Where am I?

Tracking your Rocket…

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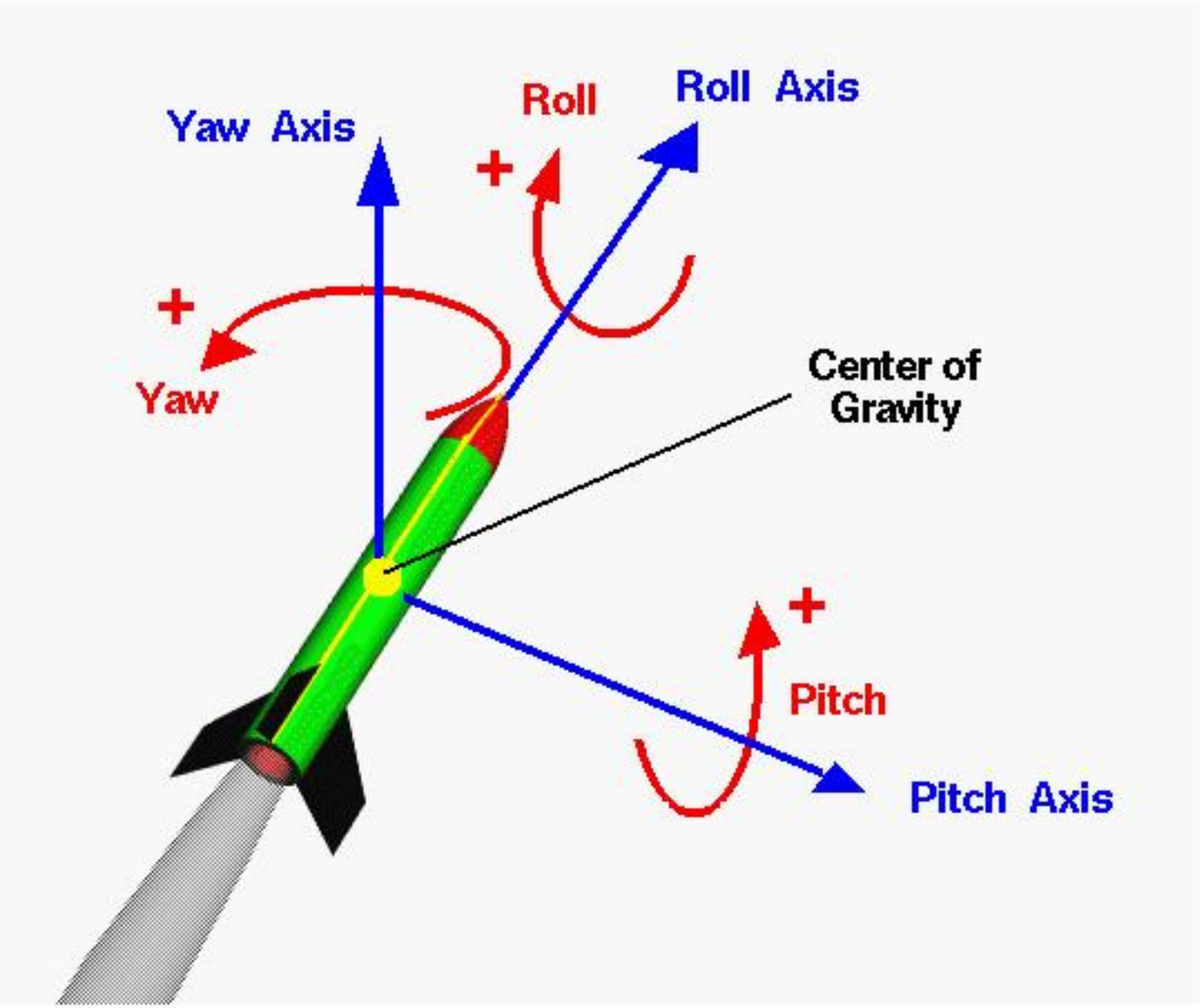
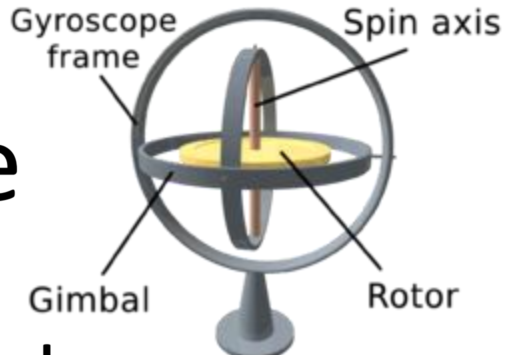
When I Lift Off I get Lost



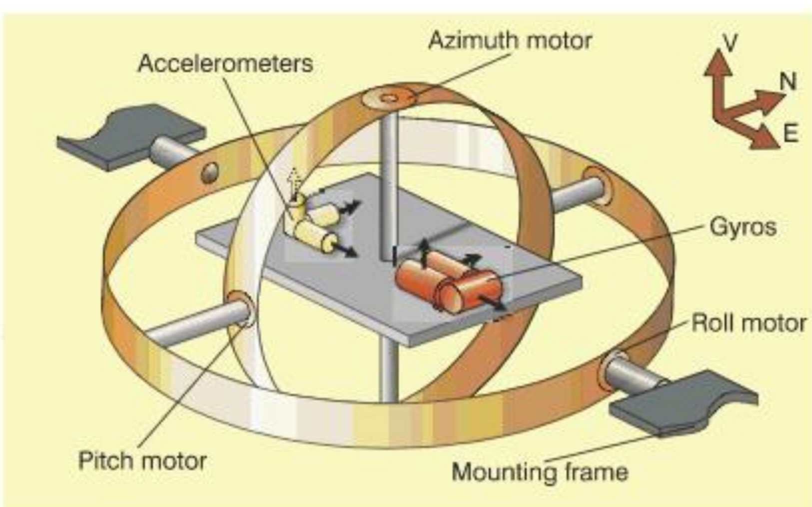
I Need a Reference

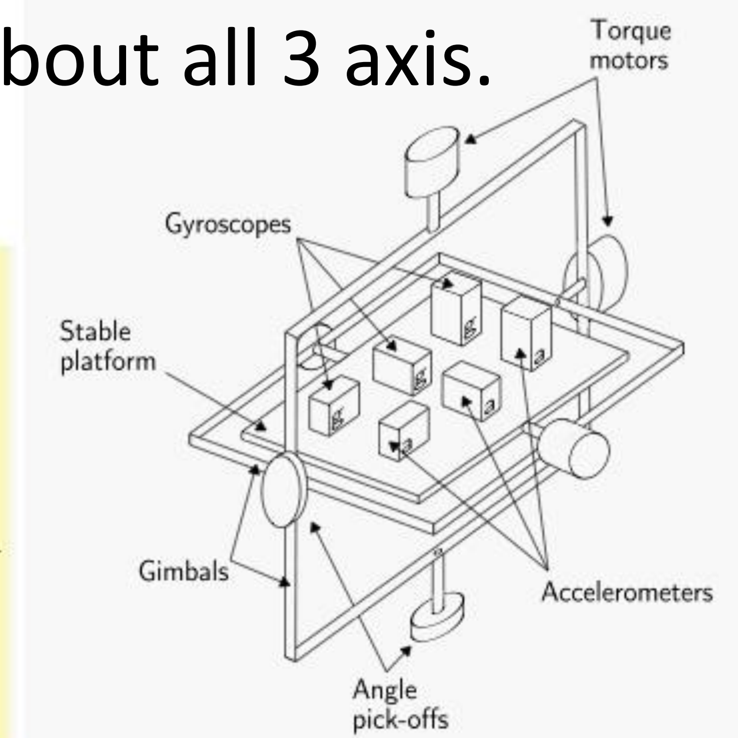


- In order to keep track of my rocket, I need a reference . I need something that stays stationary even though my rocket is maneuvering.
- I can use a spinning rotor and gimbals to build a gyroscope.
  - Flight maneuvers introduce torques that try to change the orientation of the device, but the spinning rotor stays fairly undisturbed since the torques don’t transfer well through the gimbals.

Gimbaled Gyro

- An instrumentation (navigation) platform can be isolated from external disturbances through the use of gimbals which allow the platform to rotate freely about all 3 axis.

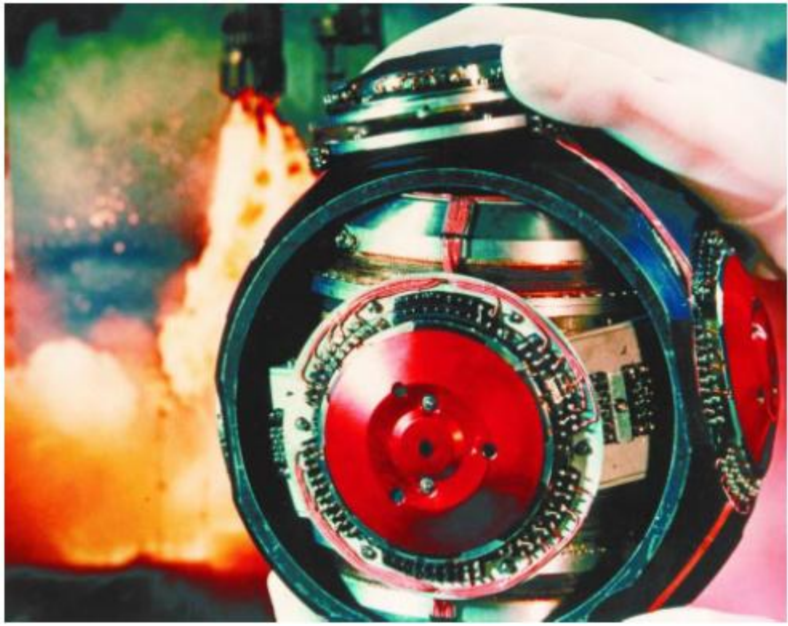




Keeping it Level

- Each of the gimbals can be rotated by a motor.
- A set of three accelerometers mounted on the platform sense the accelerations in three perpendicular directions: N-S, E-W, V.
- A set of three gyros on the platform detect platform rotations and output signals proportional to the rotation angle about the three perpendicular axes.
  - These signals drive torque motors (at gimbal bearings) that rotate the gimbals to ensure the platforms stays level.

Mechanical Gyros



- The gimbal arrangement is mechanically complex.
  - delicate slip rings
  - motors dissipate power
  - varying thermal environment as the gimbals move can affect instrumentation
  - mechanical resonances may couple with fixtures

Strap Down Systems

- Eliminates the need for gimbals.
- Instruments are instead “strapped down” onto rocket (avionics section).
- Instrumentation includes both accelerometers and rate gyros.
  - Rate Gyros are used to measure rate of rotations to help us track the orientation of the accelerometer axes as a function of time.

MEMS Gyro

- MEMS (Micro-machined Electro-Mechanical Systems) devices that are easily available commercially, affordable, and very small in size.
- Fundamental to an understanding of the operation of an vibrating structure gyroscope is an understanding of the Coriolis force.
- Your rate gyro accounts for this Coriolis effect.
  - *Refer to the specification sheet for more info*

Coriolis Effect

- http://www.classzone.com/books/earth_science/terc/content/visualizations/es1904/es1904page01.cfm
- http://www.youtube.com/watch?v=mcPs_OdQOYU

Great MEMS Overview

- http://www.ett.bme.hu/memsedu/cd/menu.html

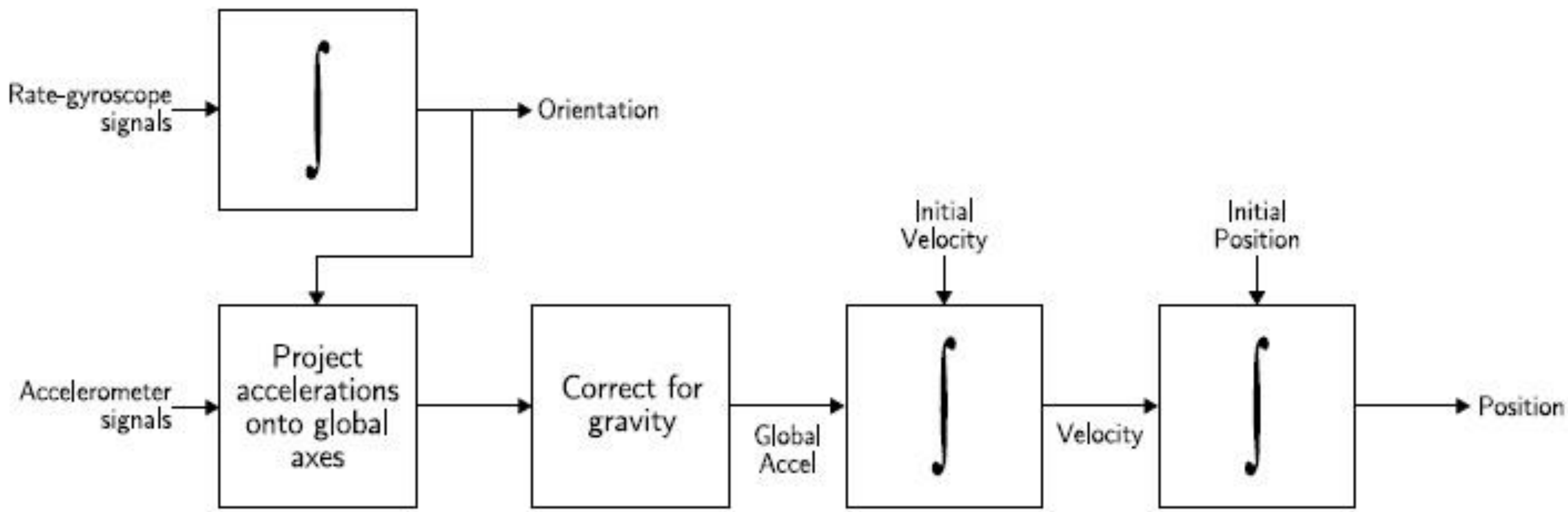
Hardware Measures…

- Rate Gyros measure angular velocities in three orthogonal axes $\omega = \dot{\varphi} = \left[\dot{\varphi}_{x,} \dot{\varphi}_{y,} \dot{\varphi}_{z}\right]^T$
- Accelerometers measure the linear accelerations in each of the three orthogonal directions $a_{global}(t) = R(t)a_{local}(t)$

Where am I?

- To find the rocket’s location, we need to know the relative location of the local coordinate system of the rocket relative to the global reference frame.
- Then, we can transform the acceleration measurements properly, integrate once and twice and you’ve got it!

Block Diagram



Transformation Matrices

$$\mathbf{R}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_x & -\sin\phi_x \\ 0 & \sin\phi_x & \cos\phi_x \end{bmatrix}, \quad \mathbf{R}_y = \begin{bmatrix} \cos\phi_y & 0 & \sin\phi_y \\ 0 & 1 & 0 \\ -\sin\phi_y & 0 & \cos\phi_y \end{bmatrix}, \quad \mathbf{R}_z = \begin{bmatrix} \cos\phi_z & -\sin\phi_z & 0 \\ \sin\phi_z & \cos\phi_z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The overall rotation can be treated as the result of making the three individual rotations one at a time in the order of R_x, R_y and R_z, and the rotation matrix is:

$$\mathbf{R} = \mathbf{R}_z\mathbf{R}_y\mathbf{R}_x = \begin{bmatrix} \cos\phi_z\cos\phi_y & \cos\phi_z\sin\phi_y\sin\phi_x - \sin\phi_z\cos\phi_x & \cos\phi_z\sin\phi_y\cos\phi_x + \sin\phi_z\sin\phi_x \\ \sin\phi_z\cos\phi_y & \sin\phi_z\sin\phi_y\sin\phi_x + \cos\phi_z\cos\phi_x & \sin\phi_z\sin\phi_y\cos\phi_x - \cos\phi_z\sin\phi_x \\ -\sin\phi_y & \cos\phi_y\sin\phi_x & \cos\phi_y\cos\phi_x \end{bmatrix}$$

When ϕ is small, we have these approximations:

$$\cos\phi \approx 1, \qquad \sin\phi \approx \phi$$

and any product of two or more sin functions is approximately zero. Applying this approximation to the expression for $\mathbf{R}$ above, we get:

$$\mathbf{R} = \mathbf{R}_z\mathbf{R}_y\mathbf{R}_x = \begin{bmatrix} 1 & -\phi_z & \phi_y \\ \phi_z & 1 & -\phi_x \\ -\phi_y & \phi_x & 1 \end{bmatrix} = \mathbf{I} + \mathbf{\Phi}$$

where

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{\Phi} = \begin{bmatrix} 0 & -\phi_z & \phi_y \\ \phi_z & 0 & -\phi_x \\ -\phi_y & \phi_x & 0 \end{bmatrix}$$

Computing Position

After converting the acceleration $\mathbf{a}_l = [a_x, a_y, a_z]^T$ from local to global system and subtracting gravitational acceleration, it is integrated to get velocity

$$\mathbf{v}_g(t) = \mathbf{v}_g(0) + \int_0^t [\mathbf{a}_g(t) - \mathbf{g}_g] dt$$

which is integrated one more time to get displacement (translation):

$$\mathbf{t}_g = \mathbf{t}_g(0) + \int_0^t \mathbf{v}_g(t) dt$$

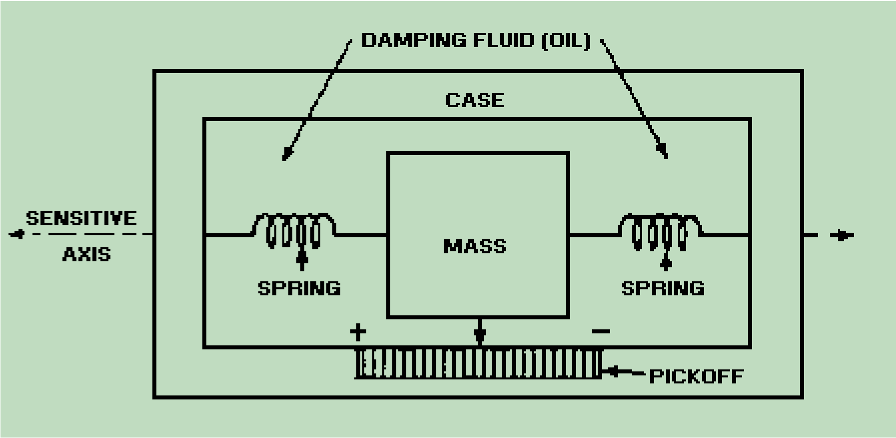
In implementation, again, the velocity and translation vectors are updated whenever a set of new acceleration samples are available:

$$\mathbf{v}(t + \delta t) = \mathbf{v}_g(t) + \delta t[\mathbf{a}_g(t + \delta t) - \mathbf{g}_g]$$

$$\mathbf{t}(t + \delta t) = \mathbf{t}_g(t) + \delta t[\mathbf{v}_g(t + \delta t)]$$

Accelerometers

- Recall E59



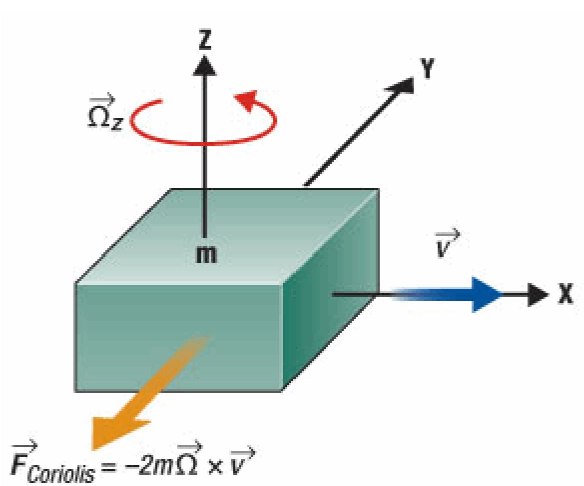
- A systems overview…

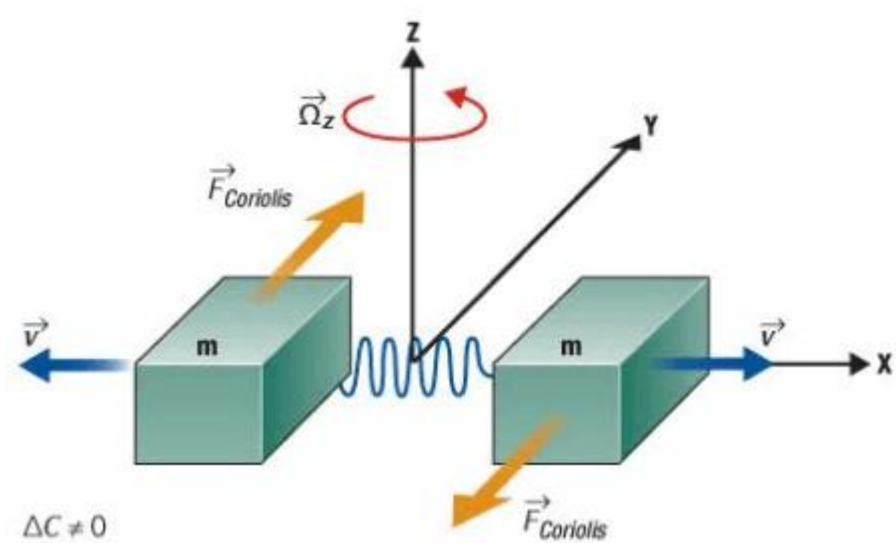
More Coriolis Effect

- To travel in a straight line while on a rotating system, lateral speed must be increased or decreased to maintain the same relative angular position (longitude) on the body.
- The act of slowing down or speeding up is acceleration, and the Coriolis force is this acceleration times the mass of the object whose longitude is to be maintained.
- The Coriolis force is proportional to both the angular velocity of the rotating object and the velocity of the object moving towards or away from the axis of rotation.

MEMS Rate Gyro

- A MEMS gyroscope based on the Coriolis effect is composed of a pair of masses vibrating in opposite directions (along x-axis). When an angular velocity (along z-axis) is applied, the difference between the Coriolis forces on the two mass in opposite directions (along y-axis) is detected. However, when a linear acceleration is applied along y-axis, both masses experience the same force with zero difference. A signal conditioning circuitry converts this signal to angular rate output.





Mechanical Sketch - Gyro