

# Basic Electrical Measurements

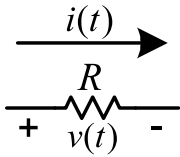
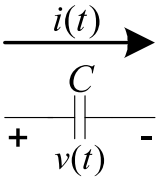
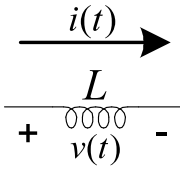
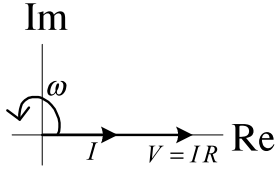
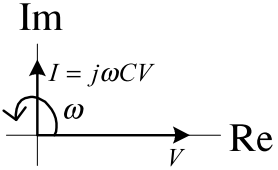
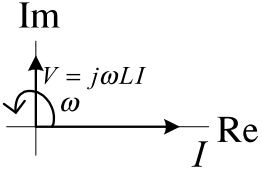
E80 – Feb 26, 2012 – Prof. Katherine Candler  
 (Notes adapted from Prof. Sarah Harris’s lecture from 2009)

## Overview

- **Circuits**
  - Electrical Building Blocks: R, L, C
  - Voltage Division
  - Experimental Plots
    - Time Domain
    - Frequency Domain (Bode Plots)
- **Instrumentation**
  - Instruments that generate signals
  - Instruments that measure signals
  - Measuring AC signals
  - Instruments affect the measurement!

## Circuits

### Electrical Building Blocks: R, L, C

	Resistor (R)	Capacitor (C)	Inductor (L)
Units	[ $\Omega$ ]	[F]	[H]
Symbol			
I-V relationship	$v(t) = i(t)R$	$i(t) = C \frac{\partial v(t)}{\partial t}$	$v(t) = L \frac{\partial i(t)}{\partial t}$
Impedance	$Z_R = \frac{V}{I} = R$	$Z_C = \frac{V}{I} = \frac{1}{j\omega C}$	$Z_L = \frac{V}{I} = j\omega L$
Phasor representation			
	Current and voltage are in phase.	Current is 90° ahead of voltage.	Voltage is 90° ahead of current.

Notes:

- Electrical building blocks are characterized by their current-voltage (I-V) relationship.

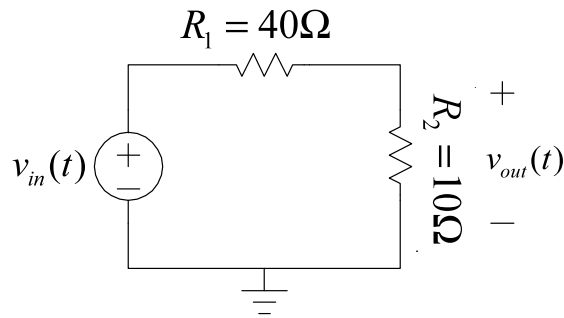
- $V$  and  $I$  are phasors, e.g., of the form  $V = Ae^{j\theta}$ .
- See Appendix for:
  - Derivation of impedance
  - I-V phase relationship obtained through time-domain analysis

### Voltage Division

Given a voltage divider circuit, there are two methods for solving for the output voltage, given an input voltage  $v_{in}(t)$ .

**Method 1:** Use voltage division if the circuit contains purely resistors.

Example:



Since the circuit is purely resistive, we can use simple voltage division.

$$v_{out}(t) = \left( \frac{R_2}{R_2 + R_1} \right) v_{in}(t) = \left( \frac{10\Omega}{10\Omega + 40\Omega} \right) v_{in}(t) = \frac{1}{5} v_{in}(t)$$

**Method 2:** Use impedances to find the frequency response function  $H(j\omega) = \frac{V_{out}}{V_{in}}$ , where

$V_{out}$  and  $V_{in}$  are the output and input phasors of the circuit. With  $H(j\omega)$ , if we have a sinusoidal input, e.g.,  $x(t) = X \cos(\omega t + \phi)$ , then we know the output will also be sinusoidal, e.g.,  $y(t) = X |H(j\omega)| \cos(\omega t + \phi + \angle H(j\omega))$ .

Example 1:

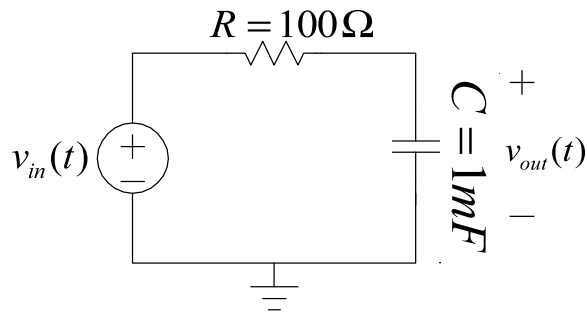
For the same circuit above, but now using impedances,

$$V_{out} = \left( \frac{Z_{R2}}{Z_{R2} + Z_{R1}} \right) V_{in}$$

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_{R2}}{Z_{R2} + Z_{R1}} = \frac{10}{10 + 40} = \frac{1}{5}$$

Since the *FRF* is completely real in this case, there is no phase shift of the output relative to the input and the output is  $1/5^{\text{th}}$  the magnitude of the input, i.e.,  $v_{out}(t) = \frac{1}{5} v_{in}(t)$ .

Example 2: Let's look at a more complex circuit.



Using impedances:

$$V_{out} = \frac{Z_c}{Z_c + Z_R} V_{in}$$

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_c}{Z_c + Z_R} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\omega(0.1)}$$

The gain and phase depend on the frequency of the input.

- Case 1: Suppose  $v_{in}(t) = \sin(2\pi(50)t)$ . Then

$$H(j\omega) = \frac{1}{1 + j(2\pi(50))(0.1)} = \frac{1}{1 + j31}$$

$$|H(j\omega)| = \left| \frac{1}{1 + j31} \right| = \frac{1}{\sqrt{1 + 31^2}} = \frac{1}{31} = 0.03$$

$$\angle H(j\omega) = \angle \left( \frac{1}{1 + j31} \right) = -\tan^{-1} \left( \frac{31}{1} \right) \approx -\frac{\pi}{2}$$

Thus, the magnitude of  $v_{out}(t)$  is 0.03 smaller than  $v_{in}(t)$  and its phase is shifted by about  $-\frac{\pi}{2}$ .

$$v_{out}(t) \approx 0.03 \sin(2\pi(50)t - \frac{\pi}{2})$$

- Case 2: Suppose  $v_{in}(t) = \sin(2\pi(0.05)t)$ , then

$$H(j\omega) = \frac{1}{1 + j(2\pi(0.05))(0.1)} = \frac{1}{1 + j0.031}$$

$$|H(j\omega)| = \left| \frac{1}{1 + j0.031} \right| = \frac{1}{\sqrt{1 + 0.031^2}} \approx 1$$

$$\angle H(j\omega) = \angle \left( \frac{1}{1 + j0.031} \right) = -\tan^{-1} \left( \frac{0.031}{1} \right) \approx 0$$

Thus, the magnitude and phase of  $v_{out}(t)$  are the same as  $v_{in}(t)$ .

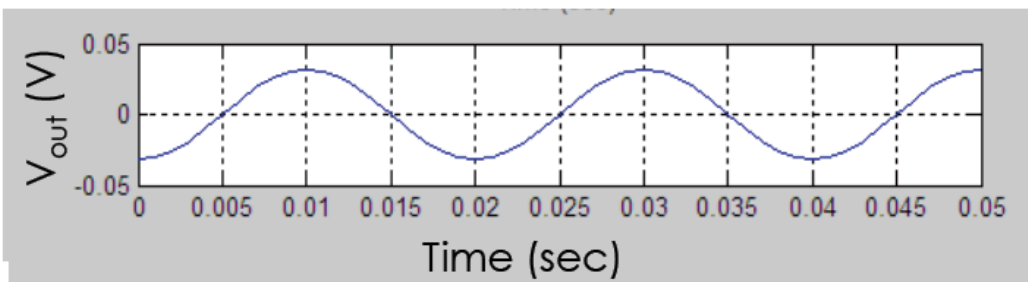
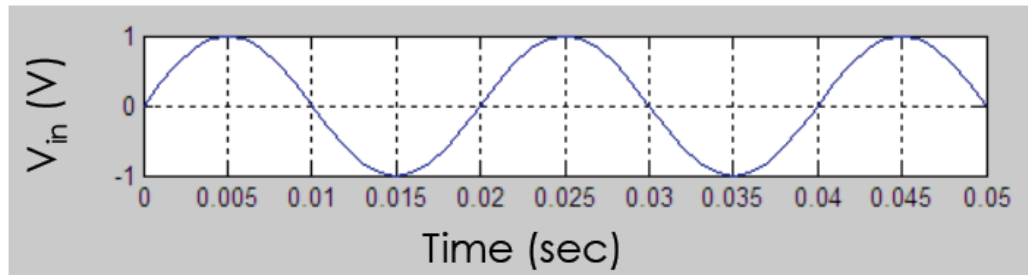
$$v_{out}(t) = v_{in}(t) = \sin(2\pi(0.05)t)$$

### Experimental Plots

Plotted experimental results give insights into the characteristics of a system. Here we discuss two useful experimental plots: time domain plots and Bode plots.

- **Time Domain**

For a given, time domain plots can be experimentally obtained by using a signal generator to generate  $v_{in}$  and using an oscilloscope to show both  $v_{in}(t)$  and  $v_{out}(t)$ .

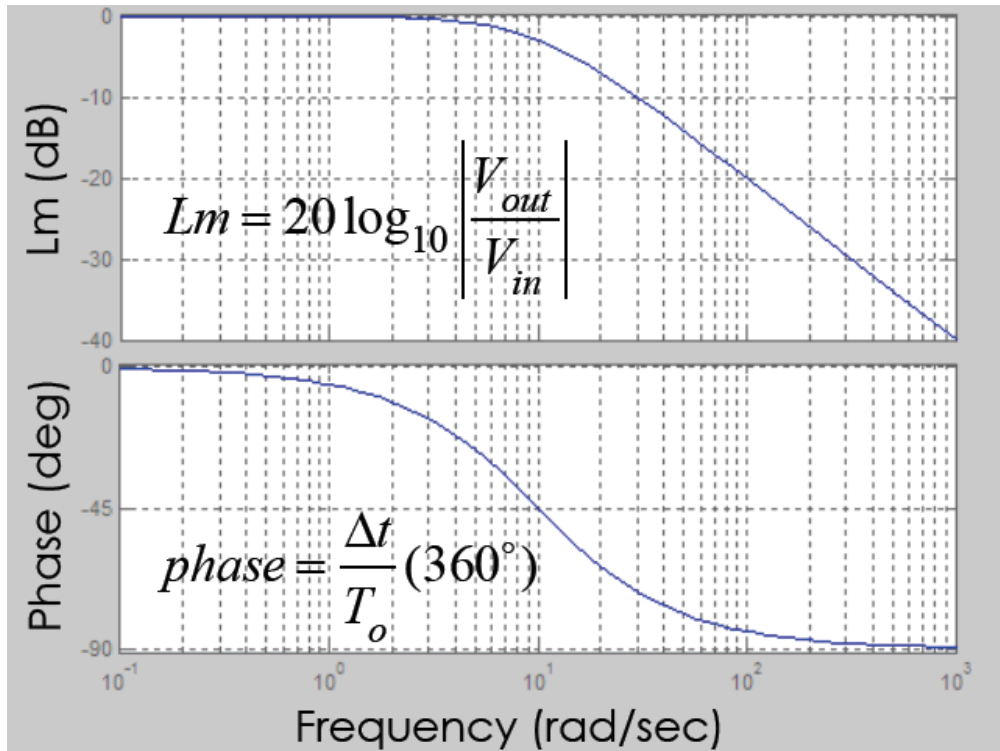


- **Frequency Domain (Bode Plots)**

A Bode plot shows a system's response at different frequencies. Bode plots can be experimentally determined by using a signal generator to generate  $v_{in}$  at various frequencies, and then using an oscilloscope to measure the gain and phase at each frequency. The magnitude plot shows the log magnitude of the gain, i.e.,

$$Lm = 20 \log_{10} \left| \frac{V_{out}}{V_{in}} \right|$$

while the phase plot shows the phase difference between the output and input at each frequency.



Remember that to make an experimental Bode plot you must measure **both** the output and the input.

### Instrumentation

Instrumentation is used to generate and measure signals and devices. Some useful instruments are: signal generators, power supplies, multimeters (ohmmeters, ammeters, voltmeters), and oscilloscopes. Specifications and operation of a specific instrument are found in its manual.

### Instruments that GENERATE signals:

- **Signal generators** (e.g., HP/Agilent 33120A)
  - **Signals:** Signal generators (also called function generators) generate sine waves, square waves, and other waveforms. Because the signals change over time, the signal generator is called an AC (alternating current) supply. The 33120A can also generate a constant DC (direct current) voltage.
  - **Parameters:** The interface allows the user to select the signal amplitude (Ampl), frequency (Freq), etc.
    - **V<sub>pp</sub>:** The HP 33120A allows you to set the peak-to-peak voltage (V<sub>pp</sub>). This measures the signal from its maximum to its minimum. So, the peak voltage is twice the signal's magnitude:  $V_{pp} = 2V_m$ . For example, for  $v_s(t) = 3 \sin(2\pi(50)t)$ , V<sub>pp</sub> = 6 V.
    - **Out Term:** The signal generator has a setting called Out Term that selects among two modes: High Z (large impedance) or 50 Ω. In the 50 Ω mode, the signal generator will produce internally 2X the voltage that you set on

the front panel. In the High Z mode, the signal generator will produce internally the voltage that you set on the front panel. See “Using the HP 33120A Function Generator”

[http://www.ece.utah.edu/~harrison/ece6721/Function\\_gen\\_tutorial.pdf](http://www.ece.utah.edu/~harrison/ece6721/Function_gen_tutorial.pdf) for a good explanation of the two modes.

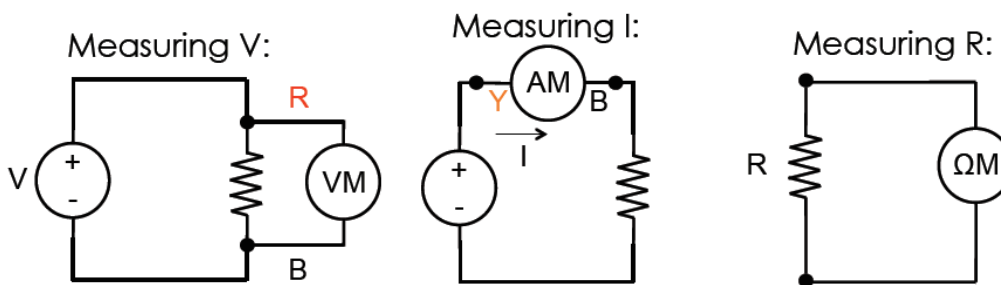
• **Power supplies** (e.g., HP/Agilent 6236)

- **Signals:** Power supplies generate a DC signal.
- **Parameters:** The HP 6236 power supply is a triple power supply that can supply voltages from in the range of 0-6V, 0-20V, or 0-(-20)V.
- **COM vs GND ( $\perp$ ):** COM is the common reference (usually referred to as  $\perp$  in your circuit) to which the output voltage is referenced. GND (the symbol  $\perp$  on the power supply) is the chassis ground (the ground reference coming in from the wall). COM and GND are **not** connected internally. If COM is connected to GND, the output will become referenced to the chassis ground. Otherwise, it is independent of the chassis ground.

**Instruments that MEASURE signals:**

• **Multimeters** (e.g., Elenco). Using different settings, multimeters can measure voltage, resistance, or current.

- **Ammeter:** measures current. Place multimeter leads in series with circuit. Internal impedance is low to avoid changing the behavior of the measured circuit.
- **Voltmeter:** measures voltage. Place multimeter leads in parallel with circuit. Internal impedance is large to avoid changing the behavior of the measured circuit. Digital voltmeters have a fixed impedance, typically  $\sim 10\text{M}\Omega$ .
- **Ohmmeter:** measures resistance. Place multimeter leads in parallel with resistor. An ohmmeters use an internal battery to supply a voltage and then measure the current through the meter, so disconnect the resistor from any other external power supply.



• **Oscilloscopes** (e.g., Tektronix 2212)

- **AC signals:** It is most straightforward to use an oscilloscope to monitor an AC signal as a function of time. When an oscilloscope is not available, multimeters can also be used to measure the AC voltage (or current).
- **Internal impedance:** Just like multimeters, an oscilloscope (or “scope”) has finite internal impedance. These numbers can often be read directly off the port of the oscilloscope.

- **1x and 10x probes:** A 1x probe reads the measured signal directly (with some instrument loading effect due to the probe and internal impedance). A 10x probe increases the impedance of the probe and scope by 10 times (10x), thus decreasing the instrument loading effect. This attenuates the signal 10 times, so the signal read on the scope is decreased by a factor of 10. Some oscilloscopes have a setting to indicate the kind of probe being used.

### Measuring AC signals

Oscilloscopes are the best instruments to use to measure AC signals (i.e., signals that vary over time, like a sine wave). However, multimeters (like the Elenco) can also be used.

The difference is that an oscilloscope shows the actual waveform (so that you can extract details like the amplitude  $A$  or the peak-to-peak ( $2A$ ) voltage of a signal), whereas a multimeter will show just the RMS value.

The RMS value is computed as:

$$v_{rms} = \sqrt{\frac{1}{T_0} \int_{T_0} v^2(t) dt} \quad (\text{it is the square \textbf{root} of the \textbf{mean} of the \textbf{squares} of the values})$$

So, for example, if you have a sine wave of the form:  $v(t) = A \cos(\omega t)$

where

$A$  = amplitude

$\omega = 2\pi / T_0$  = fundamental frequency

$T_0$  = period

Then,

$$v_{rms} = \sqrt{\frac{1}{T_0} \int_{T_0} v^2(t) dt} = A \sqrt{\frac{1}{T_0} \int_{T_0} \cos^2\left(\frac{2\pi}{T_0} t\right) dt} = \frac{A}{\sqrt{2}} = 0.707A$$

So, for example if a sinusoidal voltage of amplitude  $A = 1$  V is input into the multimeter, the displayed RMS voltage will read 0.707 V. If it is input into the oscilloscope,  $V_{pp}$  will be 2 V.

### Summary:

If  $A$  = amplitude of signal

$V_{pp} = 2A$  ← oscilloscope reading

$v_{rms} = 0.707A$  ← multimeter reading

$$\frac{v_{rms}}{V_{pp}} = \frac{0.707}{2} = 0.3536 \quad \text{or} \quad v_{rms} = 0.3536V_{pp}$$

**Side note:** The RMS voltage is a useful value for calculating average power dissipation. For example, the power through a resistor is:

$$P = iv = \frac{v^2}{R}$$

With an AC voltage, you could measure the instantaneous power (using the equation above) or the average power, which is usually most useful, using the equation below.

$$P_{ave} = \left( \frac{v^2}{R} \right)_{ave} = \frac{(v^2)_{ave}}{R} = \frac{\frac{1}{T_0} \int_{T_0} v^2(t) dt}{R} = \frac{v_{rms}^2}{R}$$

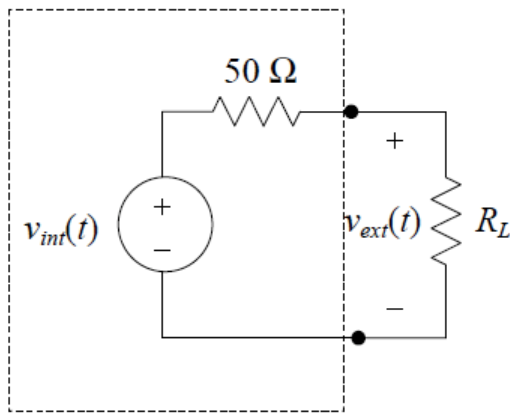
**Instruments affect the measurement!**

Both signal generation (signal generators, power supplies) and signal measurement (multimeters, scopes) instruments are non-ideal. To accurately model our systems, we must take non-idealities into account.

Below are some examples of how the instruments themselves change our measurements.

**Example: Signal Generator**

Recall the signal generator has an output impedance of 50 Ω. Let’s say we connect it to a load resistor as such:



HP 33120A

The voltage that will be seen at the output of the signal generator is:

$$v_{ext} = \left( \frac{R_L}{R_L + 50\Omega} \right) v_{int}$$

Let’s see at what happens with four different Out Term settings and different  $R_L$  values:



- Case 1: Out Term = 50  $\Omega$  and  $R_L = 50 \Omega$
- Case 2: Out Term = 50  $\Omega$  and  $R_L \neq 50 \Omega$
- Case 3: Out Term = High Z and  $R_L \gg 50 \Omega$
- Case 4: Out Term = High Z and  $R_L$  is not  $\gg 50 \Omega$

Case 1: Out Term = 50  $\Omega$  and  $R_L = 50 \Omega$

Let's say we set the signal generator to be 5  $V_{pp}$ .

1. Internally, the signal generator will supply  $v_{int} = 10V_{pp}$
2. And the signal generator display will show

$$v_{ext} = \left( \frac{50\Omega}{50\Omega + 50\Omega} \right) v_{int} = \frac{1}{2} v_{int} = \frac{1}{2} (10V_{pp}) = 5V_{pp}$$

(The display will match what we set the signal generator to be)

Case 2: Out Term = 50  $\Omega$  but  $R_L \neq 50 \Omega$

Again, let's say we set the signal generator to be 5  $V_{pp}$ .

1. Internally, the signal generator will supply  $v_{int} = 10V_{pp}$
2. But the signal generator display will show

$$v_{ext} = \left( \frac{10\Omega}{10\Omega + 50\Omega} \right) v_{int} = \frac{1}{6} v_{int} = \frac{1}{6} (10V_{pp}) = \frac{5}{3} V_{pp}$$

(The display will **not** match what we set the signal generator to be)

Case 3: Out Term = High Z and  $R_L \gg 50 \Omega$

Again, let's say we set the signal generator to be 5  $V_{pp}$ .

1. Internally, the signal generator will supply  $v_{int} = 5V_{pp}$
2. And the signal generator display will show

$$v_{ext} = \left( \frac{R_L}{R_L + 50\Omega} \right) v_{int} \approx v_{int} = 5V_{pp}$$

(The display will match what we set the signal generator to be)

Case 4: Out Term = High Z but  $R_L$  is not  $\gg 50 \Omega$

Again, let's say we set the signal generator to be 5  $V_{pp}$ .

1. Internally, the signal generator will supply  $v_{int} = 5V_{pp}$
2. But the signal generator display will show

$$v_{ext} = \left( \frac{R_L}{R_L + 50\Omega} \right) v_{int} < v_{int}$$

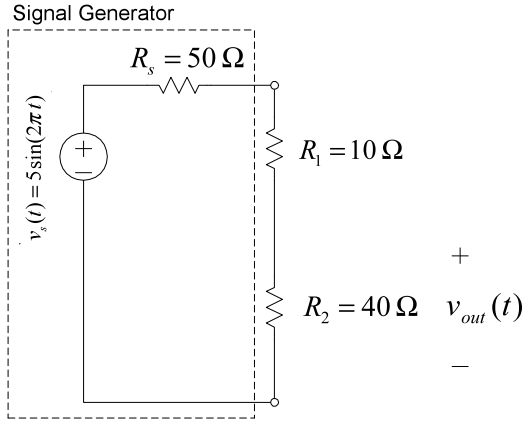
(The display will **not** match what we set the signal generator to be)

**Example: Measurement Instruments (e.g., Oscilloscope or Multimeter)**

Now let's examine another place where we introduce instrument loading effects: signal measurement.

Case 1:  $R_{instr} \gg R_2$

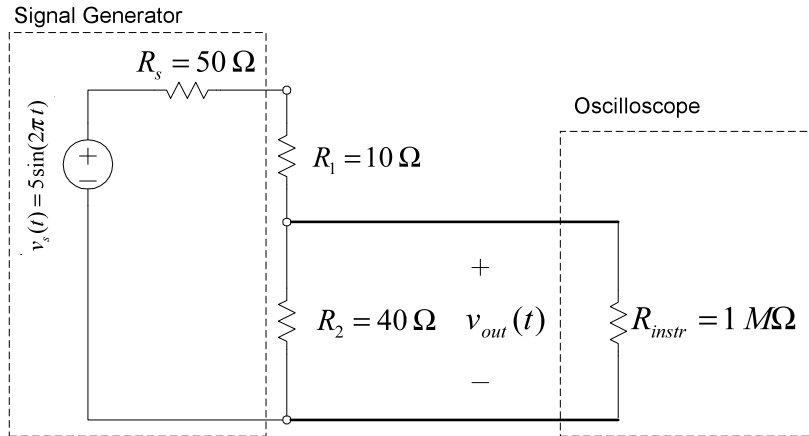
Consider the circuit below:



We expect:

$$v_{out} = \frac{R_2}{R_2 + R_1 + R_s} v_s = \frac{2}{5} v_s$$

Now let's connect an oscilloscope to measure  $v_{out}$ . Suppose our scope and probe have a combined impedance of  $1\text{ M}\Omega$ . Then, we can represent the above circuit as:



What is  $v_{out}$ ?

$$v_{out} = \frac{R_2 \parallel R_{instr}}{(R_2 \parallel R_{instr}) + R_1 + R_s} v_s :$$

Since  $R_{instr} \gg R_2$ ,  $R_2 \parallel R_{instr} \approx R_2$

Therefore,

$$v_{out} = \frac{R_2}{R_2 + R_1 + R_s} v_s = \frac{2}{5} v_s$$

(The scope will output what we expect)

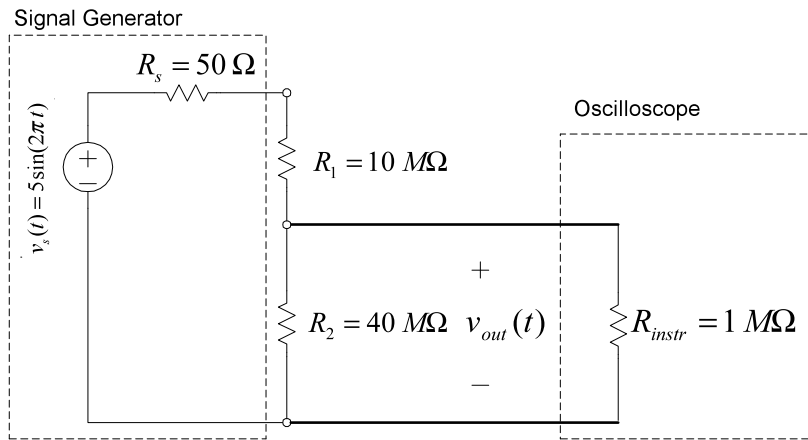
Case 2:  $R_{instr} \ll R_2$

Now suppose  $R_1$  and  $R_2$  are replaced by larger resistors.

Without the oscilloscope connected, we expect:

$$v_{out} = \frac{R_2}{R_2 + R_1 + R_s} v_s \approx \frac{R_2}{R_2 + R_1} = \frac{4}{5} v_s$$

Now let's connect the oscilloscope to measure  $v_{out}$ :



Now what is  $v_{out}$ ?

$$v_{out} = \frac{R_2 \parallel R_{instr}}{(R_2 \parallel R_{instr}) + R_1 + R_s} v_s \approx \frac{R_{instr}}{R_{instr} + R_1} = \frac{1}{11} v_s$$

(The scope will not output what we expected for  $v_{out}$ )

Lesson:

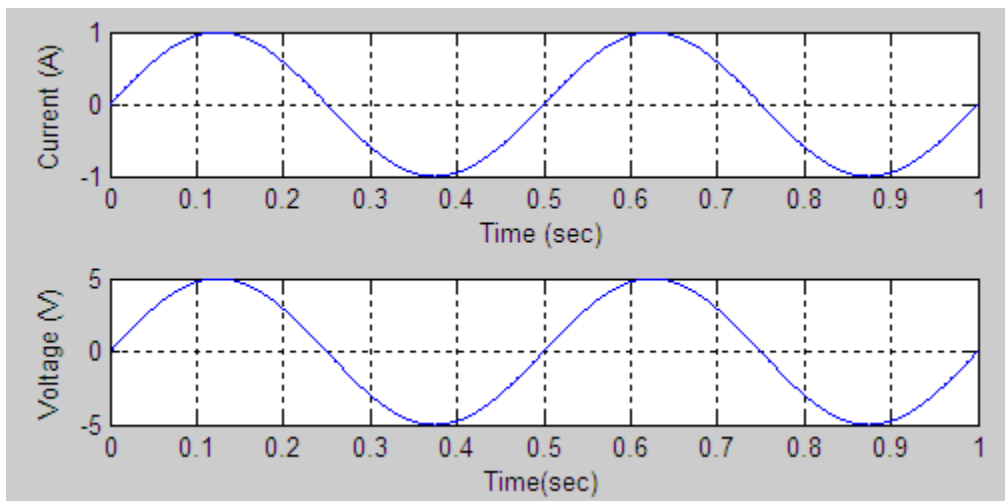
When connecting measurement instruments to the circuit, make sure that the input impedance of the instrument (oscilloscope or multimeter) is much larger than the impedance of the part of the circuit you're measuring. (i.e.,  $R_{instr} \gg R_{circuit}$ )

## Appendix

### I-V Phase Relationship for R/L/C's Using Time-Domain Analysis.

- *Resistor:* Current and voltage are **in phase**.

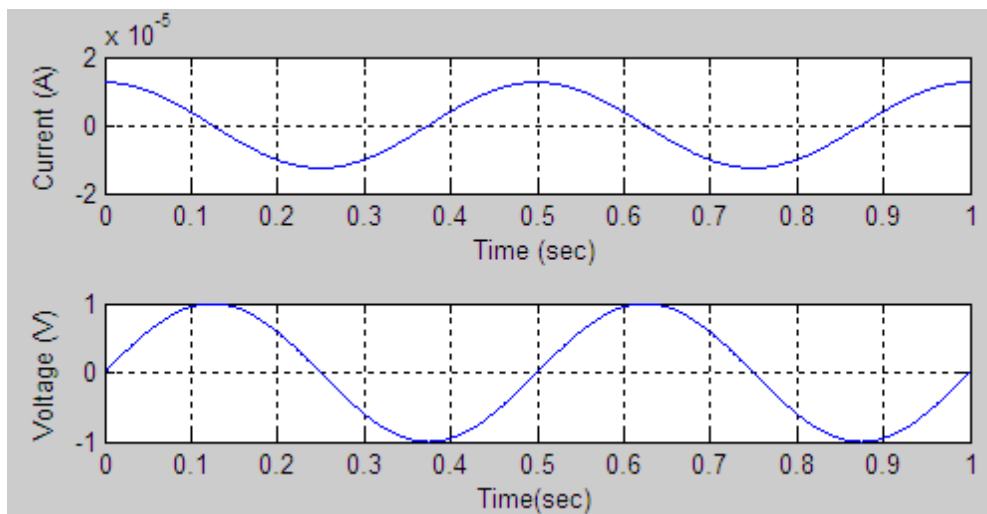
Example: Given  $R = 5\Omega$  and  $i(t) = \sin(4\pi t)$ , then  $v(t) = i(t)R = 5\sin(4\pi t)$



- *Capacitor:* **Current is  $90^\circ$  ahead of voltage**.

Example: Given  $C = 1\mu F$  and  $v(t) = \sin(4\pi t)$ , then

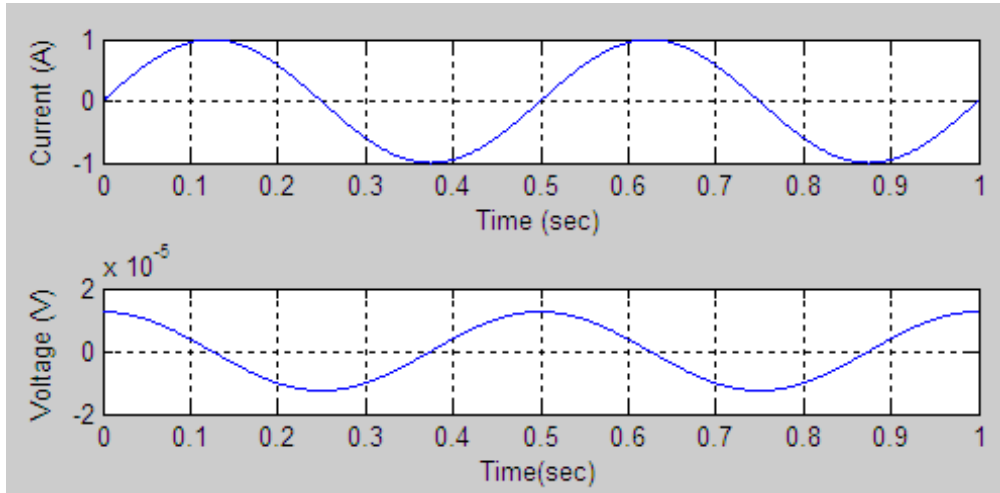
$$i(t) = C \frac{\partial v(t)}{\partial t} = 1\mu F [4\pi \cos(4\pi t)] \approx 1.25 \times 10^{-5} \cos(4\pi t) = 1.25 \times 10^{-5} \sin(4\pi t + \frac{\pi}{2})$$



- **Inductor: Voltage is 90° ahead of current.**

Example: Given  $L = 1\mu H$  and  $i(t) = \sin(4\pi t)$ , then

$$v(t) = L \frac{\partial i(t)}{\partial t} = 1\mu H [4\pi \cos(4\pi t)] \approx 1.25 \times 10^{-5} \cos(4\pi t) = 1.25 \times 10^{-5} \sin(4\pi t + \frac{\pi}{2})$$



Impedance derivations:

- Impedance is the ratio of the voltage to current:  $Z = \frac{V}{I}$
- Suppose current and voltage are represented as complex exponentials:  
 $v(t) = Ve^{j\omega t}$ ,  $i(t) = Ie^{j\omega t}$ , where  $V$  and  $I$  are phasors, e.g. of the form  $V = Ae^{j\theta}$ .

• <b>Resistor:</b>	$v(t) = i(t)R$ $Ve^{j\omega t} = Ie^{j\omega t} R$	$Z_R = \frac{V}{I} = R$
• <b>Capacitor:</b>	$i(t) = C \frac{\partial v(t)}{\partial t}$ $Ie^{j\omega t} = C \frac{\partial Ve^{j\omega t}}{\partial t}$ $Ie^{j\omega t} = C[j\omega Ve^{j\omega t}]$	$Z_C = \frac{V}{I} = \frac{1}{j\omega C}$
• <b>Inductor:</b>	$v(t) = L \frac{\partial i(t)}{\partial t}$ $Ve^{j\omega t} = L \frac{\partial Ie^{j\omega t}}{\partial t}$ $Ve^{j\omega t} = L[j\omega Ie^{j\omega t}]$	$Z_L = \frac{V}{I} = j\omega L$