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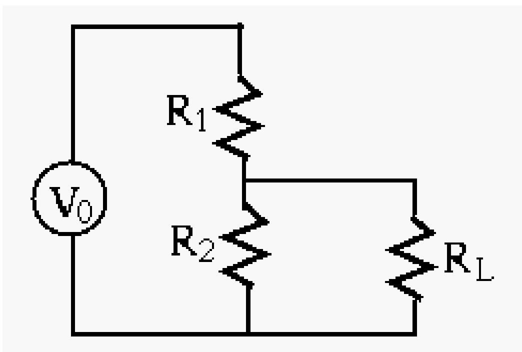
E84 Homework 4

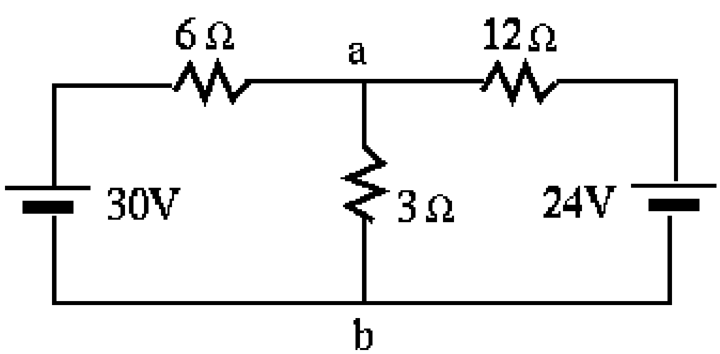
1. In the voltage divider circuit below, $V_0 = 20V$, $R_1 = R_2 = 500\Omega$. Use Thevenin's theorem to find the current through and voltage across the load resistor R_L when it is 100Ω , 200Ω , 300Ω , respectively.



Solution:

- $R_T = 250$, $V_T = 10V$
- When $R_L = 100$, $I_L = V_T/(R_T + R_L) = 1/35$, $V_L = I_L R_L = 20/7$
- When $R_L = 200$, $I_L = V_T/(R_T + R_L) = 1/45$, $V_L = I_L R_L = 40/9$
- When $R_L = 300$, $I_L = V_T/(R_T + R_L) = 1/55$, $V_L = I_L R_L = 60/11$

2. Use Thevenin's and Norton's theorems to determine the current in the 3Ω resistor of the following figure.



Solution: move two voltage sources to left, and 3 Ω resistor to the right as load find equivalent voltage Vo and internal resistance and Ro.

- Use Thevenin's theorem:

The current going clockwise around the loop (without load) is $(30-24)/(6+12)=1/3$, the voltages across the 6 Ω resistor and 12 Ω resistor are -2V and 4V, respectively.

$$V_T = 30 - 6/3 = 24 + 12/3 = 28\ V, \qquad R_T = 6//12 = 4\Omega$$

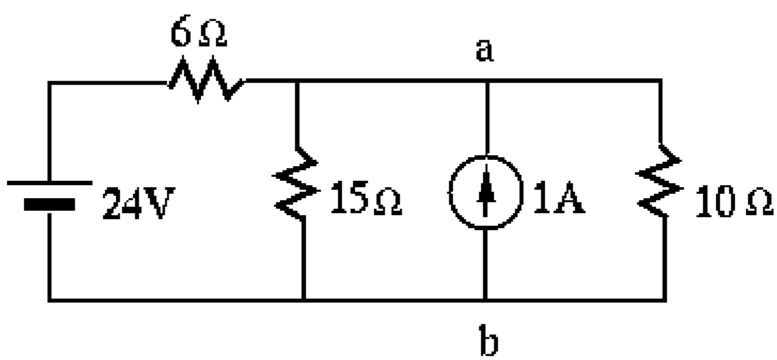
The current through the load resistor is

$$I_L = \frac{V_T}{R_T + R_L} = \frac{28}{4+3} = 4\ A$$

- Use Norton's theorem: Turn the two voltage sources to current sources $I_1 = 30/6 = 5\ A, \quad I_2 = 24/12 = 2\ A$. These two current sources are in parallel to form a total current source of $I_N = I_1 + I_2 = 7\ A$. The two internal resistances are also in parallel $R_N = R_1//R_2 = 6//12 = 4\Omega$. The current through the load resistor is obtained by current dividor:

$$I_L = I_N \frac{R_N}{R_N + R_L} = 7\frac{4}{4+3} = 4\ A$$

3. Find voltage across and the current through the 10 Ω resistor.



Solution:

- Use superposition principle. When 24V is acting alone with 1A open, parallel resistors 15 and 10 become $15//10=6$, $V'ab = 24 \times 6/(6+6) = 12V$, $I' = 12/10 = 1.2A$. When 1A is acting alone with 24V closed, parallel resistors 6 and 15 become $90/21 = 30/7$, $I'' = (30/7)/(10+30/7) = 3/10 = 0.3A$, $V''ab = I'' \times 10 = 3V$, overall $V = V' + V'' = 12 + 3 = 15$,

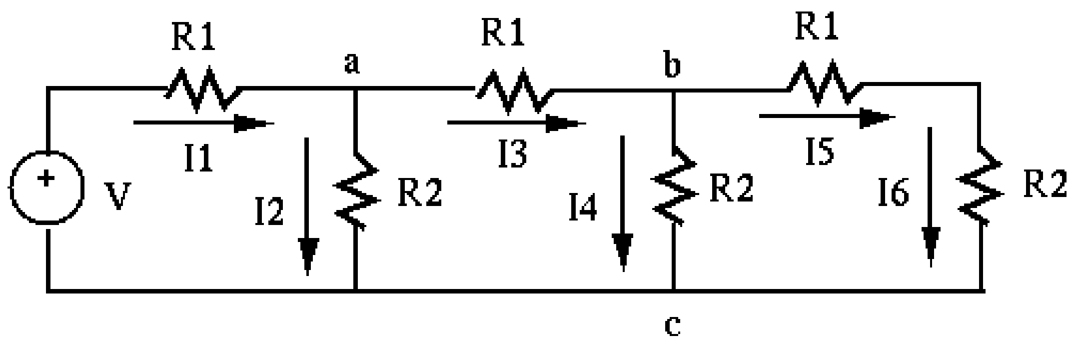
$I = I' + I'' = 1.2 + 0.3 = 1.5A$.

- Use Norton's theorem. Turn the voltage source to a current source with $I = 24/6 = 4\ A$ and $R = 6\ \Omega$. Then the two current sources in parallel form a total current source of $I_N = 5\ A$ with internal resistance $R_N = 15//6 = 30/7$. By current devidor, we get

$$I_L = 5\frac{30/7}{10+30/7} = 1.5, \qquad V_L = 1.5 \times 10 = 15\ V$$

4. Find all currents in the diagram in which $V = 120V$, $R_1 = 2\Omega$, $R_2 = 20\Omega$.

Hint: It is very hard to solve the problem by finding the currents in the order of I_1, I_3, I_5, as computing the resistances of the resistor network is tedious. However, it is much more straight forward to find the currents in the order of I_5, I_3, I_1, if you assume I_5 is known, e.g., $I_5 = I_6 = 1A$. However, the voltage for the voltage source obtained based on this assumption is of course not as given (120V). In this case, the linearity property $F(ax+by) = aF(x) + bF(y)$ can be applied. In particular, given $y = F(x)$, then $ay = F(ax) = aF(x)$. Use this relationship to find the actual values of the currents.



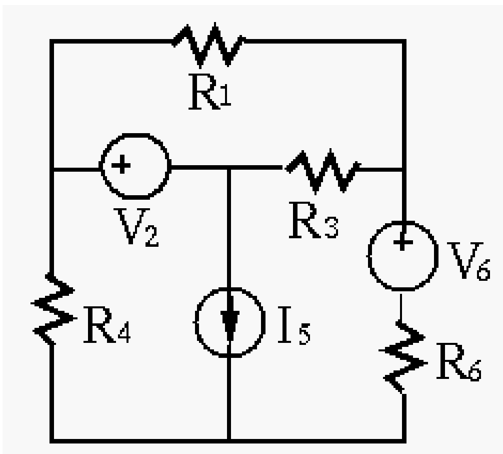
Solution:

- use node c as reference (ground), assume $I_5 = 1$, then $V_b = (20+2) = 22V$, $I_4 = 22/20 = 1.1A$, $I_3 = I_4 + I_5 = 1.1 + 1 = 2.1A$, $V_a = 2.1 \times 2 + V_b = 4.2 + 22 = 26.2V$, $I_2 = V_a/20 = 26.2/20 = 1.31A$, $I_1 = I_2 + I_3 = 1.31 + 2.1 = 3.41A$, $V_0 = I_1 \times 2 + V_a = 2 \times 3.41 + 26.2 = 33.02$.

- But the given voltage source is 120V, all currents should be scaled up by a factor $120/33.02 = 3.634$, $I_1 = 12.39$, $I_2 = 4.76$, $I_3 = 7.63$, $I_4 = 4.00$, $I_5 = 3.63$

5. In the circuit below, $R_1 = 1\Omega$, $V_2 = 2V$, $R_3 = 1\Omega$, $R_4 = 3\Omega$ $I_5 = 5A$, $V_6 = 2V$, and $R_6 = 1\Omega$. Find
    1. current through voltage souce V_2.
    2. current through resistor R_3

   (Hint: consider superposition theorem.)



Solution: Consider each of the three sources alone:

- V2 alone (I5 open, V6 short): $I' = V2/[R1 + R3//(R3 + R6)] = 2/(1 + 1//4) = 2/(1 + 4/5) = 10/9$ (left).
- V6 alone (I5 open, V2 short): $I'' = 0.5xV6/(R6 + R4 + R1//R3) = 1/(1 + 3 + 0.5) = 1/4.5 = 2/9$ (left)
- I5 alone (V2, V6 both short):

  current thru R6: $I5 \times R4/[R4 + (R6 + R1//R3)] = 5 \times 3/4.5 = 10/3$

  current thru R4: $I5 \times (R6 + R1//R3)/[R4 + (R6 + R1//R3)] = 5 \times 1.5/4.5 = 5/3$

  current thru R1 is (half of that thru R6): 5/3

- I''' is the sume of current thru R4, current thru $R1 = 5/3 + 5/3 = 10/3$ (right)

  and current thru V2: $I' + I'' + I''' = 10/9 + 2/9 - 10/3 = -2$ (left) or 2 (right)

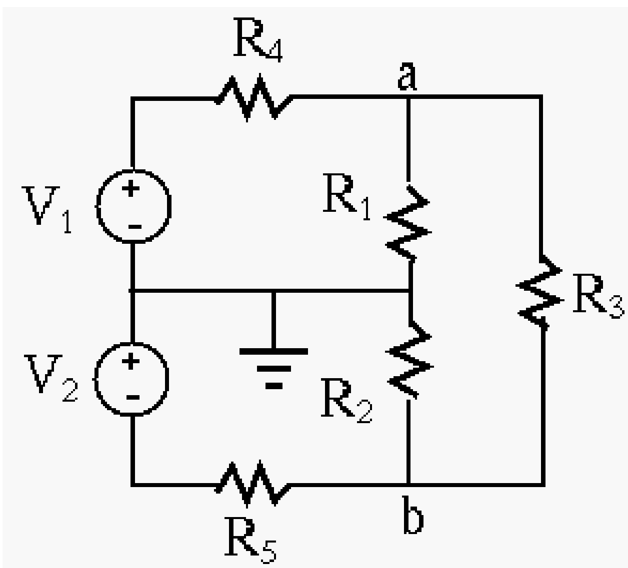
- currents thru R3 due to V2 and V6 respectively are the same as that thru V2 currents thru R3 due to I5 is (half of current thru R6 above): 5/3 (left)

- total current thru R3: $10/9 + 2/9 + 5/3 = 3$ (left)

- The sum of two currents is equal to $I5 = 2 + 3 = 5$

6. In the circuit below, $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, $R_4 = 4\Omega$, $R_5 = 5\Omega$, $V_1 = 4V$, $V_2 = 5V$. Find voltage V across and current I through the load resistor R_3 by converting the rest of the circuit into a
   - Thevenin's voltage source V_T with R_T
   - Norton's current source I_N with R_N

   Verify your results by converting the current source into a voltage source (or vise versa). Then find voltage V and current I associated with load R_3.



Solution:

- Find Thevenin's voltage source V_T with R_T. First turn voltage sources off and find R_T between nodes a and b:

$$R_T = R_1||R_4 + R_2||R_5 = \frac{R_1 R_4}{R_1 + R_4} + \frac{R_2 R_5}{R_2 + R_5} = 78/35$$

Find V_{oc}:

$$V_a = V_1 \frac{R_1}{R_1 + R_4} = 4/5, \qquad V_b = -V_2 \frac{R_2}{R_2 + R_5} = -10/7$$

$$V_{oc}=V_a-V_b=78/35$$

- Norton's current source I_N with R_N. First $R_N=R_T=78/35$. Then use loop current method to find $I_N=I_{sc}$. Assume loop currents I_a (top-left loop), I_b (bottom left loop) and $I_c=I_{sc}=I_N$ (right loop), and we have these loop current equations:

$$\left\{\begin{array}{rrr} 5I_a & & -I_c=4 \\ & 7I_b & -2I_c=5 \\ -I_a & -2I_b & +3I_c=0 \end{array}\right.$$

  Solving this to get $I_N=I_{sc}=I_c=1$.

- Verify your results by converting the current source into a voltage source (or vise versa).

$$I_N R_N=78/35=V_T$$

Finally,

$$I=\frac{V_T}{R_T+R_3}=\frac{78/35}{78/35+3}=\frac{78}{183}=0.426$$

$$V=IR_3=1.28V$$

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