Thrust Curve from Acceleration and Velocity Flight Data

Determination of the thrust curve from the acceleration and velocity flight data is relatively straightforward in principle, but can become quite involved depending on the level of detail and accuracy you want to use. We begin with either the rocket equation and include gravity and drag, or Equation 1.3 from the 1-D analytical model. For upward vertical motion in the positive x direction, they both end up as

$$ma = T_x - \frac{1}{2} C_D A_p \rho v^2 - mg. \tag{1.1}$$

The difference is that during the thrust phase (the phase of interest) m is a function of time for the rocket equation and not for the 1-D analytical model. The choice of whether to include a time-varying mass affects the ease of calculation and the accuracy of the thrust curve, especially when the propellant mass is a significant fraction of the rocket mass.

Rearranging Equation (1.1) yields

$$T_x = m\left(a + g\right) + \frac{1}{2} C_D A_p \rho v^2 \tag{1.2}$$

If one assumes constant mass and drag coefficient, evaluation is straightforward for $a(t)$ (or $a(t)+g$) data and $v(t)$ data. The drag coefficient data are usually available as a function of velocity, either from the post-boost phase, or from Rocksim or Open Rocket, so a varying C_D is usually not too hard to account for. However, the time varying mass is much more challenging. The usual assumption in the rocket equation is that the exit velocity of the exhaust, v_e, is constant, and that the relationship between thrust and mass is

$$T_x = v_e \frac{dm}{dt}, \text{ or } m(t) = m_0 + \frac{1}{v_e} \int_{t_0}^{t} T_x dt \tag{1.3}$$

In other words, if we knew the mass as a function of time, we would know the thrust and vice versa. One way to proceed is to assume a known easy-to-calculate $m(t)$, and use it in Equation(1.2) to calculate a tentative T_x, which is then used with Equation (1.3) to calculate a better $m(t)$, and the process is iterated until the mass curve and thrust curve don't change much from iteration to iteration. In the iterations, one would want to make sure that the calculated change of mass was equal to the known amount of propellant on board, perhaps by adjusting the value of v_e. The initial value of v_e is determined by $v_e = g_0 I_{sp}$ where I_{sp} is the specific impulse of the propellant.