E80

Experimental Engineering

Lecture 8

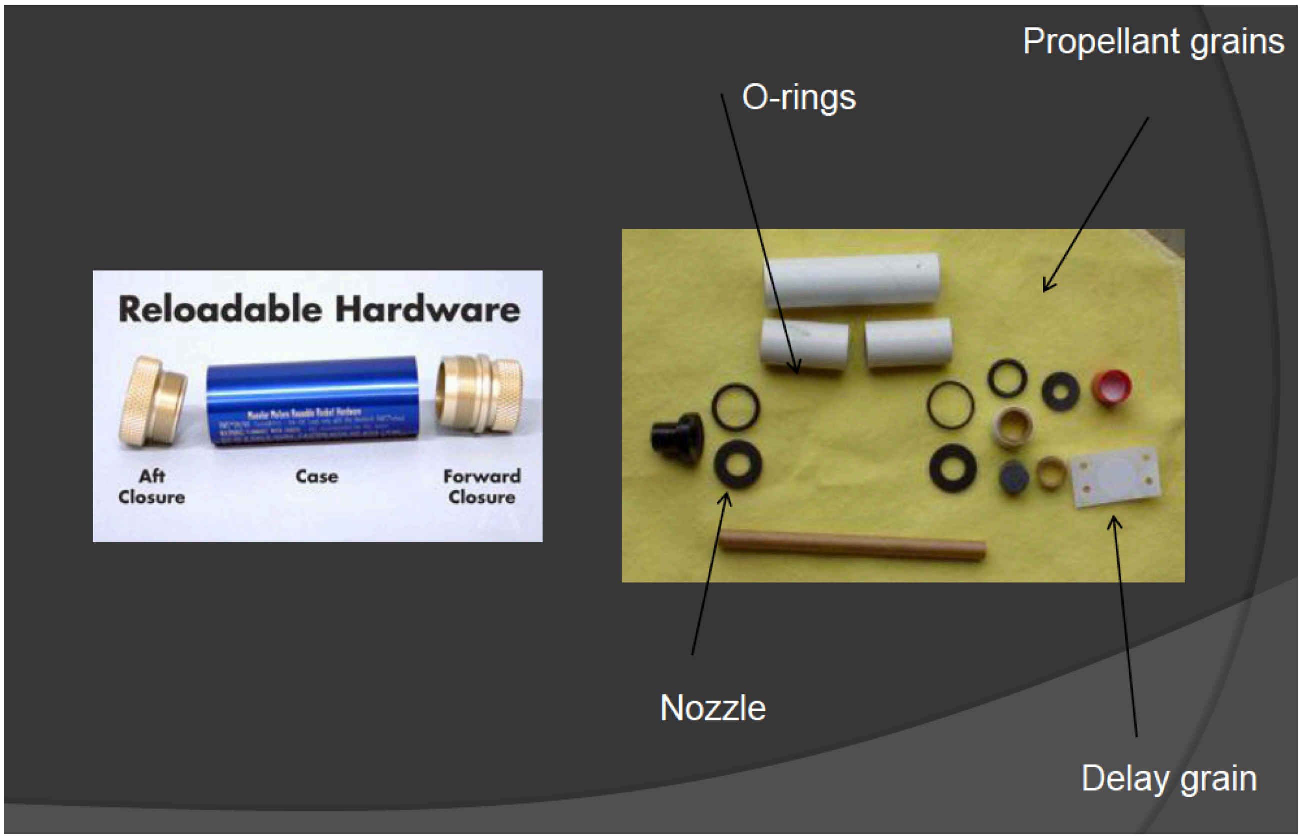
Thrust and light Modeling

http://twistedsifter.com/2012/10/red-bull-stratos-space-jump-photos/

Outline

- Static Motor Rotation Lab
- Overview of Modeling
- 1DOF Model
- 3DOF Model

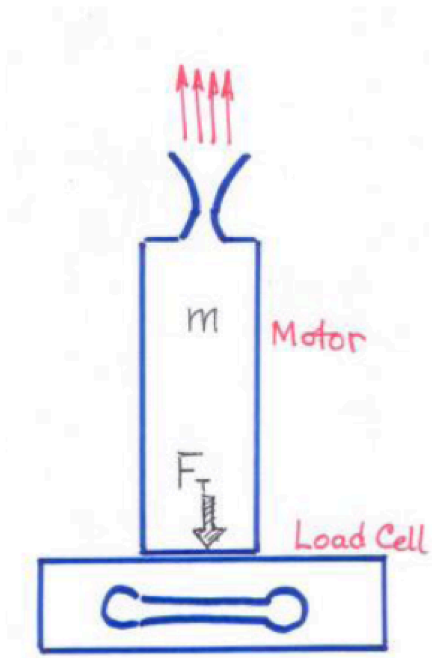
Static Motor Rotation Lab

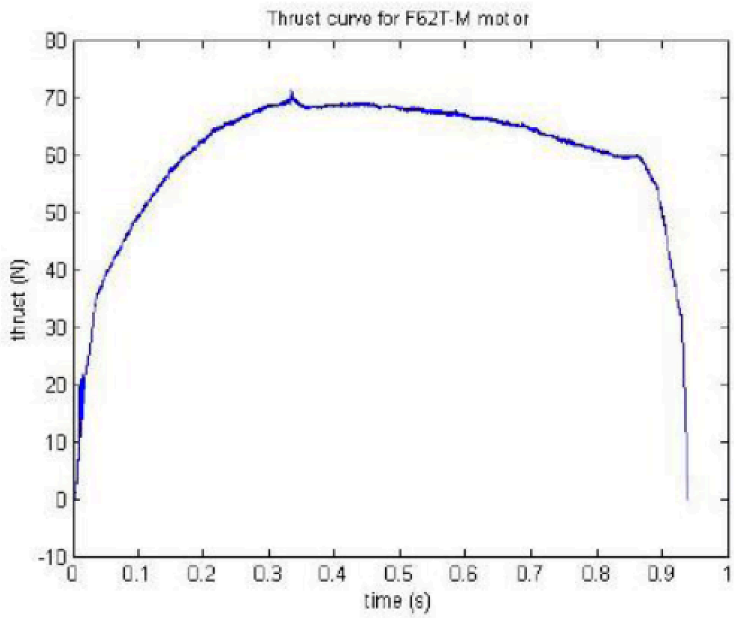


Static Motor Rotation Lab

- To Dos…
  - Calibrate Load Cell
  - Measure Thrust Curve of Rocket Motor
  - Measure average mass flow rate
  - Model 1DOF and 3DOF flight trajectory
  - Compare with OpenRocket or Rocksim

Static Motor Rotation Lab





Static Motor Rotation Lab

- You can calculate the average flow rate

$$\dot{m} = \frac{m_{final} - m_{initial}}{t_{burn}} = \frac{\Delta m}{t_{burn}}$$

- You can measure the average thrust, hence you can estimate the exit velocity

$$V_{eq} \cong \frac{F_{T,average}}{\dot{m}}$$

Outline

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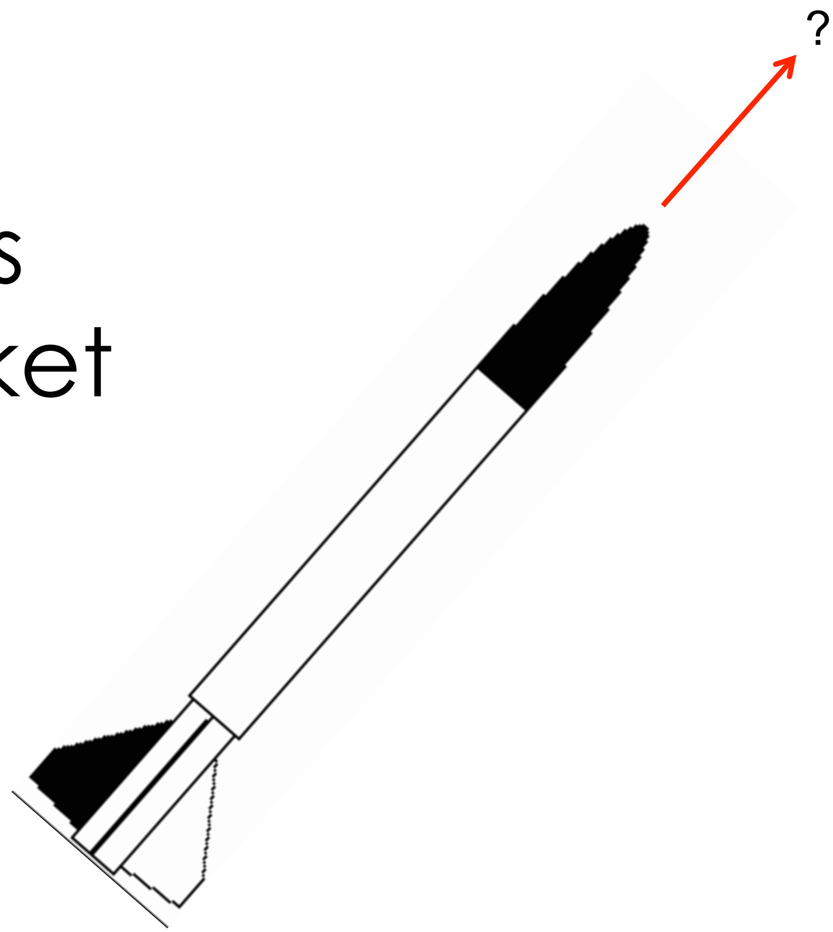
Where is the rocket going?

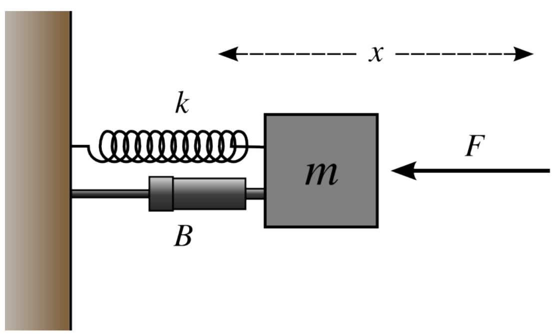


Outline

- Overview of Modeling
- 1DOF Model
- 3DOF Model
- More about the motor…

Strategy

- Use our Newtonian Physics
  1. Draw our Free Body Diagram
  2. Derive Equations of Motion
  3. Solve equations



Outline

- Static Motor Rotation Lab
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One DOF Model

- Free Body Diagram

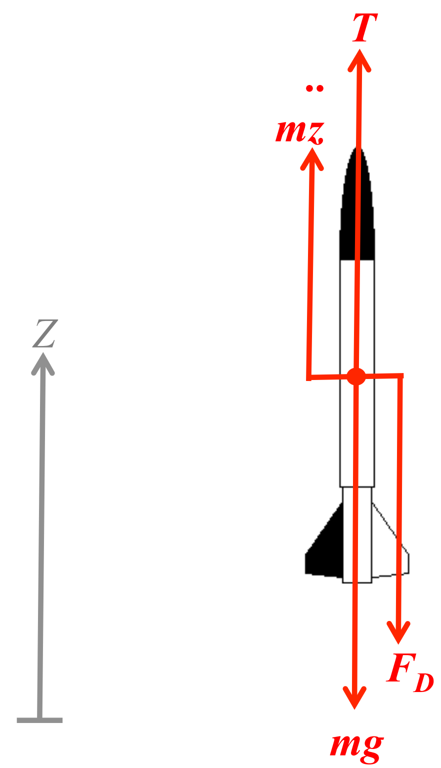
z = Acceleration in Z direction

m = mass

T = Thrust Force

g = 9.81 ms^{-2}

F_D = Drag Force

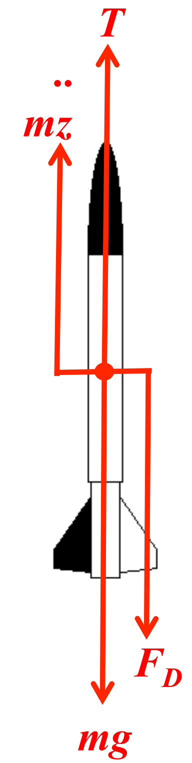


One DOF Model

- Equations of Motion

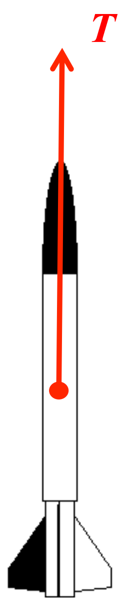
$$m\ddot{z} = T - mg - F_D$$

$$m = m(t)$$



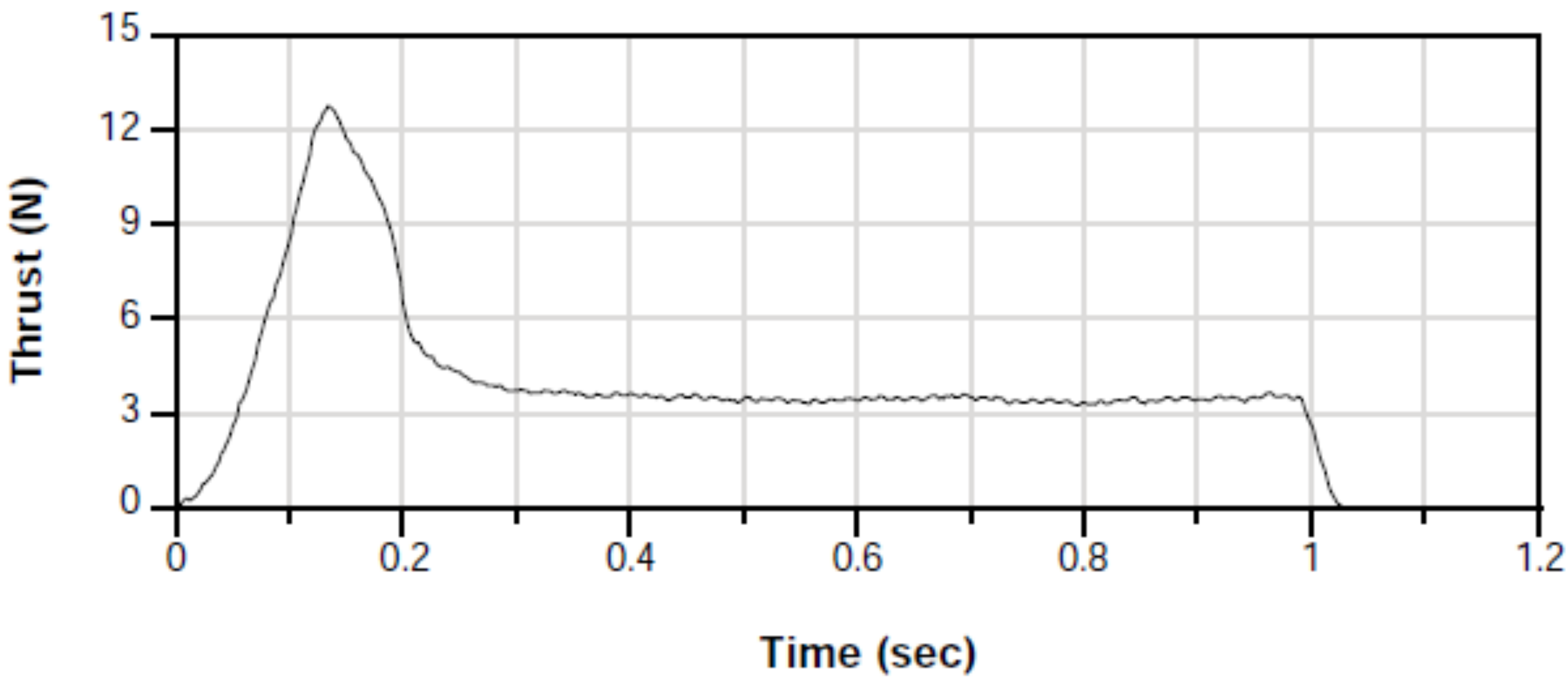
One DOF Model

- Thrust
  - We know that motor thrust…
    - -Is not constant
    - -Is dependent on the motor
    - -Can be modeled experimentally
    - -Can be extracted from simulations (e.g. OpenRocket)



One DOF Model

- Thrust



One DOF Model

- Drag

$$F_d = \tfrac{1}{2}\rho A V^2 C_D$$

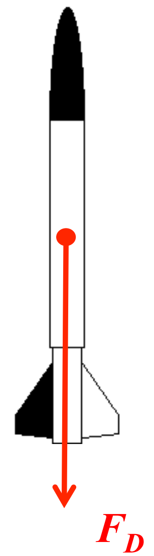
where

ρ = density of fluid

V = velocity of rocket

A = Functional area

C_D = Drag Coefficient



One DOF Model

- Solving the Equations

$$m\ddot{z} = T - mg - F_D$$

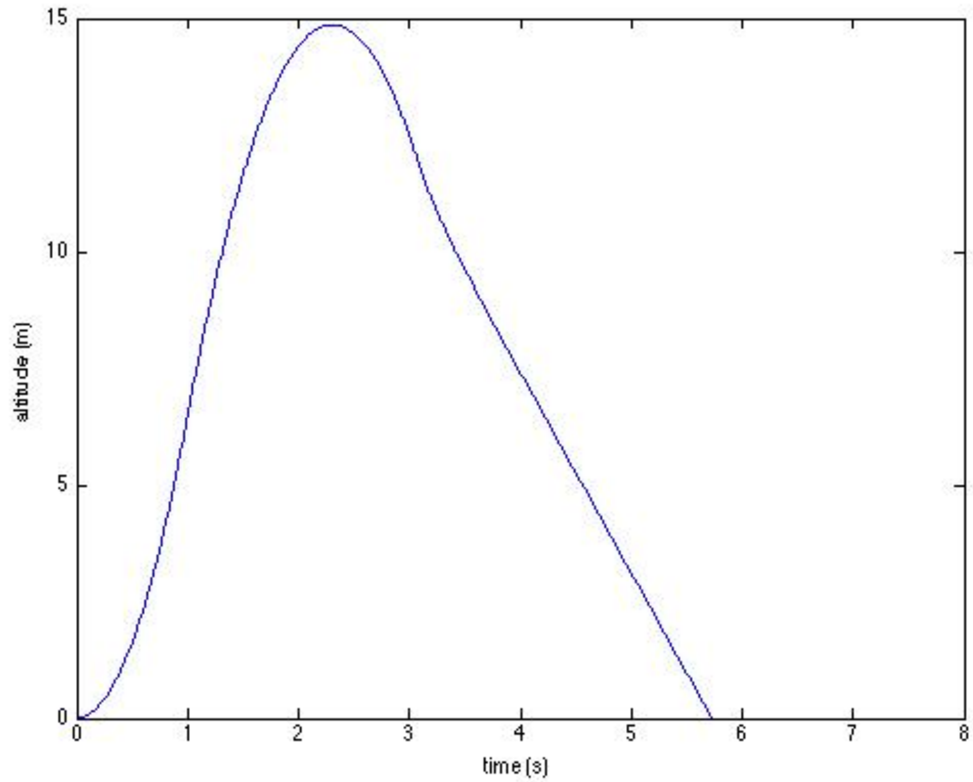
  - OpenRocket uses **Runge-Kutta** (RK4)
  - We can use **Euler equations** that ignore high order terms…

One DOF Model

```
for t = 0 to maxTime
{
        T = …
        m = …
        Fd = …

        z_dd(t) = 1/m*(T-m*g-Fd);
        z_d(t) = z_d + z_dd*Δt
        z(t) = z + z_d*Δt
}
```

One DOF Model



Outline

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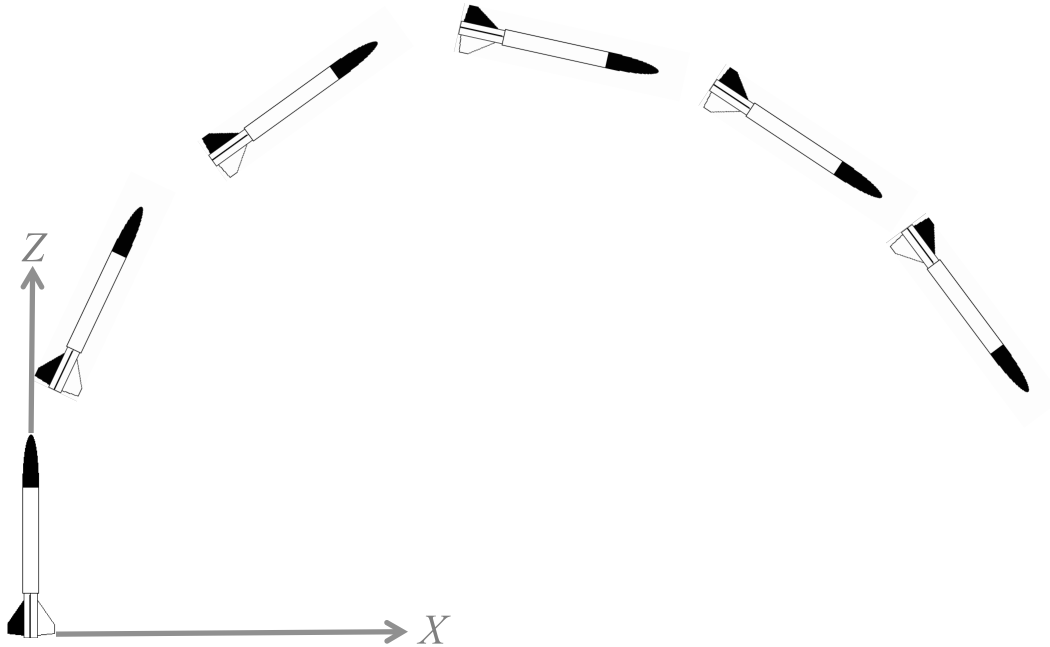
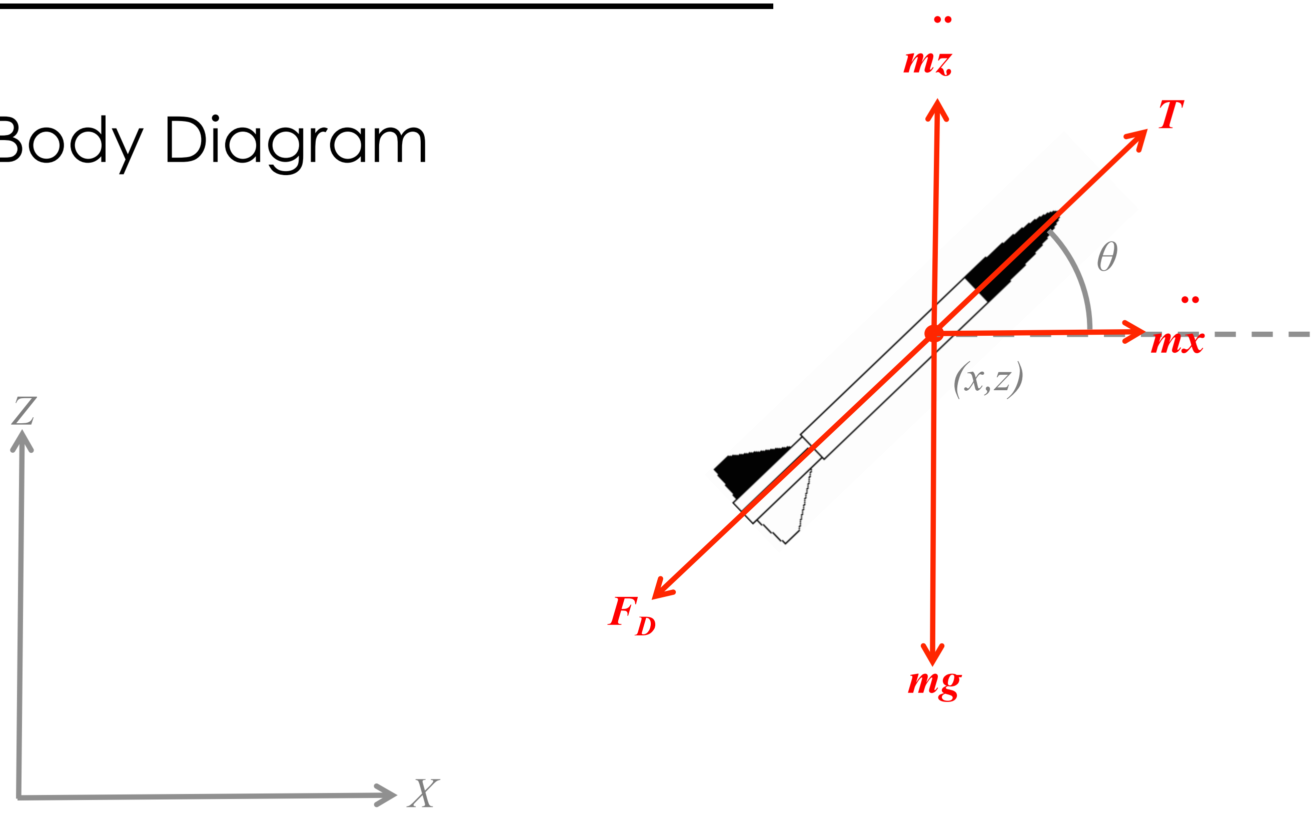
Three DOF Model

- What are the 3DOF?



Three DOF Model

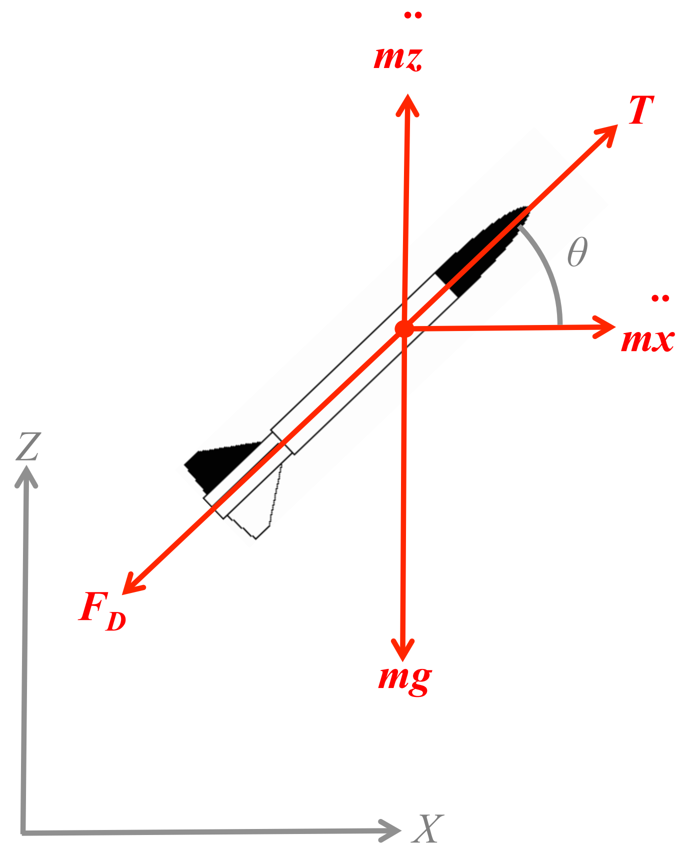
- Free Body Diagram



Three DOF Model

- Equations of Motion
  - Must now sum forces in 2 directions

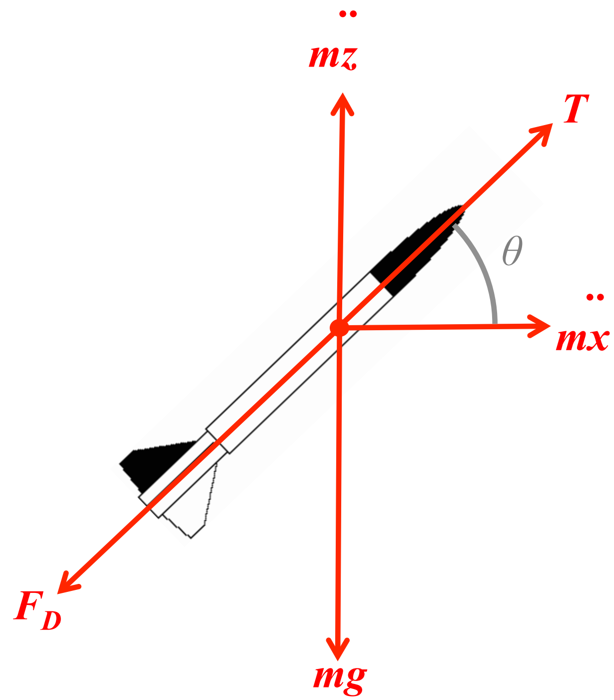
$$m\ddot{z} = T\sin\theta - mg - F_D\sin\theta$$
$$m\ddot{x} = T\cos\theta - F_D\cos\theta$$



Three DOF Model

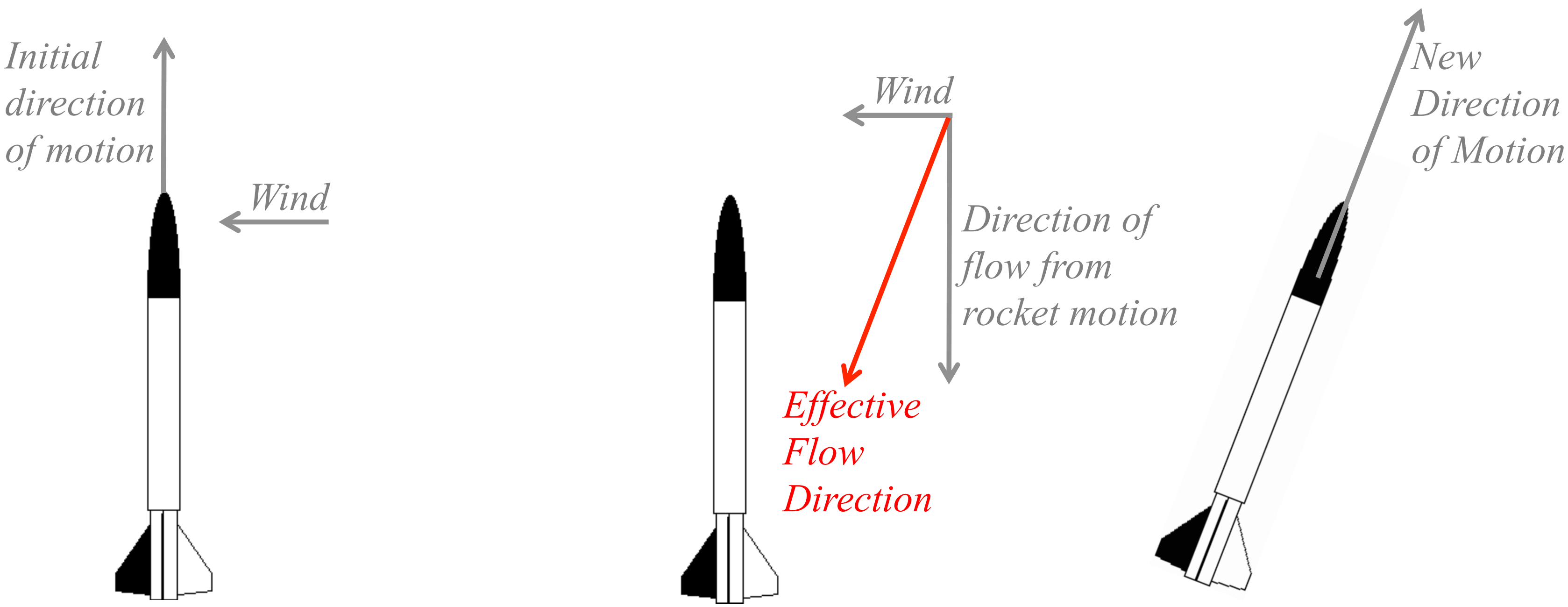
- Free Body Diagram

Is it this simple?



Three DOF Model

- Free Body Diagram



Three DOF Model

- Free Body Diagram

L = Lift Force

T_D = Damping Torque

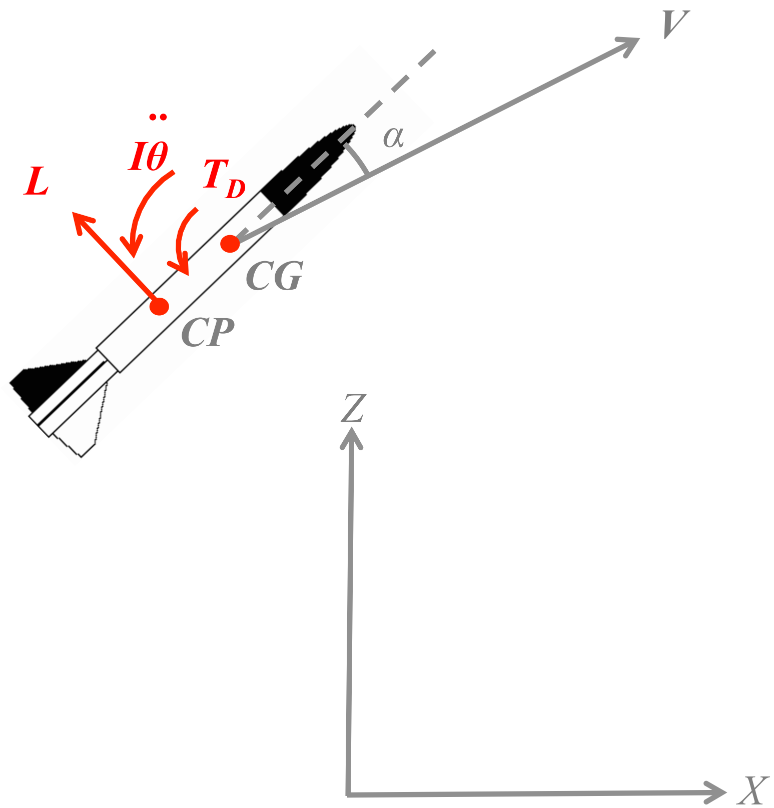
V = velocity vector

α = angle of attack

CG = Center of Gravity

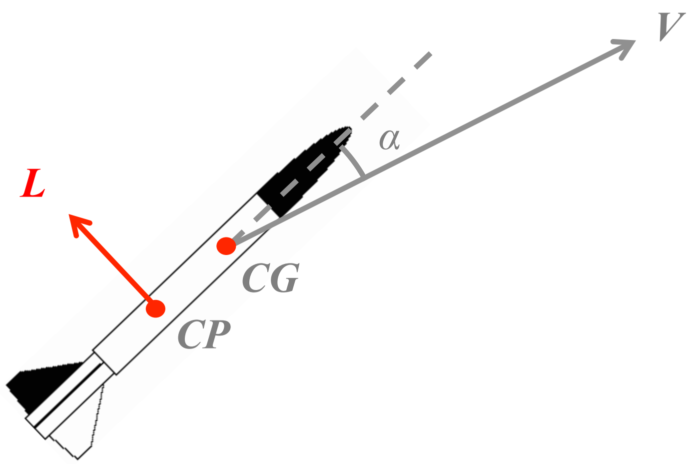
CP = Center of Pressure

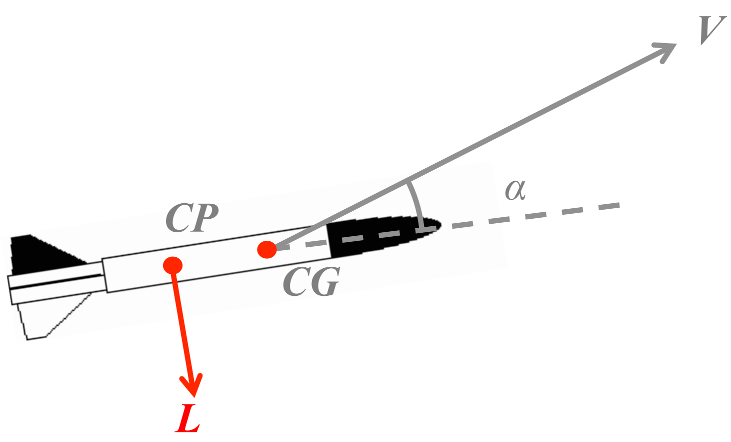
I = moment of Inertia



Three DOF Model

- Is this stable?
  - Depends on location of CP versus CG





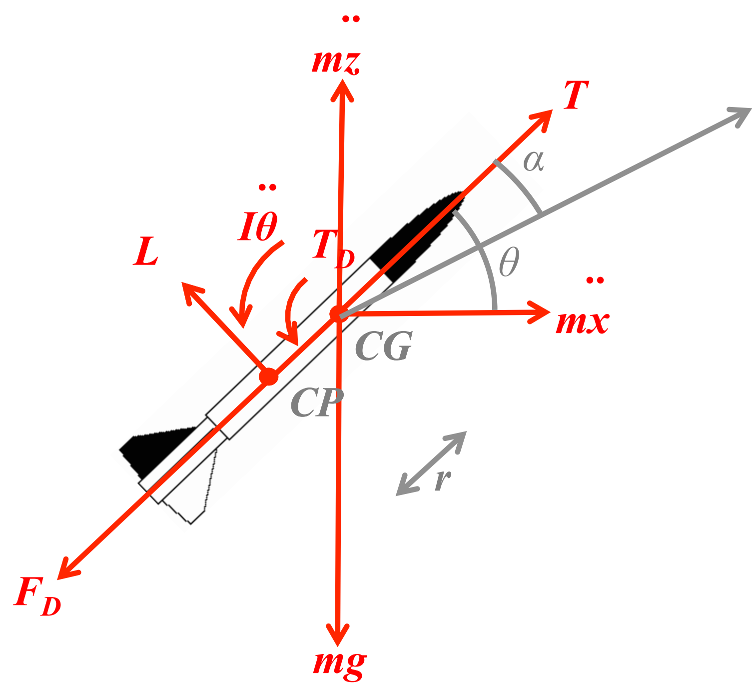
Three DOF Model

- Can we demonstrate this
  - If CP is between CG and nose the rocket will be ______
  - If CP is between CG and tail the rocket will be ________

Three DOF Model

- Free Body Diagram
  - Must also balance torques

$$I\ddot{\theta} = T_D - rL$$



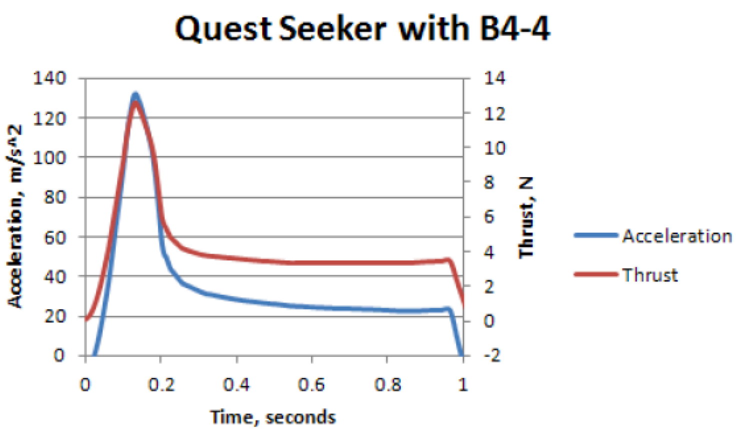
Three DOF Model

- Free Body Diagram
  - Resulting Equations

$$I\ddot{\theta} = T_D - rL$$
$$m\ddot{z} = T sin\theta - mg - F_D sin\theta + L cos\theta$$
$$m\ddot{x} = T cos\theta - F_D cos\theta - L sin\theta$$

Three DOF Model

- Motor thrust…
  - Is not constant
  - Is dependent on the motor
  - Can be modeled experimentally
  - Can be extracted from simulations (e.g. OpenRocket)



Three DOF Model

- The **drag** force can be calculated with

$$F_d = \tfrac{1}{2}\rho A V^2 C_D$$

where

ρ = density of fluid

V = velocity of rocket wrt fluid

A = Functional area

C_D = Drag Coefficient (function of α)

Three DOF Model

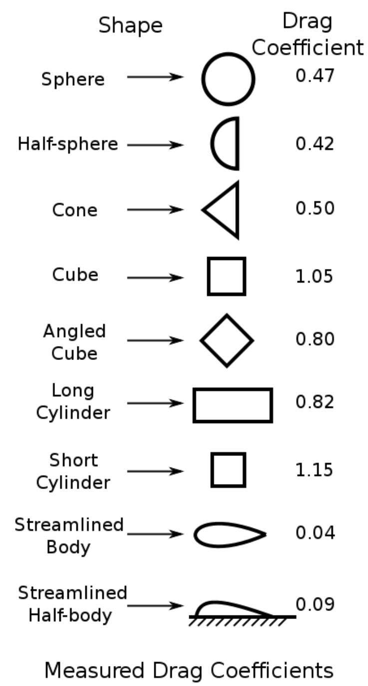
- Notes on Drag

$$F_d = \tfrac{1}{2}\rho A V^2 C_D$$

  - Drag is a function of Velocity
  - C_D, hence drag, is a function of Angle of attack
  - The calculation of C_D is a function of how A is defined
  - The direction of Drag is coordinate system dependent…

Three DOF Model

- Notes on Drag

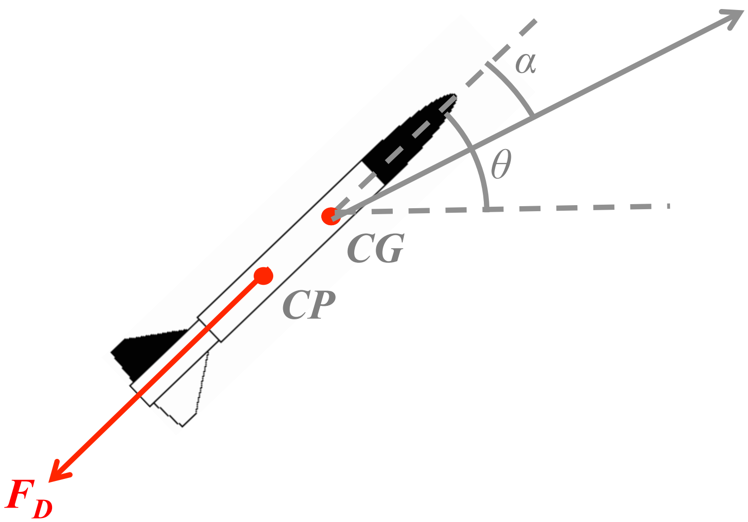


www.wikipedia.com

Three DOF Model

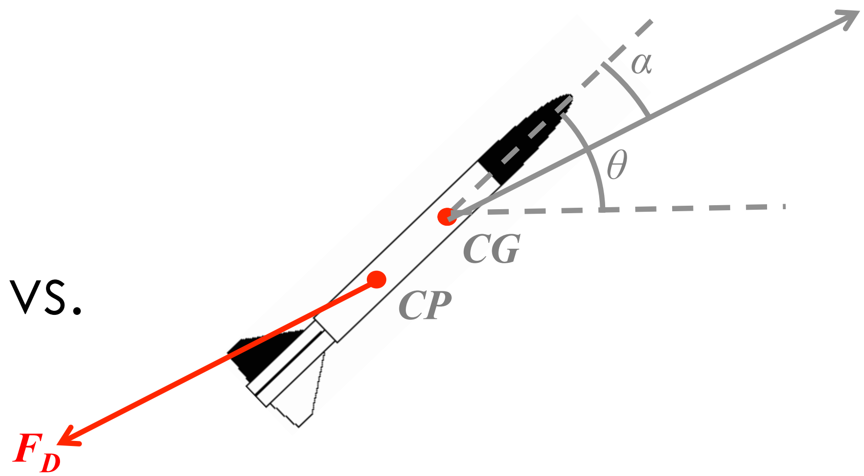
- Drag can be calculated w.r.t.

pitch of rocket



vs.

rocket velocity vector



Three DOF Model

- The **lift** force can be calculated with

$$L = \tfrac{1}{2}\rho A V^2 C_L$$

where

ρ = density of fluid

V = velocity of rocket wrt fluid

S = Functional area

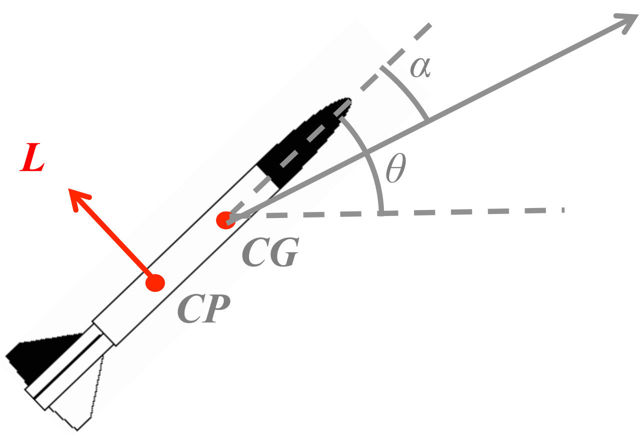
C_L = Lift Coefficient (function of α)

E80
Experimental Engineering

Lift Forces

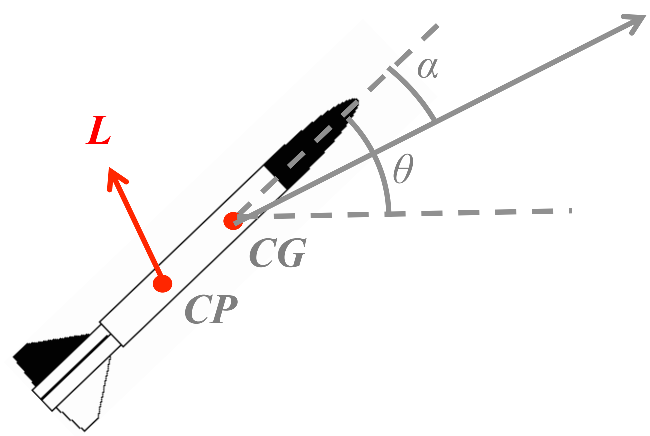
- Lift can be calculated perpendicular to

pitch of rocket



vs.

rocket velocity vector



Three DOF Model

- The **rotational damping** can be modeled as

$$T_D = c\,\dot{\theta}$$

where

c is a damping coefficient

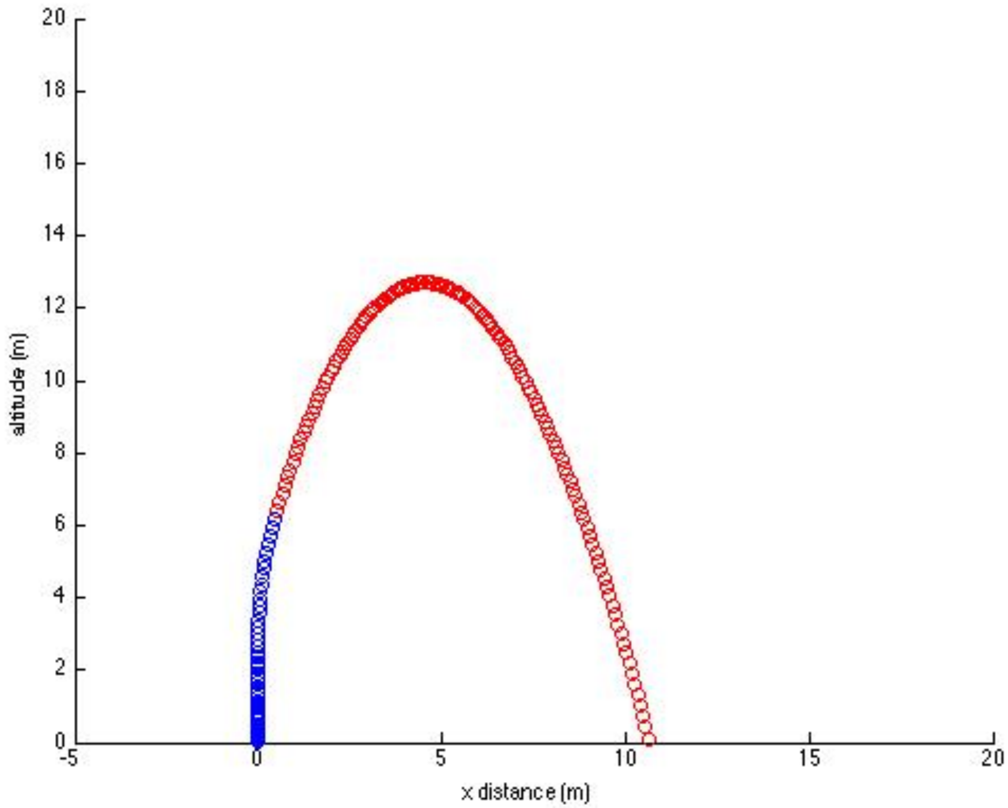
$\dot{\theta}$ is the rotational velocity of the rocket

Three DOF Model

```
for t = 0 to maxTime
{
        T = …
        m = …
        Fd = …
        L = …
        Td = …
        alpha = …

        z_dd(t) = …
        x_dd(t) = …
        θ_dd(t) = …
        …
}
```

Three DOF Model



To Linde Field

- Good luck!