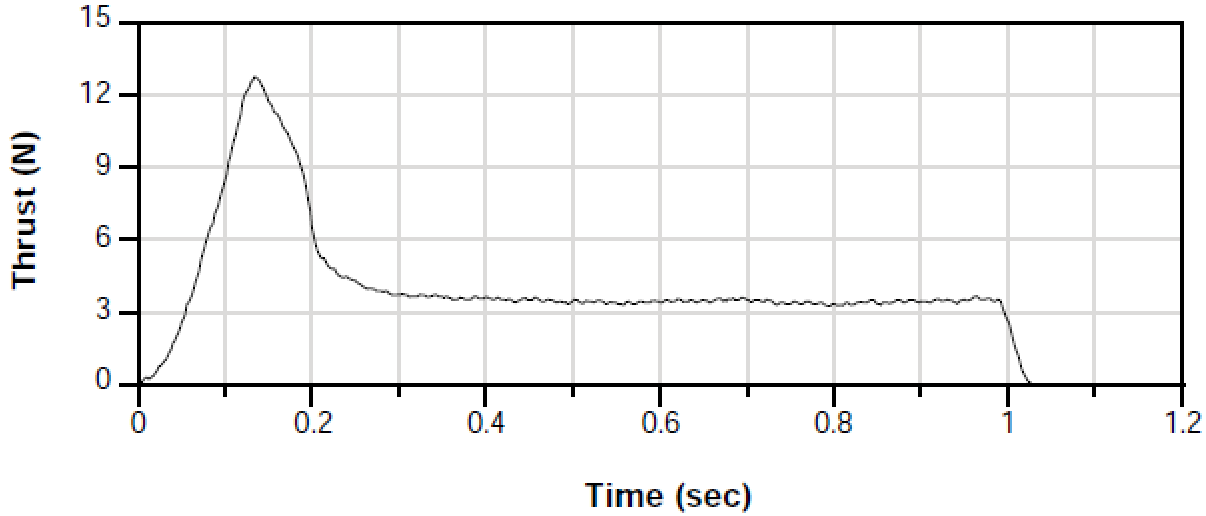
Thrust and Flight Modeling

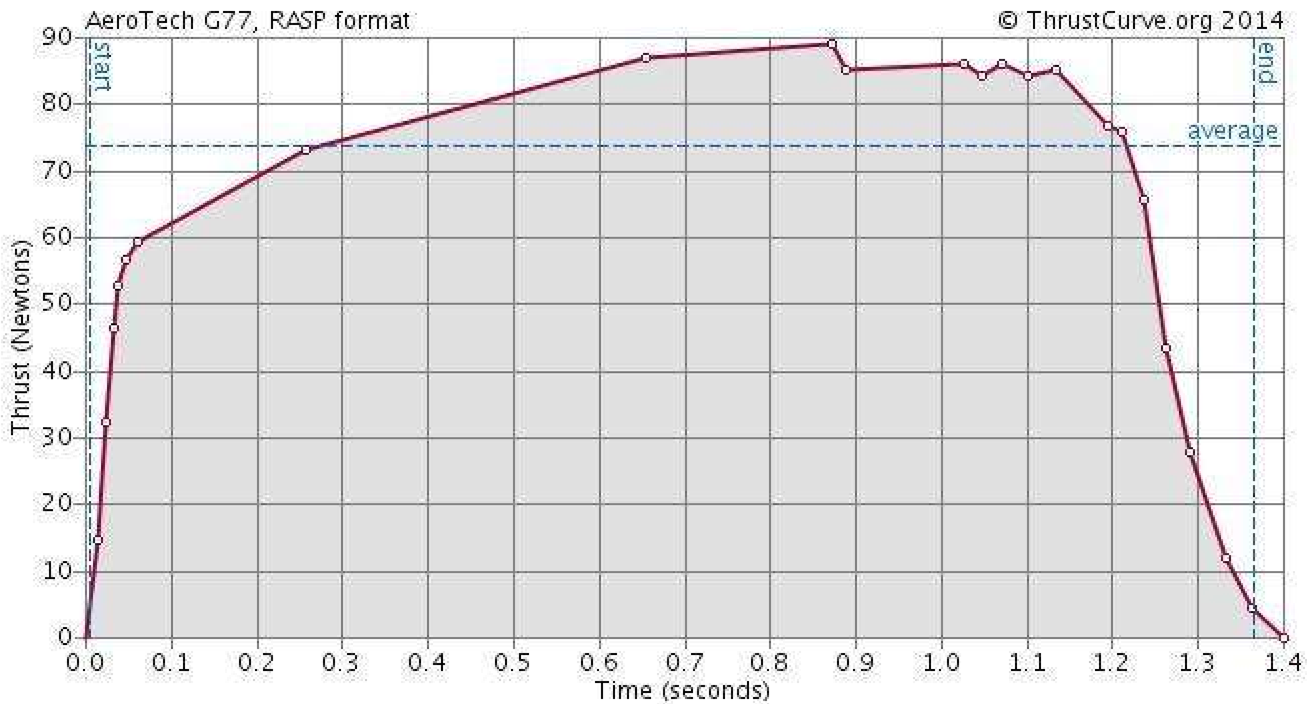
- How did OpenRocket or Rocksim predict the trajectory, velocity, and acceleration of our vehicle?
- What did we input into these numerical tools?

Thrust Curve of an Estes B4 Motor



AeroTech G77

Data Graph



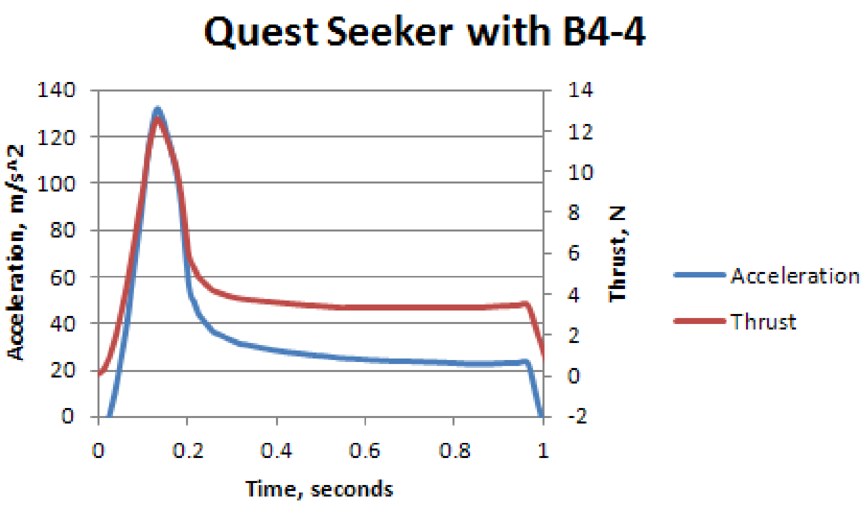
http://www.thrustcurve.org/simfilesearch.jsp?id=288

Thrust Curve Input

- We need the thrust over the time the rocket motor is firing
- OpenRocket and Rocksim have thrust curves for many motors built in
- See thrustcurve.org as well
- Also thrust curve data are available on E80 web site for previous static motor firings
  - http://www.eng.hmc.edu/NewE80/StaticTestData.html

What does thrust do to our vehicle?

- OpenRocket prediction of acceleration vs. time for Quest Seeker with a B4-4 motor
- Secondary y-axis shows thrust of B4-4 motor



E80 Static Motor Test Lab

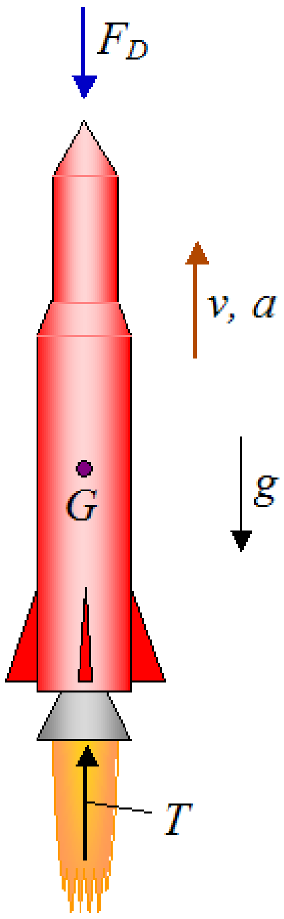
- “Measure the thrust curves, mass flow rate of combustion gases and specific impulse for two rocket motors.”
- “Construct analytical and 1-D and 2-D numerical models of rocket flight.”
- “Compare the analytical and numerical models with the output of RockSim or OpenRocket.”

http://www.eng.hmc.edu/NewE80/StaticTestLab.html

What will affect our rocket's flight?

- So we know thrust is important.
- What else?

Vertical Powered Flight



http://www.real-world-physics-problems.com/rocket-physics.html

Vertical Powered Flight

- Thrust
  - From thrust curve
- Weight
- Drag force
  - How do we get this?
    - Coefficient of drag
    - $F_D = \frac{1}{2}\rho V^2 \mathrm{A} C_D$
    - NOTE: Drag is a function of velocity

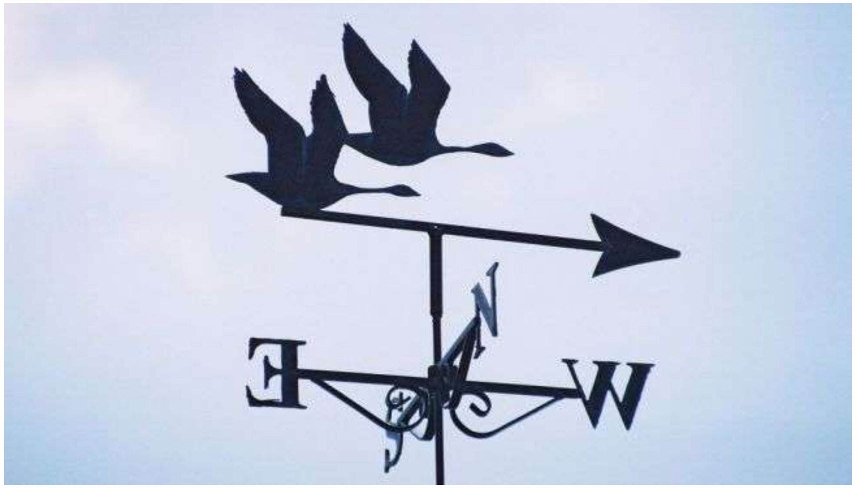
1DOF Equations of Motion

- Newton's 2nd Law: sum of forces = time rate of change of linear momentum
- Not too difficult to analyze when the rocket is in vertical flight
  - Vehicle acceleration is along what we think of as "vertical"
  - Thrust, weight, drag all lined up
- $\mathrm{m}\frac{dv}{dt}$ = ma =Thrust – Drag – Weight
  - We assumed constant mass here
- Does the rocket always fly vertically during flight?

E80 Videos

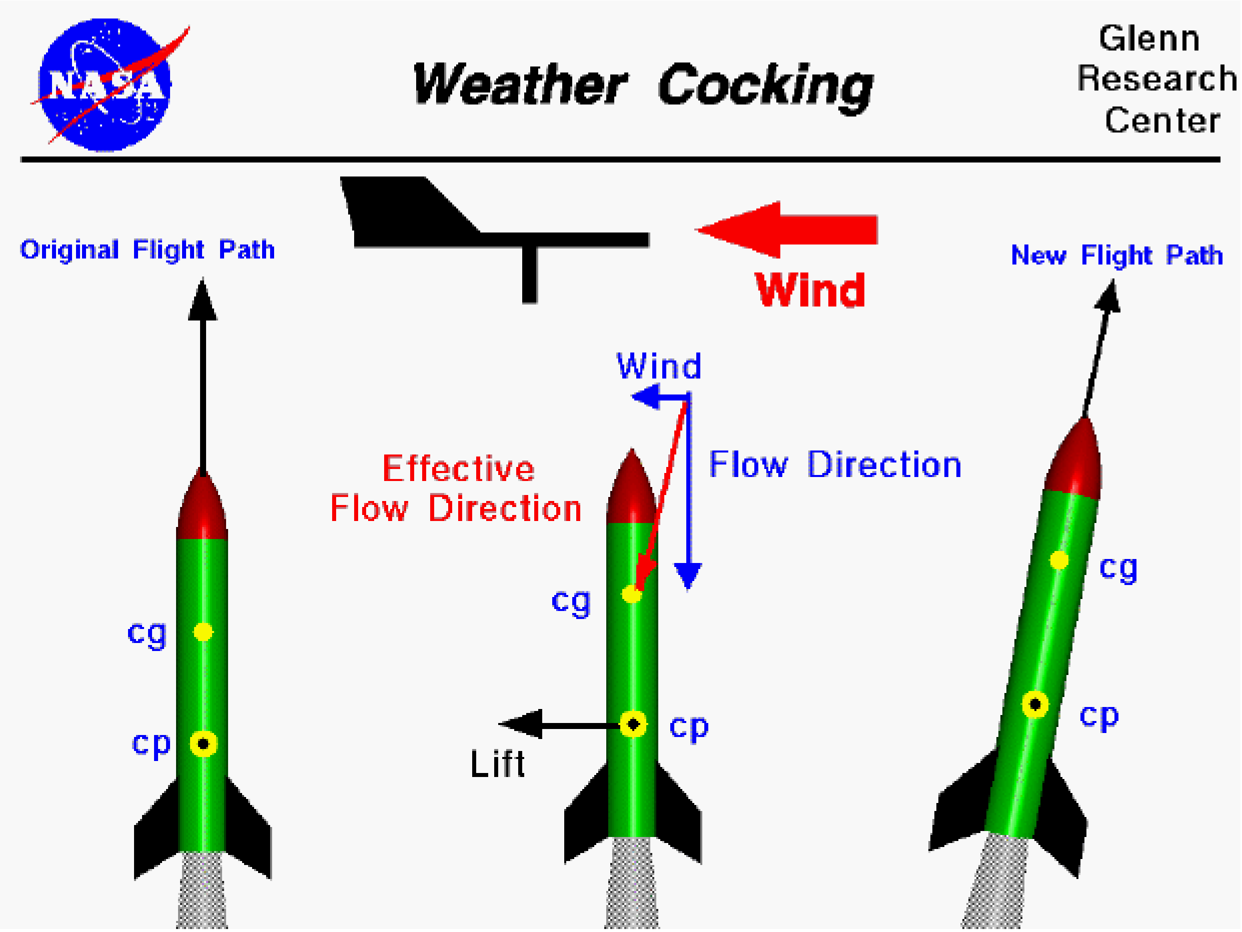
- http://www.eng.hmc.edu/NewE80/FlightVideos.html

Weather Cocking



http://g3nqf.co.uk/weather-vane-not-a-weather-cock

Rocket path is not always vertical



http://www.grc.nasa.gov/WWW/k-12/VirtualAero/BottleRocket/airplane/rktcock.html

Weather Cocking

- Lift is generated, since the rocket is at an angle of attack to the wind
- Where does the lift force act?
  - Center of Pressure (Cp)
  - Is this the same thing as the center of mass?
- Due to the lift force and location of the Cp, a moment is generated about the CG and the rocket **rotates**

Weather Cocking

- When does it stop rotating?
  - When the rocket is aligned in the direction of the relative wind (when the angle of attack is zero), the lift force goes to zero, and so does the moment due to that lift force
- When designing our vehicle we need to take into account damping, as a well-designed rocket is probably not critically damped or overdamped

Damping coefficient (E59 stuff)

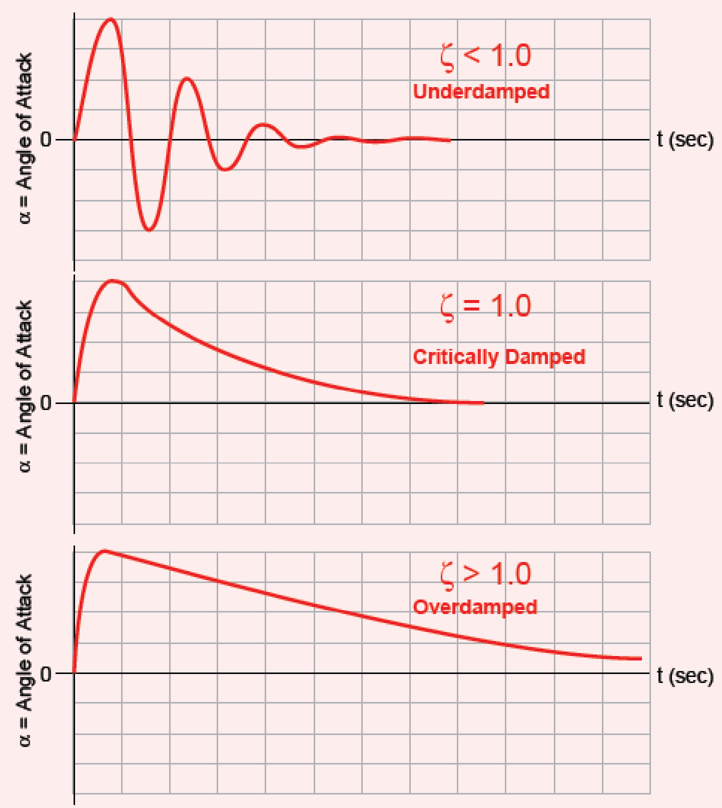


Figure 1: The damping ratio tells us how a rocket is going to respond to a disturbance in flight. For example, whether or not it will oscillate.

http://www.apogeerockets.com/downloads/Newsletter197.pdf

Dynamic characteristics

- Usually we have to trade-off between increase in drag (maybe bigger fins) and living with some oscillations
- Topics in Advanced Rocketry recommends a damping coefficient between 0.05 and 0.3
- Rocksim can calculate the damping ratio explicitly
- Look into the functional form for torsional damper (rotational analogy to linear damper)

Back to Trajectory Modeling

- So, in order to predict our trajectory if we have a side wind, we'll need
  - Aerodynamic forces
  - Angle of attack
  - Rocket geometry
  - Windspeed

Trajectory Modeling

- We still write Newton's 2nd Law, but what do we also have to write now to describe the rotation?
- What else happened that we need to account for?

3DOF Equations of Motion

- Need to solve angular momentum equation until the rocket has rotated into the direction of the relative wind (until angle of attack is zero)
- Need to solve two linear momentum equations
- But now we have local and global coordinate systems, and as the rocket rotates, it continually changes orientation

Coordinate Frames

- Thrust and Aerodynamic Forces are relative to the vehicle (local) frame
  - Note: we are now taking “Lift” as the force normal to the rocket long axis and not perpendicular to the relative wind
  - Why?
- Weight is easy in the global frame (though not at all difficult in the local frame)
- However, we are after a trajectory (position) in the global frame
  - Euler Rotation Theorem
  - Quaternions

Numerical solution of eqns of motion

- Could do
  - Explicit Euler
  - Implicit Euler
  - Runge-Kutta (OpenRocket does RK4)
- Recall your differencing equations from the First Flight Lab
- $v(t+\Delta t) = v(t) + \frac{dv}{dt}\Delta t + \frac{d^2v}{dt^2}\frac{\Delta t^2}{2} + \cdots$

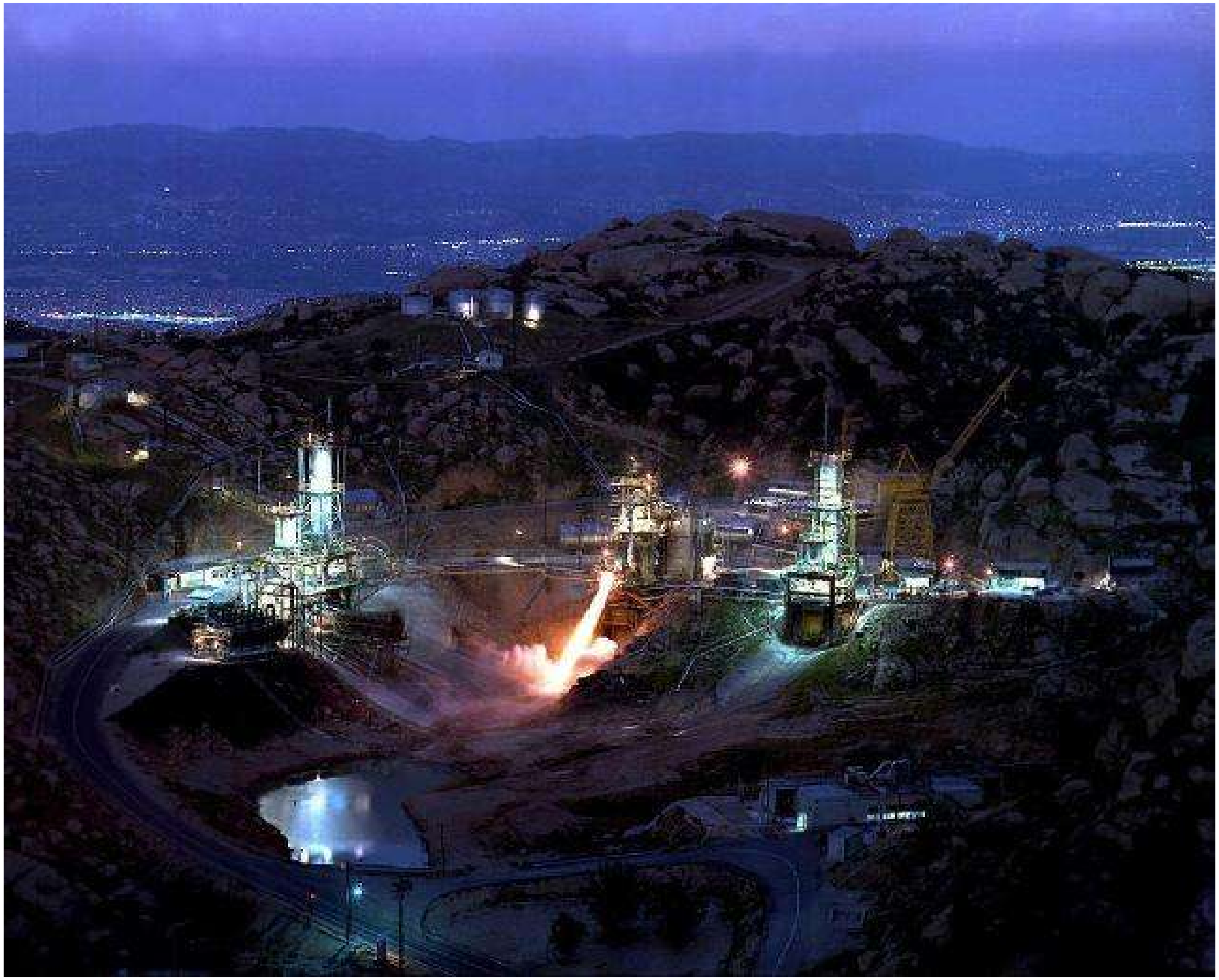
Explicit Euler

- $\frac{dv}{dt}$=$\frac{v(t+\Delta t)-v(t)}{\Delta t}$ (ignoring H.O.T.)
- m($\frac{v(t+\Delta t)-v(t)}{\Delta t}$)=Thrust(t)-Drag(t)-Weight(t)
- $v(t + \Delta t)$=$\frac{\Delta t}{m}$ [Thrust(t)-Drag(t)-Weight(t)] +$v(t)$
- We know everything on the RHS at time t
- We calculate our new velocity at time $t + \Delta t$
- Then march forward in time
  - Consider the size of your time step if you choose Explicit Euler (Prof. Spjut's video lecture from 2012 does a demo)
  - Also see the VIs and an excel spreadsheet linked to the lab for examples

1DOF vs. 3DOF

- 1DOF = one degree of freedom
  - Rocket goes up vertically, no side wind, therefore no rotation
  - Solve one linear momentum equation
    - Can predict maximum altitude
- 3DOF = three degrees of freedom
  - Side wind causes rocket to rotate into wind
  - Need to solve one angular momentum equation as well as two linear momentum equations
    - End up predicting trajectory in two coordinates
- 6DOF = six degrees of freedom
  - When you absolutely positively got to nail down every $^*#& variable in the room[1] (3 linear momentum equations; 3 angular momentum equations)

1.) Apologies to Quentin Tarantino ("Jackie Brown")



http://www.rocketdynearchives.com/areai/bo
wlarea.html

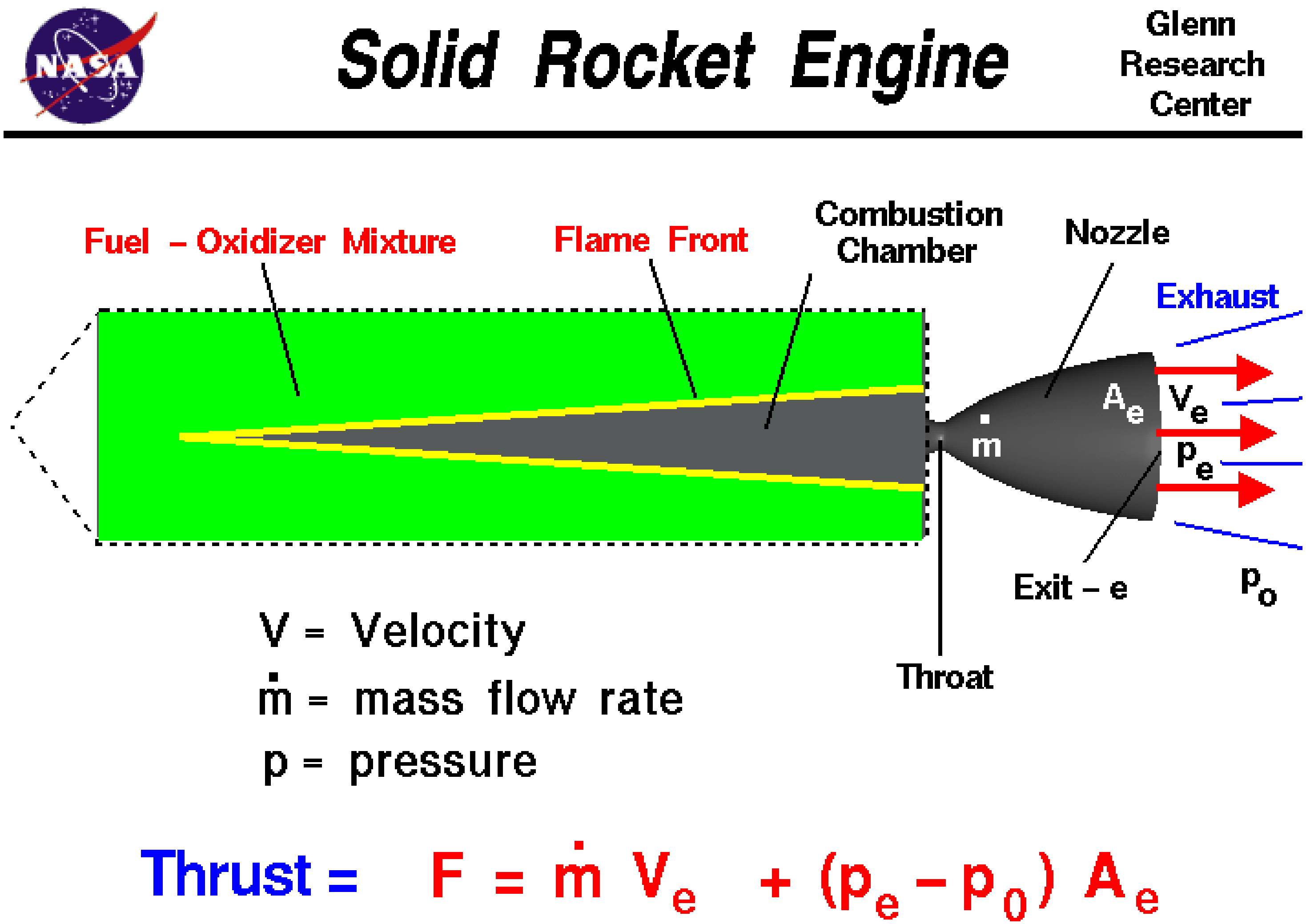
http://crgis.ndc.nasa.gov/historic/File:01-086-30.jpg

Test Stand Video

- https://www.youtube.com/watch?v=TaJhrFQoxh8
- https://www.youtube.com/watch?v=0ON88MC8BQs
- https://www.youtube.com/watch?v=jH0SHe8u2FE

Rocketdyne Santa Susana Field Laboratory

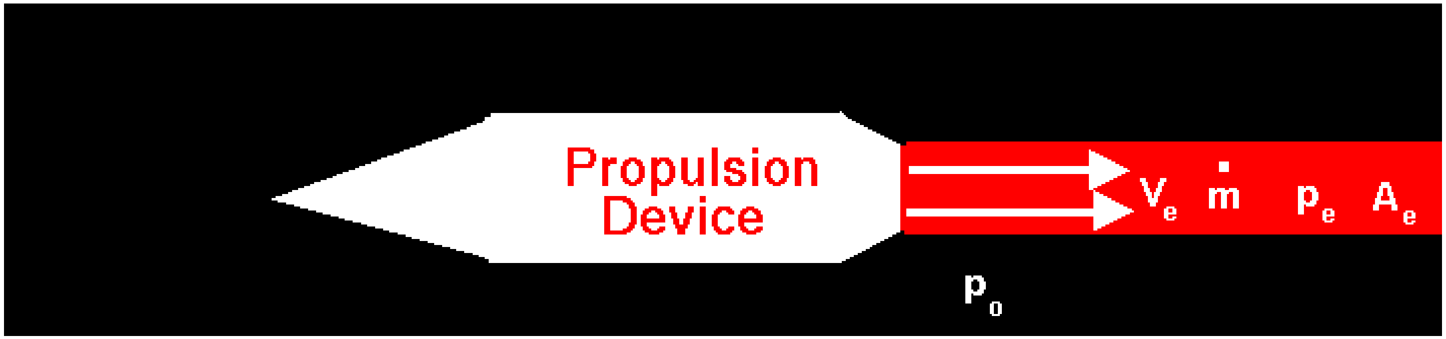
- Aside: Horribly contaminated site, Simi Valley residents and Rocketdyne workers justifiably upset about this
- Nuclear reactor test site—uranium fuel rods ruptured and partially melted in 1959
  - No containment
- Runoff of various contaminants
  - Groundwater contamination (perchlorate)



http://www.grc.nasa.gov/WWW/k-12/airplane/Images/srockth.gif

Specific Impulse

Glenn Research Center



Rocket Thrust Equation $F = \dot{m} V_e + (p_e - p_o) A_e$

where p = pressure, V = velocity, A = area, $\dot{m}$ = mass flow rate, F = thrust

Define: Equivalent Velocity: $V_{eq} = V_e + \frac{(p_e - p_o) A_e}{\dot{m}}$ $F = \dot{m} V_{eq}$

Define: Total Impulse: $I = F \Delta t = \int F \, dt = \int \dot{m} V_{eq} \, dt = m V_{eq}$

Define: Specific Impulse: $\frac{\text{Total Impulse}}{\text{Weight}}$ $Isp = \frac{I}{m g_o} = \frac{V_{eq}}{g_o}$ units = sec

$$Isp = \frac{F}{\dot{m} g_o}$$

http://www.grc.nasa.gov/WWW/k-12/airplane/Images/specimp.gif

Solid Motors

O-rings

Propellant grains

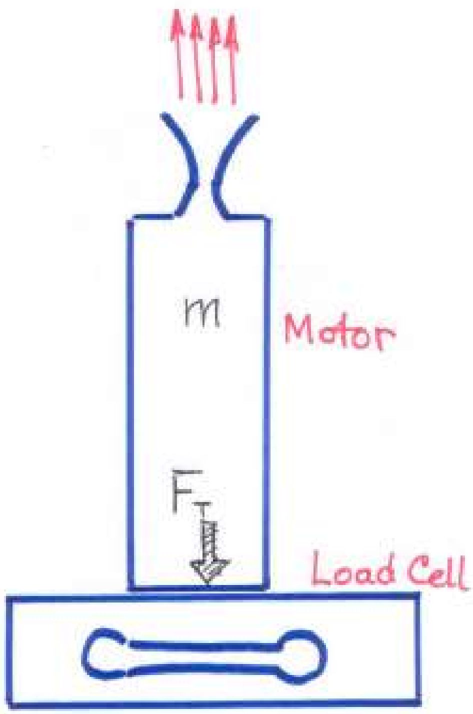
Nozzle

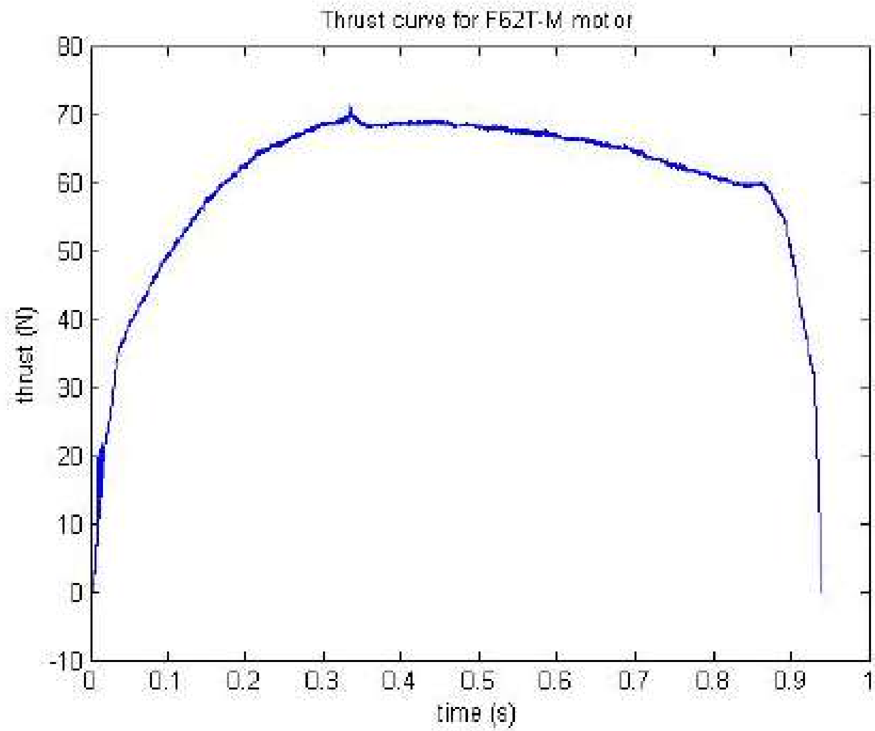
Delay grain

Solid rocket motor

- https://www.youtube.com/watch?v=SZgxEdajwdE

Static Motor Test





http://www.eng.hmc.edu/NewE80/PDFs/E80StaticMotorTestMotion2012.pdf

Static Motor Test

$$F_T = \dot{m}V_e + \left(P_e - P_0\right)A_e = \dot{m}V_{eq}$$

$$\dot{m} = \frac{m_{final} - m_{initial}}{t_{burn}} = \frac{\Delta m}{t_{burn}}$$

$$V_{eq} \cong \frac{F_{T,average}}{\dot{m}}$$

http://www.eng.hmc.edu/NewE80/PDFs/E80StaticMotorTestMotion2012.pdf

Propellant

- Estes motors: black powder--charcoal, sulfur, and saltpeter (potassium nitrate)
- Aerotech motors: composite propellant--ammonium perchlorate, synthetic rubber and a metal fuel, like aluminum

E80 videos

- http://www.eng.hmc.edu/NewE80/StaticTestVideos.html

Area-Velocity Ratio (Nozzles)

- Boardwork