E82

The Carnot Cycle

E82 – Two ways to implement a Carnot cycle in the physical world.

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Mathematical Statement of the Second Law:

$$dW_{\text{lost}} \geq 0$$

or

$$\Delta S_{\text{system}} + \Delta S_{\text{surroundings}} \geq 0$$

Remember:

$$\Delta S_{\text{system}} \text{ or } \Delta S_{\text{surroundings}}$$

can be less than zero. But

$$\Delta S_{\text{system}} + \Delta S_{\text{surroundings}} \geq 0$$

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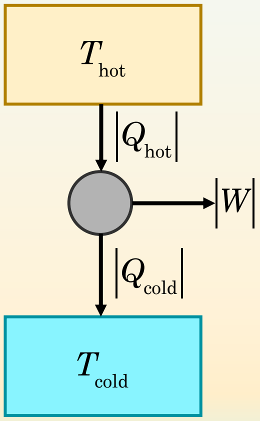
A Carnot engine operates between two isothermal heat reservoirs and produces the greatest possible work while rejecting the minimum amount of heat. The efficiency of a Carnot engine is:

$$\eta \equiv \frac{|W|}{|Q_{\text{hot}}|} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$$



The four steps of any Carnot engine are:

1. Adiabatic temperature change from T_{cold} to T_{hot}.
2. Isothermal heat addition of Q_{hot} at T_{hot}.
3. Adiabatic temperature change from T_{hot} to T_{cold}.
4. Isothermal heat removal of Q_{cold} at T_{cold}.

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Air-standard Carnot cycle

Carnot cycle

1. Adiabatic compression from T_{cold} to T_{hot} (State a to State b).
2. Isothermal heat addition (State b to State c).
3. Adiabatic expansion from T_{hot} to T_{cold} (State c to State d).
4. Isothermal heat removal (State d to State a).

For air-standard assume:

A) Ideal Gas

B) $C_v = \frac{5}{2}R \Rightarrow C_p = \frac{7}{2}R$

<u>For step 1</u> (Review from 1st Law for Ideal Gas video)

Closed system adiabatic reversible.

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1st Law

$$\Delta U = W$$
$$nC_v \Delta T = W$$
$$W = nC_v \left(T_{\text{hot}} - T_{\text{cold}}\right)$$

How about P and $\hat{V}$?

If $\Delta S = 0$ (why?)

$$\left(\frac{P_b}{P_a}\right)^{\frac{R}{C_p}} = \frac{T_{\text{hot}}}{T_{\text{cold}}}$$

$$P\hat{V} = RT$$

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So

$$\frac{\hat{V}_a}{\hat{V}_b} = \left(\frac{P_b}{P_a}\right)^{\frac{C_p - R}{C_p}}$$

or

$$P_a \hat{V}_a^{\gamma} = P_b \hat{V}_b^{\gamma} = \text{ constant}$$

where

$$\gamma \equiv \frac{C_p}{C_v} = \frac{C_p}{C_p - R}$$

For Step 2

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1st law – closed system isothermal

$$\Delta U = Q + W$$

$$\Delta U = 0 \text{ (Why?)}$$

$$Q = -W$$

$$W = -n\int_{P_i}^{P_f} P\, d\hat{V}$$

Since

$$P = \frac{RT}{\hat{V}}$$

$$W = -nRT\int_{\hat{V}_i}^{\hat{V}_f} \frac{d\hat{V}}{\hat{V}} = nRT\ln\frac{\hat{V}_i}{\hat{V}_f} = nRT\ln\frac{P_f}{P_i} = -Q$$

For Step 3

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Adiabatic expansion

$$W = nC_v\left(T_{\text{cold}} - T_{\text{hot}}\right)$$

For Step 4

Isothermal cooling

$$W = nRT\ln\frac{\hat{V}_i}{\hat{V}_f} = nRT\ln\frac{P_f}{P_i} = -Q$$

The Complete Cycle

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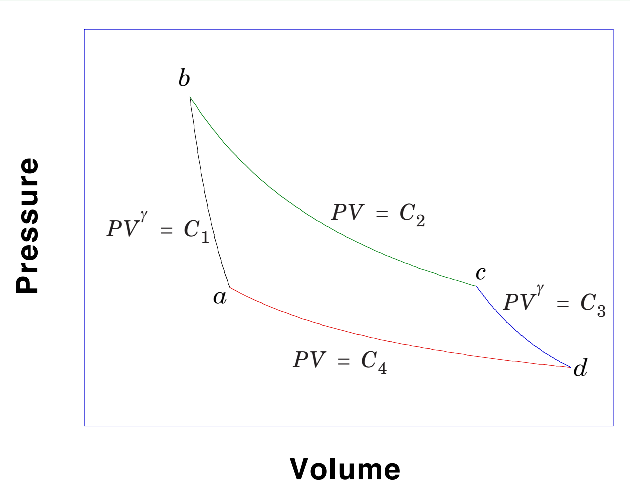

Note that

$$\frac{P_b}{P_a} = \left(\frac{T_{\text{hot}}}{T_{\text{cold}}}\right)^{\frac{C_p}{R}}$$

and

$$\frac{P_c}{P_d} = \left(\frac{T_{\text{hot}}}{T_{\text{cold}}}\right)^{\frac{C_p}{R}}$$

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so

$$\frac{P_b}{P_a} = \frac{P_c}{P_d}$$

or rearranged

$$\frac{P_d}{P_a} = \frac{P_c}{P_b}$$

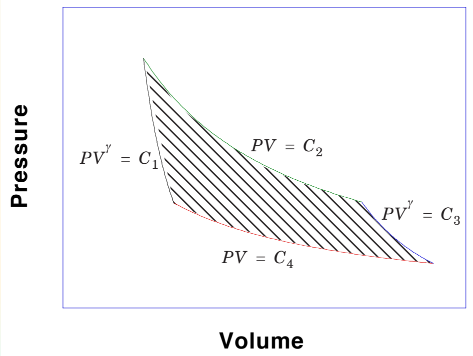
$$-\frac{W_{\text{net}}}{n} = C_v\left(T_{\text{cold}} - T_{\text{hot}}\right) + RT_{\text{hot}} \ln\frac{P_b}{P_c} + C_v\left(T_{\text{hot}} - T_{\text{cold}}\right) + RT_{\text{cold}} \ln\frac{P_d}{P_a}$$

$$-\frac{W_{\text{net}}}{n} = R\left(T_{\text{hot}} - T_{\text{cold}}\right)\ln\frac{P_b}{P_c}$$

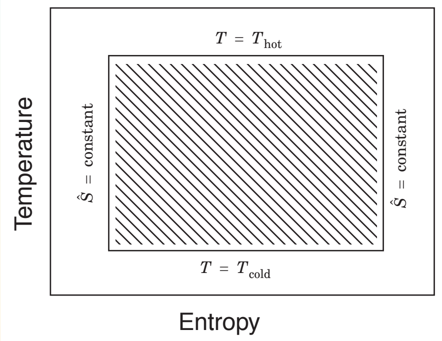
$$\frac{Q_{\text{hot}}}{n} = RT_{\text{hot}} \ln\frac{P_b}{P_c}$$

$$\eta = \frac{-W_{\text{net}}}{Q_{\text{hot}}} = \frac{R\left(T_{\text{hot}} - T_{\text{cold}}\right)\ln\dfrac{P_b}{P_c}}{RT_{\text{hot}}\ln\dfrac{P_b}{P_c}} = \frac{T_{\text{hot}} - T_{\text{cold}}}{T_{\text{hot}}} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$$

Note area enclosed on $P\hat{V}$ diagram is $-W_{\text{net}}$



What does a Carnot cycle look like on a *TS* diagram?



enclosed area $= Q_{\text{net}}$

For cycle

$$-W_{\text{net}} = Q_{\text{net}}$$

The Carnot efficiency does not depend on the working fluid.

Carnot steam cycle

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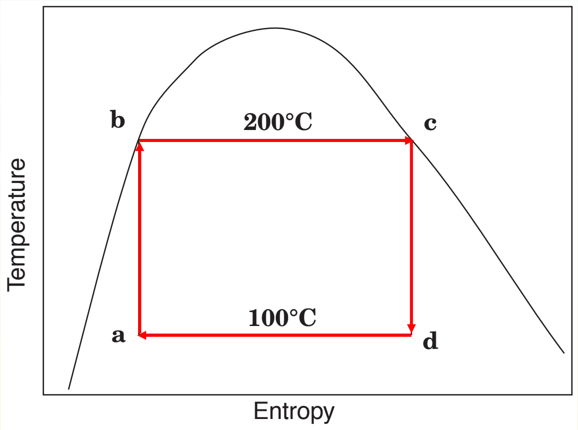
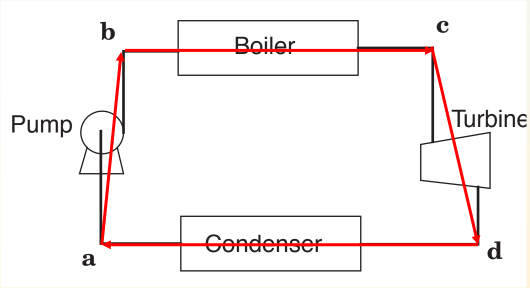




Boiler

1st Law

$$\Delta\dot{H} = \dot{Q} = \dot{m}\Delta\hat{H} = \dot{m}\left(2792.0 - 852.3\right)$$

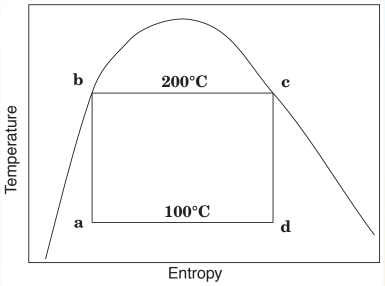
Turbine

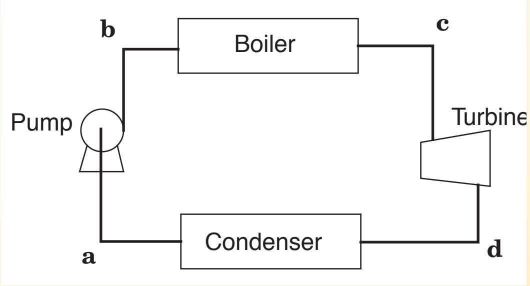
1st Law

$$\Delta\dot{H} = \dot{W}_s = \dot{m}\Delta\hat{H} = \ ?$$

2nd Law

$$\Delta\dot{S} = 0 = \dot{m}\Delta\hat{S}$$





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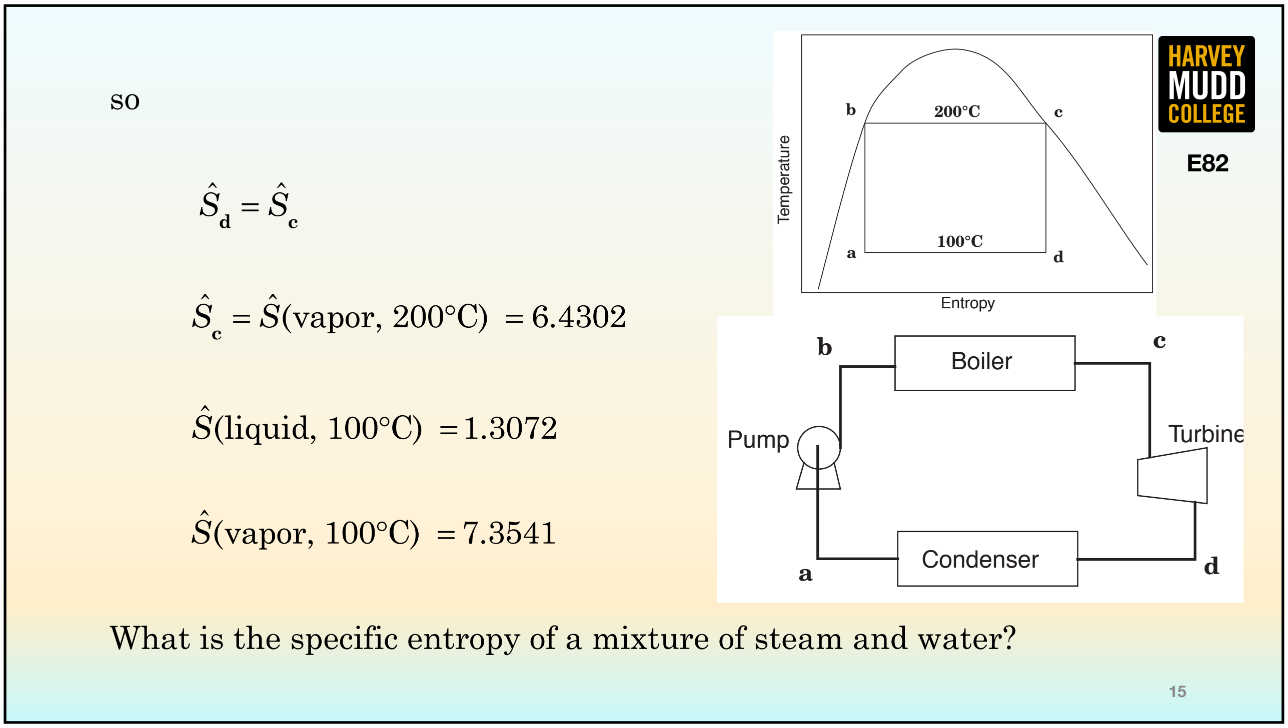

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so

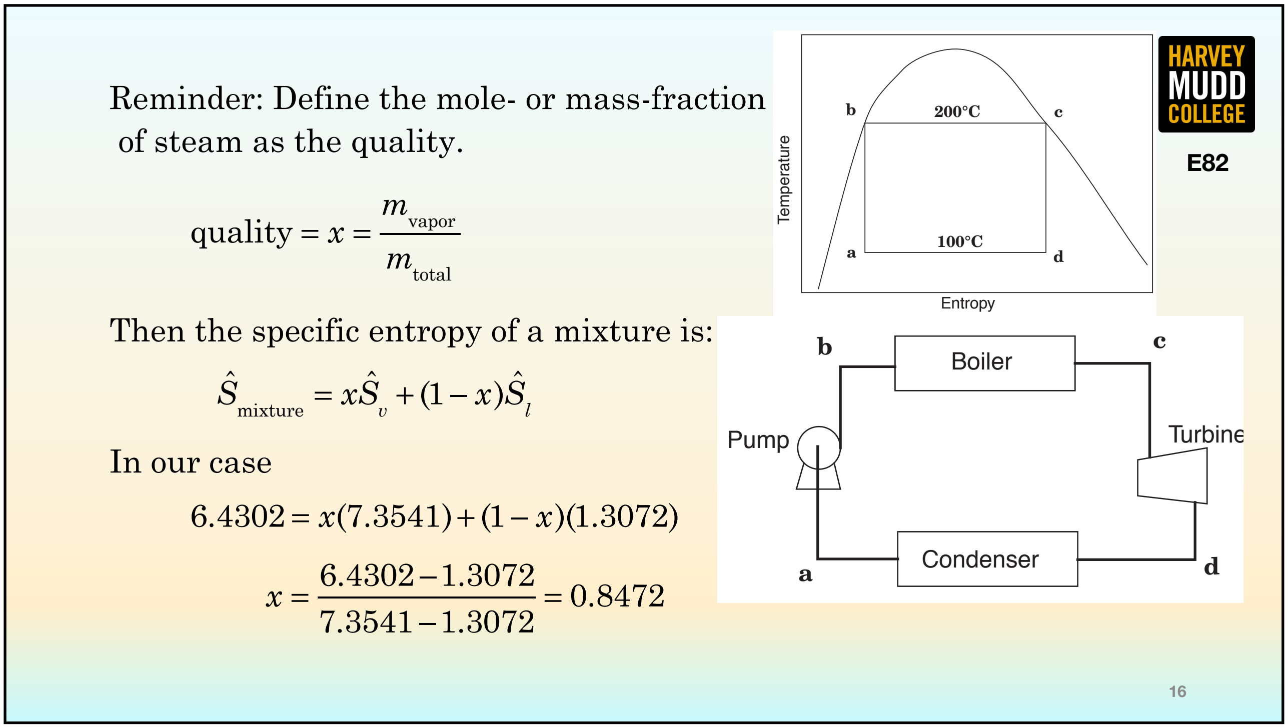
$$\hat{S}_{\mathbf{d}} = \hat{S}_{\mathbf{c}}$$

$$\hat{S}_{\mathbf{c}} = \hat{S}(\text{vapor, } 200°\text{C}) = 6.4302$$

$$\hat{S}(\text{liquid, } 100°\text{C}) = 1.3072$$

$$\hat{S}(\text{vapor, } 100°\text{C}) = 7.3541$$

What is the specific entropy of a mixture of steam and water?



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Reminder: Define the mole- or mass-fraction of steam as the quality.

$$\text{quality} = x = \frac{m_{\text{vapor}}}{m_{\text{total}}}$$

Then the specific entropy of a mixture is:

$$\hat{S}_{\text{mixture}} = x\hat{S}_v + (1-x)\hat{S}_l$$

In our case

$$6.4302 = x(7.3541) + (1-x)(1.3072)$$

$$x = \frac{6.4302 - 1.3072}{7.3541 - 1.3072} = 0.8472$$

To calculate the enthalpy at point **d**

$$\hat{H} = x\hat{H}_v + (1-x)\hat{H}_l$$

$$\hat{H}_{\mathbf{d}} = 2330.8$$

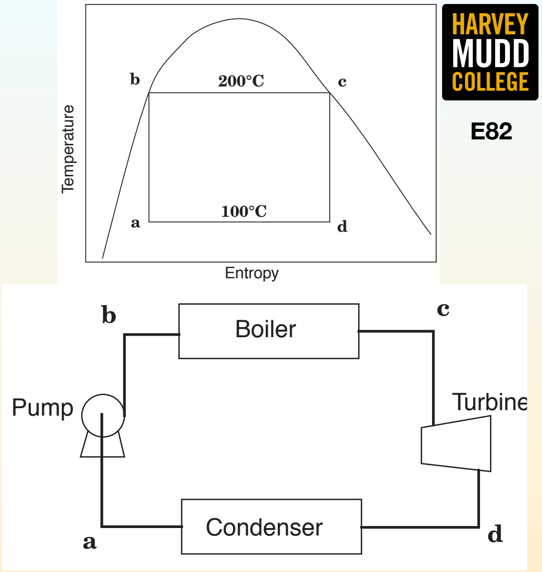
By a similar process

$$x(\text{at point } \mathbf{a}) = 0.1692$$

$$\hat{H}_{\mathbf{a}} = 801.0$$

$$\eta = \frac{|Q_{\text{hot}}| - |Q_{\text{cold}}|}{|Q_{\text{hot}}|}$$

$$= \frac{(2792.0-852.3)-(2330.8-801.0)}{(2792.0-852.3)} = 0.2113$$



From the formula for Carnot efficiency

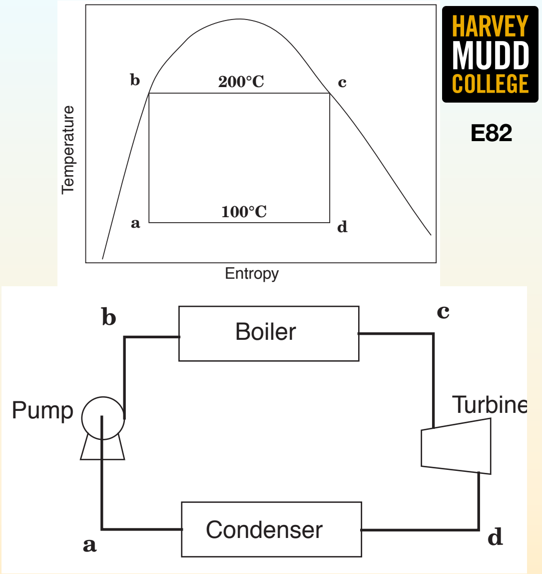
$$\eta = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}} = 1 - \frac{373.15}{473.15} = 0.2113$$

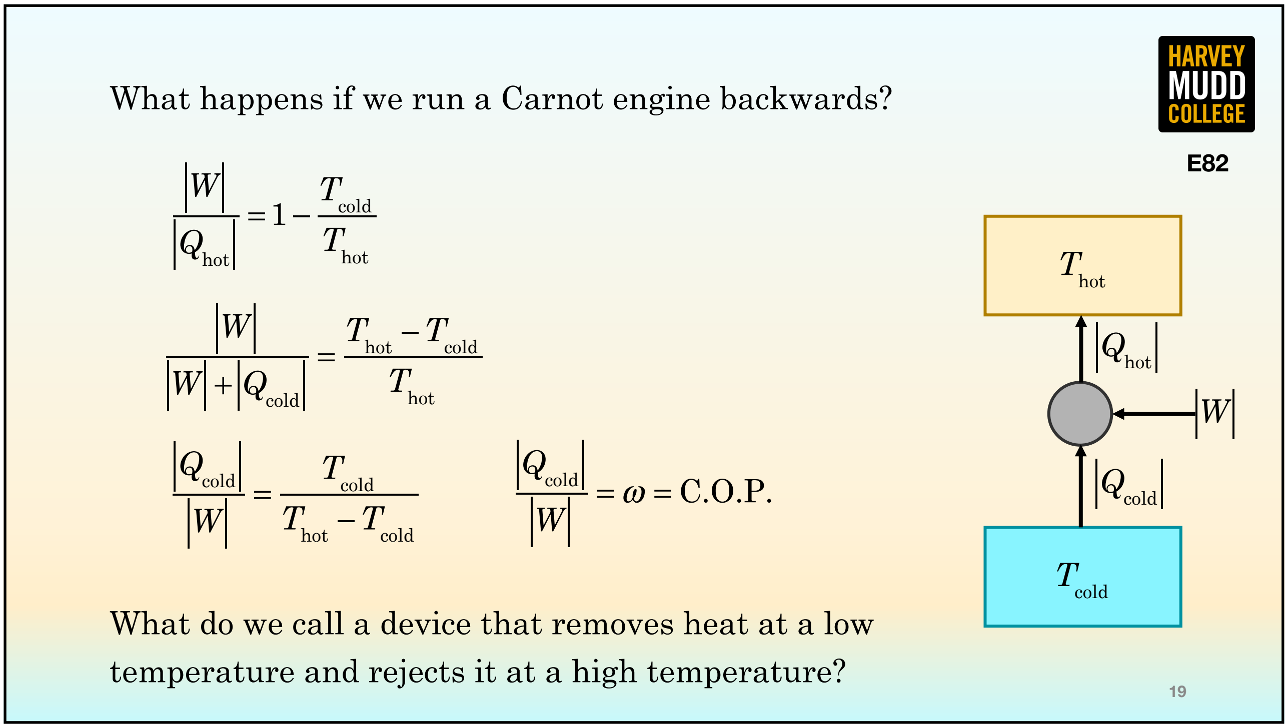
$$-W_{\text{net}} = 409.9 \text{ kJ/kg}$$

$$Q_{\text{hot}} = 1939.7 \text{ kJ/kg}$$

$$Q_{\text{cold}} = -1529.8 \text{ kJ/kg}$$

What are the engineering problems with the Steam Carnot cycle?





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What happens if we run a Carnot engine backwards?

$$\frac{|W|}{|Q_{\text{hot}}|} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$$

$$\frac{|W|}{|W| + |Q_{\text{cold}}|} = \frac{T_{\text{hot}} - T_{\text{cold}}}{T_{\text{hot}}}$$

$$\frac{|Q_{\text{cold}}|}{|W|} = \frac{T_{\text{cold}}}{T_{\text{hot}} - T_{\text{cold}}} \qquad \frac{|Q_{\text{cold}}|}{|W|} = \omega = \text{C.O.P.}$$

What do we call a device that removes heat at a low temperature and rejects it at a high temperature?