C_D From Post-Boost Flight Data

To calculate the drag coefficient, C_D, from flight data in the post-boost phase we need an equation of motion. A reasonable 1-D starting point is either the rocket equation with additions for gravity and drag force, or the equation of motion from the 1-D analytical solution. They both look like

$$ma = T_x - \frac{1}{2} C_D A_p \rho v |v| - mg . \tag{1.1}$$

The difference is that during the thrust phase m is a function of time for the rocket equation and not for the 1-D analytical model. However, after the boost phase, when the thrust is 0, the mass is constant in both cases. Assuming upward vertical motion, with positive x being upward and setting the thrust to 0, we end up with

$$m(a+g) = -\frac{1}{2} C_D A_p \rho v^2 \tag{1.2}$$

It can be seen that for constant C_D a plot of a versus v^2 is a straight line with a slope related to C_D. There are two rearrrangements that make determination of C_D relatively straightforward, the first is

$$-\frac{2m}{A_p \rho}(a+g) = C_D v^2 , \tag{1.3}$$

where a plot of $-\frac{2m}{A_p \rho}(a+g)$ versus v^2 has a slope of C_D and an intercept of 0. The second is

$$-\frac{2m}{A_p \rho} a = C_D v^2 + \frac{2mg}{A_p \rho} , \tag{1.4}$$

where a plot of $-\frac{2m}{A_p \rho} a$ versus v^2 has a slope of C_D and an intercept of $\frac{2mg}{A_p \rho}$. Which is to be preferred? The answer depends on what an axial accelerometer on a rocket measures, is it a, $a+g$, or $a-g$? It also depends on which is easier to evaluate graphically, an intercept of 0 or an intercept of $\frac{2mg}{A_p \rho}$. Evaluation of the expressions requires having both $a(t)$ (or $a(t)+g$) data and $v(t)$ data. While it is possible to have a Pitot-static tube in the nose of a rocket to directly measure velocity (interesting research possibility), it is much more common to determine the velocity by numerically integrating the acceleration, or taking the numerical derivative of the altitude. This procedure works quite well to calculate an average C_D or fitting a C_D to noisy data, but it won't give you C_D as a function of v.

If you have very clean data, to calculate C_D as a function of v, we rearrange Equation (1.3) as

$$C_D = -\frac{2m}{A_p \rho} \frac{(a+g)}{v^2}, \tag{1.5}$$

and plot $-\frac{2m}{A_p \rho} \frac{(a+g)}{v^2}$ as a function of v. A quick search of the literature or Rocksim/Open Rocket will show that C_D is a fairly complicated function of v, especially in the transonic region. If your data are good enough to have this plot be meaningful, it is useful to use standard error propagation techniques to put confidence bounds on the calculated C_D versus v curve.

There is one more technique to calculate the drag coefficient from the post-boost phase: you can use a least-squares fitter on your integrated equation of motion, either velocity or altitude, to determine the best-fit value of C_D.