Kalman Filter Example

This example follows the nomenclature in the Wikipedia Kalman Filter article. The example looks at combining the roll angle from the magnetometer with the roll rate data from the *z*-axis rate gyro. The State Vector, $\mathbf{x}$, consists of two states, $\dot{\theta}$, the roll rate, and θ, the roll angle. The subscript, *k*, will indicate the state value at time step, *k*. These values are the true values of roll rate and roll. Just due to random forces and torques, we assume a value for the process noise, $w_{\theta,k}$. Although the model assumes that the process noise is a Gaussian random variable, in our case, it will be the change in rotation rate from step to step. A proper analysis would look at the frequency spectrum of the $\Delta\dot{\theta}$'s and add a shaping filter to a Gaussian white-noise signal to get $w_{\theta,k}$, but treating it as a white noise source is sufficient for this example. We assume that there is no active control, so the control vector is always 0, and we can ignore the $\mathbf{B}_k\mathbf{u}_k$ term. The dynamics of the system is described by the two equations:

$$\begin{aligned}\dot{\theta}_{k+1} &= \dot{\theta}_k + w_{\theta,k} \\ \theta_{k+1} &= \theta_k + \Delta t\dot{\theta}_{k+1} = \theta_k + \Delta t\dot{\theta}_k + \Delta t w_{\theta,k}\end{aligned}\quad. \tag{1.1}$$

We have two measured values, the roll rate, $\dot{\theta}_m$, as measured by the rate gyro, and the roll angle, θ_m, as measured by the magnetometer. The two measurements each have an associated measurement noise, v_{rg} for the rate gyro, and v_{mg} for the magnetometer, which are assumed to be normally distributed random variables with variances of σ_{rg}^2 and σ_{mg}^2 respectively. See, there was some reason to measure the standard deviation of the noise in the sensors. At any given time step, the relationship between the true state values and the measurements is:

$$\begin{aligned}\dot{\theta}_{m,k} &= \dot{\theta}_k + v_{rg,k} \\ \theta_{m,k} &= \theta_k + v_{mg,k}\end{aligned}\quad. \tag{1.2}$$

In terms of the state equation, $\mathbf{x}_k = \mathbf{F}_k\mathbf{x}_{k-1} + \mathbf{w}_k$, we have

$$\begin{bmatrix}\dot{\theta}_{k+1} \\ \theta_{k+1}\end{bmatrix} = \begin{bmatrix}1 & 0 \\ \Delta t & 1\end{bmatrix}\begin{bmatrix}\dot{\theta}_k \\ \theta_k\end{bmatrix} + \begin{bmatrix}1 \\ \Delta t\end{bmatrix} w_{\theta,k}\ . \tag{1.3}$$

In terms of the measurement equation, $\mathbf{z}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k$, we have

$$\begin{bmatrix}\dot{\theta}_{m,k} \\ \theta_{m,k}\end{bmatrix} = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}\begin{bmatrix}\dot{\theta}_k \\ \theta_k\end{bmatrix} + \begin{bmatrix}v_{rg,k} \\ v_{mg,k}\end{bmatrix}. \tag{1.4}$$

The things that are missing are the process noise covariance matrix, $\mathbf{Q}_k$, and the observation noise covariance matrix, $\mathbf{R}_k$. The observation noise covariance matrix has, as its on-diagonal terms, the variance of the measurement sensors. Unless the noise from the various sensors is somehow coupled, the off-diagonal terms are all 0, so **R**, which won't change with time step is

$$\mathbf{R} = \begin{bmatrix} \sigma_{rg}^2 & 0 \\ 0 & \sigma_{mg}^2 \end{bmatrix}. \tag{1.5}$$