leads in parallel with circuit. Internal impedance is large to avoid changing the behavior of the measured circuit. For example, the Simpson 260, DC impedance is 20 kΩ per volt. So with a 10 V scale, the impedance is 200 kΩ. Digital voltmeters have a fixed impedance, typically ~10MΩ.
Ohmmeter: measures resistance. Place multimeter leads in parallel with resistor. An ohmmeters use an internal battery to supply a voltage and then measure the current through the meter, so disconnect the resistor from any other external power supply.

- **Oscilloscopes** (e.g., Tektronix 2212)

AC signals: It is most straightforward to use an oscilloscope to monitor an AC signal as a function of time. When an oscilloscope is not available, multimeters can also be used to measure the AC voltage (or current).
Internal impedance: Just like multimeters, an oscilloscope (or "scope") has finite internal impedance. These numbers can often be read directly off the port of the oscilloscope.
1x and 10x probes: A 1x probe reads the measured signal directly (with some instrument loading effect due to the probe and internal impedance). A 10x probe increases the impedance of the probe and scope by 10 times (10x), thus decreasing the instrument loading effect. This also decreases the current through the probe by a factor of 10, so the signal read on the scope is decreased by a factor of 10. Some oscilloscopes have a setting to indicate the kind of probe being used.

- **Instruments affect the measurement!**

Both signal generation and signal measurement (scopes, multimeters) are nonideal. To accurate model our systems, we must take non-idealities into account. Below are some examples of how the instruments themselves change our measurements.

Example 1:
What is the output voltage across the load resistor given the following settings: Out Term = 50 Ω, $v_{PP} = 5V$? From the manual, we read that the output (or source) impedance, R_s, is 50 Ω. (For now, assume an ideal scope probe for measuring the output voltage.)

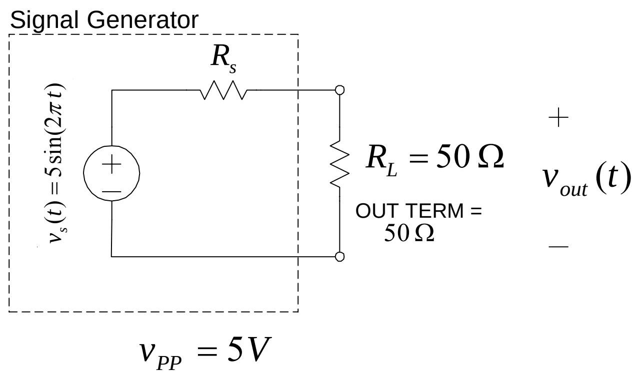


Figure 11. Example 1

Since $R_s = R_L$, half of the input voltage is dropped across the 50 Ω load resistor.

$$v_{out} = \frac{R_L}{R_L + R_S} v_s = \frac{1}{2} v_s$$

So we expect that $v_{out}(t) = 2.5\sin(2\pi t)$. The peak-to-peak voltage of the output, v_{PP}, is 5V, as displayed on the signal generator.

Example 2:
What is the output voltage across the load resistor given the following settings: Out Term = 50 Ω, $v_{PP} = 5V$? From the manual, we read that the output (or source) impedance, R_s, is 50 Ω. (For now, assume an ideal scope probe for measuring the output voltage.)

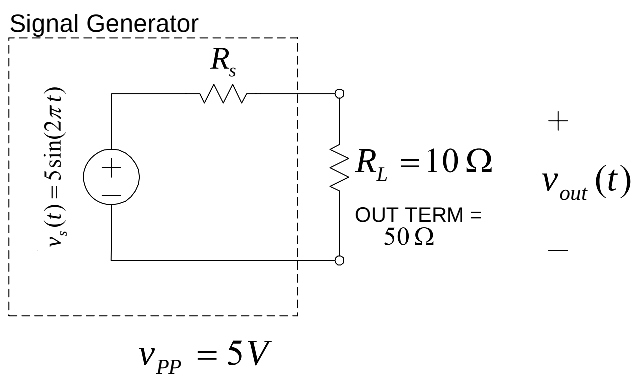


Figure 12. Example 2

$$v_{out} = \frac{R_L}{R_L + R_S} v_s = \frac{1}{6} v_s$$

So we expect that $v_{out}(t) = \frac{5}{6}\sin(2\pi t)$. The peak-to-peak voltage of the output, v_{PP}, is $\frac{5}{3}$. Note, this is **not** the value displayed on the signal generator.

Example 3:
What is the output voltage across the load resistor given the following settings: Out Term = High Z, $v_{PP} = 5V$? From the manual, we read that the output (or source) impedance, R_s, is 50 Ω. (For now, assume an ideal scope probe for measuring the output voltage.)

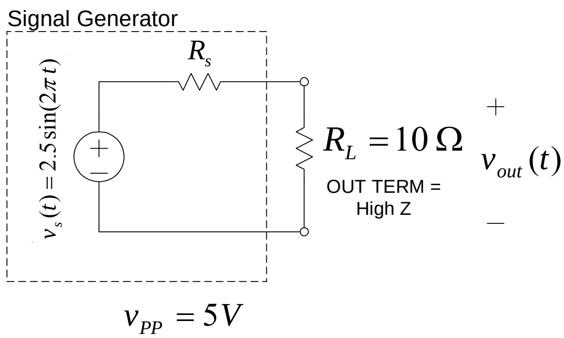


Figure 13. Example 3

Since we set the Out Term to High Z, the Vpp value would be output only if we actually had a large (approximately infinite) load impedance. However, as above, only 1/6th of the signal gets transferred to the load, so $v_{out}(t) = \frac{5}{12}\sin(2\pi t)$. The peak-to-peak voltage of the output, v_{PP}, is $\frac{5}{6}$. Again, this is **not** the value displayed on the signal generator.

Example 4:
Now let's examine another place where we introduce instrument loading effects: signal detection. First consider the ideal case:

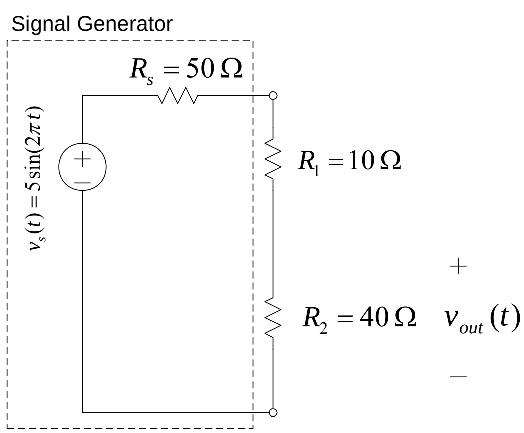


Figure 14. Example 4

$$v_{out} = \frac{R_2}{R_2 + R_1 + R_s} v_s = \frac{2}{5} v_s$$

So we expect that $v_{out}(t) = 2\sin(2\pi t)$. Now, suppose our scope and probe have a combined impedance of 1 M Ω, as shown in the Figure 15. What is the measured output voltage?

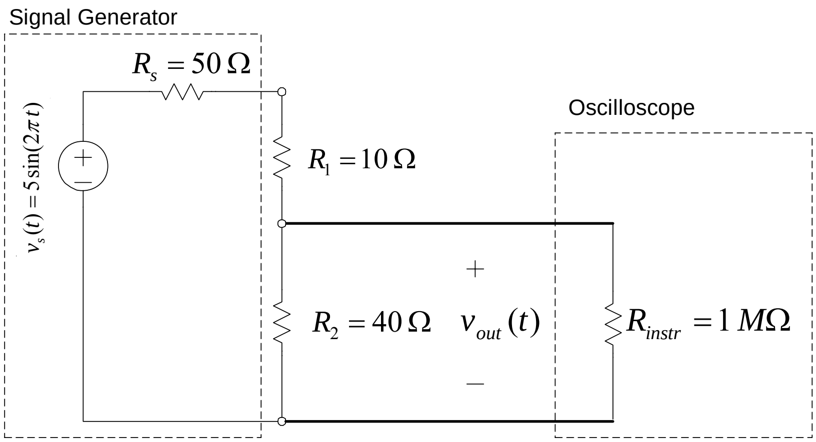


Figure 15. Example 4 with Oscilloscope

$$v_{out} = \frac{R_2 \,||\, R_{instr}}{(R_2 \,||\, R_{instr}) + R_1 + R_s} v_s = \frac{2}{5} v_s$$

Since R_{instr} is large compared to R_2, $R_2 \,||\, R_{instr}$ is about equal to R_2. So we have the same result as before: $v_{out}(t) = 2\sin(2\pi t)$.

Now suppose R_1 and R_2 are replaced by larger resistors, as shown below.

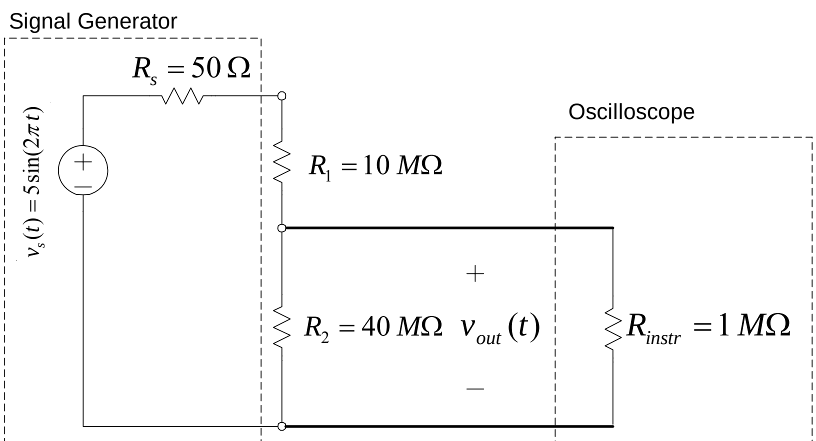


Figure 11. Example with Oscilloscope

Now we have:

$$v_{out} = \frac{R_2 \,||\, R_{instr}}{(R_2 \,||\, R_{instr}) + R_1 + R_s} v_s \approx \frac{R_{instr}}{R_{instr} + R_1} = \frac{1}{11} v_s$$

Without the instrument loading effect (R_{instr}), we would have expected:

$$v_{out} = \frac{R_2}{R_2 + R_1 + R_s} v_s \approx \frac{R_2}{R_2 + R_1} = \frac{4}{5} v_s$$

Measuring AC signals

Oscilloscopes are best instruments to use to measure AC signals. However, analog multimeters (like the Simpson 260) can also be used. The Simpson 260 measures the root-mean-squared or rms voltage.

$$v_{rms} = \sqrt{\frac{1}{T_0}\int_{T_0} v^2(t)\, dt}$$

So, for example for a sine wave,

$$v_{rms} = \sqrt{\frac{1}{T_0}\int_{T_0} v^2(t)\, dt} = \sqrt{\frac{1}{T_0}\int_{T_0} \sin^2(\frac{2\pi}{T_0}t)\, dt} = \frac{1}{\sqrt{2}} = 0.707$$

> **Side note:** the rms voltage is a useful value for calculating average power dissipation. For example, the power through a resistor is:
>
> $$P = iv = \frac{v^2}{R}$$
>
> With an AC voltage, you could measure the instantaneous power (using the equation above) or the average power, which is usually most useful, using the equation below.
>
> $$P_{ave} = \left(\frac{v^2}{R}\right)_{ave} = \frac{(v^2)_{ave}}{R} = \frac{\frac{1}{T_0}\int_{T_0} v^2(t)\, dt}{R} = \frac{v_{rms}^2}{R}$$

The Simpson 260 actually measures the voltage by first finding the average value of the rectified signal (absolute value of the signal) and then scaling the measured average value to produce the rms value. The average value is physically produced by rectifying the input signal, and storing the output onto a capacitor, which then averages the signal.

$$v_{ave} = \frac{1}{T_0}\int_{T_0} |v(t)|\, dt = \frac{1}{T_0}\int_{T_0} \left|\sin(\frac{2\pi}{T_0}t)\right| dt = \frac{2}{\pi} = 0.637$$

The Simpson 260 scales its internally measured value by 1.11 ($\frac{0.707}{0.637} = 1.11$) to produce the rms value. So, for example if a sinusoidal voltage of amplitude 1 V is input into the Simpson 260, the displayed rms voltage will read 0.707 V. The internally measured value (not output) is the average value of the rectified signal (0.637 V), and that value is internally multiplied by 1.11 to produce the reading of 0.707 V.

Note: If the AC input signal is not a sine wave, it will still take the average value of the rectified signal and scale it by 1.11. The analog voltmeter will not report the correct rms voltage for anything other than a sine wave. However, if you know the shape of the input, you can derive the correct value from the reported number.