Next: About this document ...

E84 Homework 3

1. While various voltage sources such as batteries are very common in everyday life, current sources are not widely available. One type of current source is the photocells, which generates current proportional to the intensity of the incoming light. Also, certain specially designed transistor circuits can generate to output constant current. Moreover, as discussed in class, any current source can be obtained by converting a corresponding voltage source. Design a current source with $I_0 = 10$ mA and $R_0 = 1\ K\Omega$ by converting a voltage source. Find its voltage V_0 and internal resistance R_0.

   Solution: $R_0 = 1\ K\Omega$, $V_0 = I_0 \times R_0 = 10\ mA \times 1000\ \Omega = 10\ V$.

2. The output resistance of the power amplification circuit of a Hi-Fi system is $R_{out} = 8\Omega$ and the output voltage is $V_{out} = 20V$. Find the power received by the speaker, the total power consumption, and the power efficiency of the circuit, for each of the three possible speaker resistances: $R_L = 4$, $R_L = 8$, or $R_L = 16$

   Solution: The power received by the load (speaker) is

$$P_L = \frac{V_{out}^2}{(R_{out} + R_L)^2} R_L$$

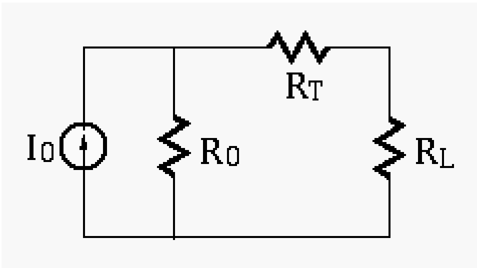
   and the power efficiency is

$$\eta = \frac{P_L}{P_{total}} = \frac{R_L}{R_{out} + R_L}$$

   where $V_{out} = 20V$, $R_{out} = 8\Omega$.

   - When $R_L = 4\Omega$, $P_L = 100/9 = 11.1$ Watts, $\eta = 1/3$, the total power is $P_{total} = 100/3 = 33.3$ Watts.
   - When $R_L = 8\Omega$, $P_L = 100/8 = 12.5$ Watts, $\eta = 1/2$, the total power is $P_{total} = 100/4 = 25$ Watts.
   - When $R_L = 16\Omega$, $P_L = 100/9 = 11.1$ Watts, $\eta = 2/3$, the total power is $P_{total} = 100/3 = 16.7$ Watts.

3. Find the optimal load resistance R_L so that it receives maximal power from the current source $I_0 = 10A$ with internal resistance $R_0 = 1\Omega$ and power transmission line resistance $R_T = 9\Omega$. Find the maximum load power and the power loss along the transmission line.

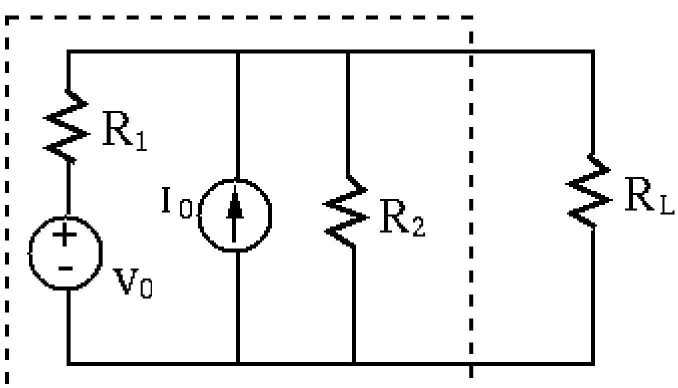


To verify your choice of load resistance, show that the power consumption of the load will always be lower than this maximum when its resistance is either increased or decreased by ten percent.

Solution:

First convert current source to voltage source with $V_0 = I_0 R_0 = 10V$ and $R_0 = 1\Omega$. To maximize load power consumption, let $R_L = R_0 + R_T = 10\Omega$. The current is $I = 10V/20\Omega = 0.5A$. Load power is $I^2 R_L = 10/4 = 2.5W$, power loss on transmission line is $I^2 R_T = 9/4A$ When $R_L = 11\Omega$, $I = 10V/21\Omega$, $W_L = I^2 R_L = 2.494$ When $R_L = 9\Omega$, $I = 10V/19\Omega$, $W_L = I^2 R_L = 2.493$

4. Convert the following circuit into (a) an equivalent current source (I_{cs}, R_{cs}) and then (b) an equivalent voltage source (V_{vs}, R_{vs}). Give an expression for the load R_L so that it will receive maximum power from the source.



Solution:

(a) Convert voltage source (V_0, R_1) on the left to a current source $(I_0' = V_0/R_1, R_1)$ in parallel with the current source (I_0, R_2). The overall current source is therefore:

$$I_{cs} = I_0 + \frac{V_0}{R_1}, \qquad R_{cs} = R_1||R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

(b) Convert the overall current source above to a voltage source:

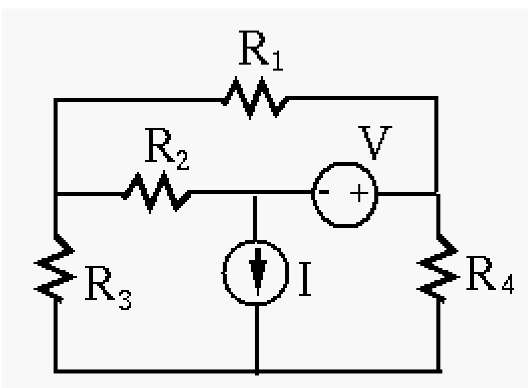
$$V_{vs} = I_{cs}R_{cs} = (I_0 + \frac{V_0}{R_1})\frac{R_1 R_2}{R_1 + R_2} = I_0\frac{R_1 R_2}{R_1 + R_2} + V_0\frac{R_2}{R_1 + R_2}, \quad R_{vs} = R_{cs}$$

(c) Thevenin't theorem:

$$R_{th} = R_1||R_2, \qquad V_{Th} = I_0\frac{R_1 R_2}{R_1 + R_2} + V_0\frac{R_2}{R_1 + R_2}$$

(d) For R_L to receive maximum power, we need $\underline{R_L = R_{cs} = R_{vs} = R_1||R_2}$

5. Find all node voltages in the circuit with respect to the bottom node as ground, where $R_1 = 100\Omega$, $R_2 = 5\Omega$, $R_3 = 200\Omega$, $R_4 = 50\Omega$, $V = 50V$, $I = 0.2A$. Use both node voltage and loop current methods to solve this circuit. Choose independent loops and nodes wisely to simplify your computation.



Note: To simplify the analysis while using node voltage or loop current method, it is preferable to

- choose independent loops so that no current source is shared by two loops;
- choose ground node so that one of the voltage sources is connected to ground.

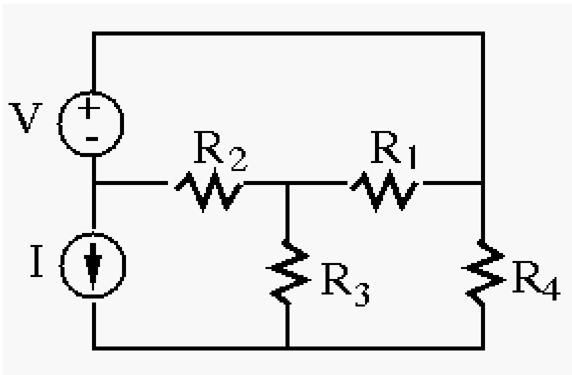
Solution:

- **Node voltage:** Let the bottom node be ground and other node voltages be V_1 (left), V_2 (middle) and V_3 (right).

$$\begin{array}{ll} \text{left node:} & V_1/200 + (V_1 - V_2)/5 + (V_1 - V_3)/100 = 0 \\ \text{right node:} & (V_3 - V_1)/100 + V_3/50 + (V_2 - V_1)/5 + 0.2 = 0 \\ \text{voltage source:} & V_3 = V_2 + V = V_2 + 50 \end{array}$$

Solving this we get:

$$V_1 = -45.23, \quad V_2 = -48.69, \quad V_3 = 1.31$$



Alternatively, rearrange the components as shown in the figure above and assume the node between the current and voltage sources is grounded $V_2 = 0$, then $V_3 = 50V$, and denote previous ground by V_0. We have

$$\begin{array}{ll} \text{middle node } V_1: & V_1/5 + (V_1 - V_0)/200 + (V_- 50)/100 = 0 \\ \text{bottom node } V_0: & (V_0 - V_1)/200 + (V_0 - 50)/50 = 0.2 \end{array}$$

Solving this we get:

$$V_1 = 3.46, \quad V_0 = 48.7, \quad V_3 = 50, \quad V_2 = 0$$

Treating V_0 as ground, we get the same result as before:

$$V_1 = -45.2, \quad V_0 = 0, \quad V_3 = 1.3, \quad V_2 = -48.7$$

We see that the second method is easier. Lesson: if one of two ends of a voltage sourse is treated as the ground, the number of equations is reduced by one.

- **Loop current:** Let the loop currents be I_a (top), I_b (left) and I_c (right).

$$\begin{array}{ll} \text{top loop:} & 100I_a + 50 + 5(I_a - I_b) = 0 \\ \text{current source:} & I_b - I_c = I = 0.2 \\ \text{bottom loop:} & 200I_b + 5(I_b - I_a) - 50 + 50I_c = 0 \end{array}$$

Solving this we get:

$$I_a = -0.465, \qquad I_b = 0.226. \qquad I_c = 0.026$$

Alternatively, use the figure on the right and assume loop currents I_a through voltage source V, $I_b = -0.2$ through current source and I_c through R_4. We have

$$\begin{array}{ll} \text{top loop:} & 100(I_a - I_c) + 5(I_a + 0.2) = 50 \\ \text{right loop:} & 100(I_c - I_a) + 50I_c + 200(I_c + 0.2) = 0 \end{array}$$

Solving this we get:

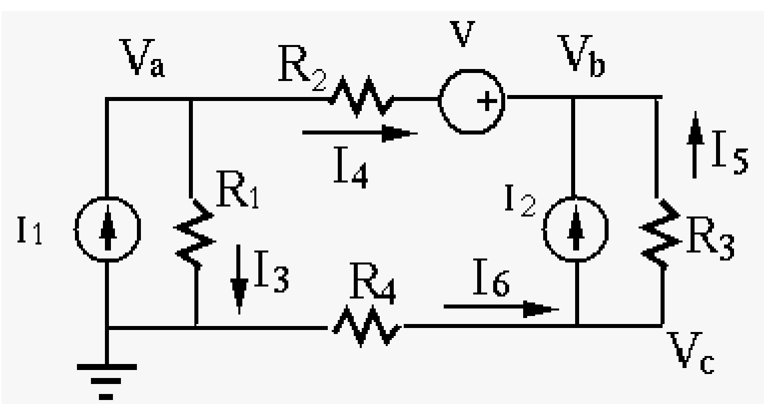
$$I_a = 0.49, \quad I_b = -0.2, \quad I_c = 0.026$$

Current through R_1 is $I_c - I_a = -0.464$, current through R_3 is $I_c + 0.2 = 0.226$, same as before.

We see that the second method is easier. Lesson: if a current source is in a single loop, then the number of equations is reduced by one.

6. Solve the circuit shown in the diagram below with $I_1 = 2$, $I_2 = 3$, $V = 9$, $R_1 = R_3 = 1$, $R_2 = 2$, $R_4 = 4$, to find V_a, V_b, V_c, I_3, I_4, I_5, and I_6 (all currents are in A, voltegae in V, and resistances in).

Resolve the problem when $V = 1$.



Solution:

- Method I: Convert I_1 and R_1 into a voltage source with $V_1 = 2$ in series with $R_1 = 1$. Convert I_2 and R_3 into a voltage source with $V_2 = 3$ in series with $R_3 = 1$. The sum of all four resistances in the loop is $2+9-3=8$. The sum of all three voltage sources in series is 8, $I_4=1$, $I_6=-1$, $V_a=1$, $V_b=8$, $V_c=4$.

- Method II - Loop current method: Identify three loops:
  1. I_1 and R_1 with loop current I_1
  2. I_2 and R_3 with loop current I_2
  3. R_1, R_2, V, R_3 and R_4 with clockwise loop current $I=I_4=-I_6$

$$(I-I_1)R_1+IR_2-V+(I+I_2)R_3+IR_4=I-2+2I-9+I+3+4I=8I-8=0, \quad I=1$$

$$I_3=I_1-I=2-1=1, \quad I_5=-(I_2+I)=-(3+1)=-4$$

- Method III - Node voltage method:

$$-I_1+\frac{V_a}{R_1}+\frac{V_a-V_b+9}{R_2}=-2+V_a+\frac{V_a-V_b+9}{2}=0$$

$$I_2+\frac{V_a-V_b+9}{R_2}+\frac{V_c-V_b}{R_3}=3+\frac{V_a-V_b+9}{2}+V_c-V_b=0$$

$$\frac{V_c}{R_4}+I_2+\frac{V_c-V_b}{R_3}=\frac{V_c}{4}+3+V_c-V_b=0$$

$$\left\{\begin{array}{lcl} 3V_a-V_b & = & -5 \\ V_a-3V_b+2V_c & = & -15 \\ -4V_b+5V_c & = & -12 \end{array}\right. \qquad \left\{\begin{array}{l} V_a=1 \\ V_b=8 \\ V_c=4 \end{array}\right.$$

If $V=1$, the sum of all three voltage sources is 0, then we get $I_4=I_6=0$, $V_a=2$, $V_b=3$, $V_c=0$, $I_3=2$, $I_5=-3$.

- About this document ...

Next: About this document ...

Ruye Wang 2016-02-10