Self-Organized Patterns in the SOM Network

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***Abstract*—This paper reports the discovery of certain self-organized patterns that develop automatically and unexpectedly during the training of a typical self-organizing map (SOM) artificial neural network. These highly structured patterns emerge and evolve gradually from the random initial state of the network as the training progresses. These patterns are web-like and characterized by some linear features at different scales, which tend to intersect at some common positions, and they gradually form a highly organized hierarchical structure as the network is continuously trained. The properties and variations of these patterns are affected by the parameters used in the training process. The specific mechanism of the formation of such self-organized patterns is still mostly unknown and currently under investigation. As a preliminary effort to understand the phenomenon, this paper also speculates and hypothesizes the possible mechanism of the phenomenon based on some qualitative and heuristic studies.**

Keywords: Neural network, Self-organizing map (SOM), Self-organized patterns

I. Introduction

A. Review of Competitive Learning of SOM Network

A *Self-organizing map (SOM)* is a classical artificial neural network ([1], [2]) that is composed of an input layer of n nodes and an output layer of m nodes organized in a 2-D map (e.g., $\sqrt{m} \times \sqrt{m}$ square grid), as shown in Fig. 1. Each output node is fully connected to all n input nodes through n weights, represented as an n-D vector $\mathbf{w} = [w_1, \cdots, w_n]^T$, and produces an activation value $\mathbf{w} \cdot \mathbf{x}$, each time an n-D sample vector $\mathbf{x} = [x_1, \cdots, x_n]^T$ is presented to the input layer. The operation of a SOM is essentially a mapping from an n-D vector space to a 2-D grid of m points. During training, the spatial vicinity of the output nodes is maintained, i.e., output nodes close to each other in the SOM respond to similar input samples. The typical application of such a network is to represent and visualize the potential cluster structure in an n-D dataset by the output nodes in the SOM.

During the iterative training process of a SOM network, a set of training samples (n-D vectors) is presented to the input layer in random order, and the output nodes gradually learn to respond selectively to different clusters in the dataset by continuously modifying their weights, which are initialized randomly. The steps of the competitive learning are listed below.

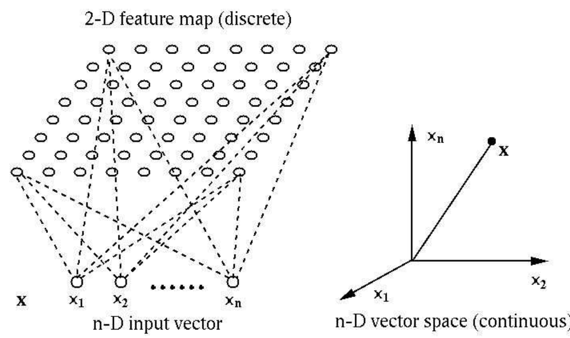

Fig. 1. The SOM network

1) Present one of the samples randomly selected from the training data to the input of the network, calculate the corresponding activation for each of the m output nodes $y_i = \mathbf{w}_i \cdot \mathbf{x}$ $(i = 1, \cdots, m)$. If $y_j \geq y_i$ for all $i = 1, \cdots, m$, the jth node becomes the winner.
2) Modify the weights of the output nodes:
$$\mathbf{w}_i^{new} = \mathbf{w}_i^{old} + g_{ij}\eta(\mathbf{x} - \mathbf{w}_i^{old}) \qquad (i = 1, \cdots, m) \quad (1)$$
where $g_{ij} = exp\left(-d_{ij}^2/\sigma^2\right)$ is a Gaussian weighting function, which determines the amount of learning for the ith node based on its distance d_{ij} to the winner (assumed to be the jth node). Update both the learning rate $0 < \eta < 1$ and the width of the weighting function σ so that they both decay exponentially over time from their initial values.
3) Repeat the steps above until the iteration converges to a steady state in which all output nodes have learned to selectively respond to certain type of input patterns and the feature map has stabilized.

Different from the typical winner-take-all competitive learning, where only the winning node gets to modify its weight vector so that it moves closer to the current sample in the n-D space, here in the training of the SOM neetwork, the weight vectors of all output nodes are modified to different extent together with that of the winner, with the amount of learning reduced as the distance to the winner in the 2-D map increases. As the result of such a learning algorithm, all nodes in a local region of the SOM become specialized in responding to a cluster of similar samples. When the training is complete, the output nodes in the 2-D array are partitioned into a set of locally homogeneous regions composed of nodes responsive to a cluster of similar input samples, i.e., the output nodes form a self-organized map.

An optional step in the learning process is the normalization of all n-D training vectors, together with all weight vectors after each training iteration. By doing so, all input and weight vectors are of unit length, i.e, they are points on the surface of the unit hyper-sphere in the n-D space.

B. The Competitive Learning of Colors

In the experiments reported here, the SOM network is trained by a set of samples representing different colors. The n=3 components of each sample vector $[r,\ g,\ b]$ are for the three primary colors of red, green, and blue, each taking one of the L intensity values (i.e., $r,\,g,\,b = 0,\cdots,L-1$). The total number of training samples is therefore L^3. The SOM can be visualized by color coding each output node according to its $n=3$ weights treated as the R, G, B components of the color. Of course this color is also the favored color of the node, as shown in Figure 2 (left). In the following this color map is referred to as image A.

Moreover, when the network is trained, the training samples can be presented to its input layer of the network one more time to identify those output nodes that become the winners for these training samples. These nodes are also color coded by the input color they represent, while the rest output nodes that never become the winner to any of the training samples remain black, as shown in Fig. 2 (right). This color map is referred to as image B. It is in this image some self-organized patterns are observed. We will be mostly concentrating our attention on image B in the discussion below.

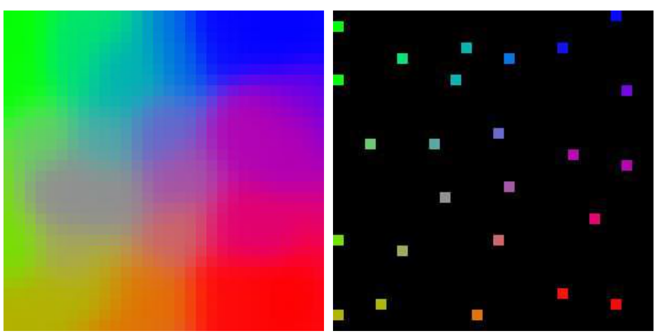

Fig. 2. The color-coded SOM *image A* (left) and the color-coded winners for the training samples *image B* (right)

II. Self-Organized Patterns in the SOM

A. Discovery of the self-organized patterns

Fig. 3 shows a set of SOMs (image B) of different sizes ($\sqrt{m}\times\sqrt{m}$), trained by different number of samples (L^3). The standard deviation (width) of the Gaussian function is typically set to be about one third of the size of the square SOM ($\sigma=\sqrt{m}/3$). It is seen that when the number of training samples increases, some linear features, a central line with narrow black strip on either side, start to emerge. Three such linear features always intersect at the same point around the central area of the SOM. Moreover, as the number of training samples and the size of the SOM array become larger, more of such linear features, but thinner and fainter, continue to emerge and intersect to form more sophisticated and better organized web-like patterns. These self-organized patterns have some hierarchical structure composed of similar line intersections of different scales, as shown in the lower-right panel of Fig. 3. A larger SOM image of size 400 is shown in Fig. 4.

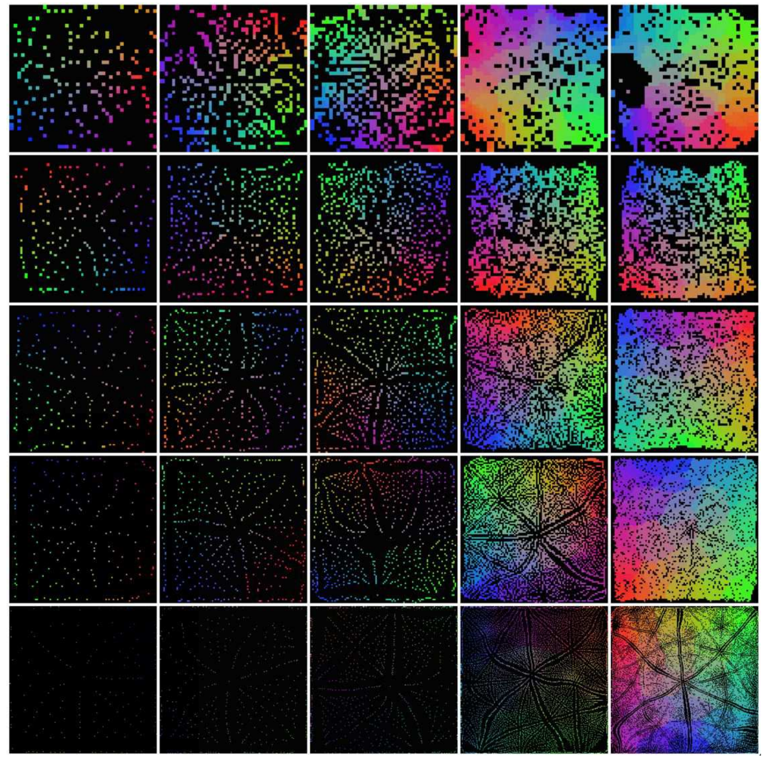

Fig. 3. SOMs of sizes $40^2,\ 60^2,\ 80^2,\ 100^2,\ \ 200^2$ (top down), trained by $6^3=216,\ 8^3=512,\ 10^3=1,000,\ 20^3=8,000$ and $40^3=16,000$ samples (left to right). Some self-organized patterns gradually emerge.

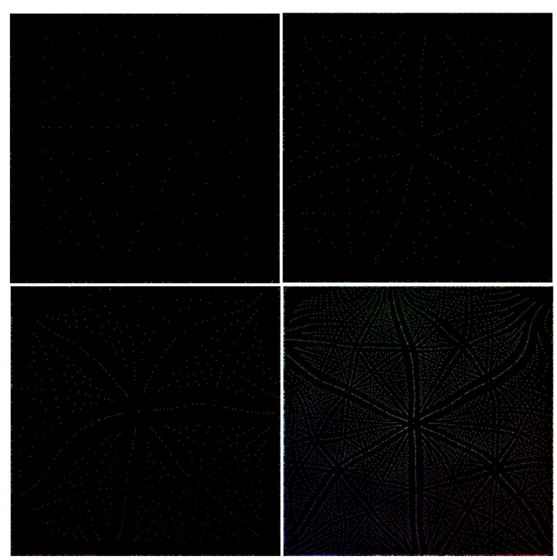

Fig. 4. SOMs of size 400×400, trained by progressively higher number of training samples: $6^3=216,\ 8^3=512,\ 10^3=1,000,\ 20^3=8,000$ and $40^3=64,000$.

B. Emergence of self-organized patterns during training

The self-organized patterns gradually emerge and evolve in image B of the SOM and they become progressively more organized and developed in terms of the fine details and the hierachical structures, as more training iterations are carried out. Fig.5 shows image B after every one of the first 20 iterations. The random nature of the SOM is obvious in the first few iterations following the random initialization of the weights. However, as the training proceeds, some primitive linear feature starts to emerge in the top-left region of the SOM after 15 iterations, and some additional linear feature starts to emerge in the lower-right region after 19 iterations. Such patterns keep evolving to form three-line intersections and the patterns become progressively more organized and complicated with some finer detailed structures, as shown in the subsequent B images in Fig. 6, after every 10 subsequent training iterations.

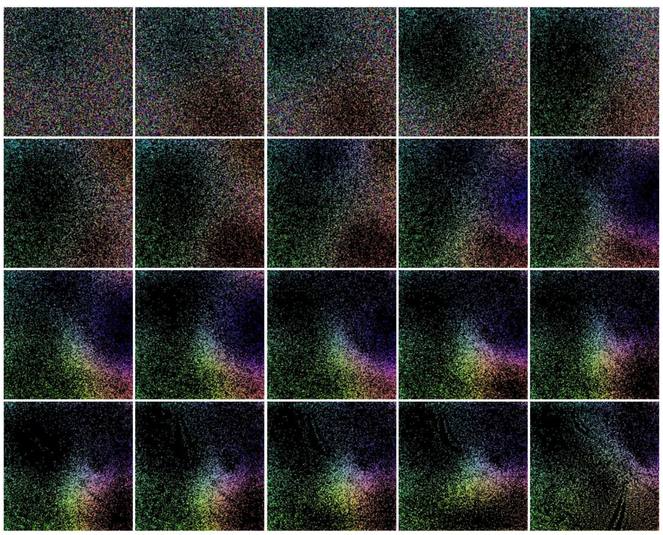

Fig. 5. The B images after each of the first 20 training iterations

When the size of the SOM is further increased (e.g., 2000^2), and the network is trained by a larger number of samples (e.g., 160^3) of different colors, the self-organized patterns become highly structured with much more sophisticated details. In particular, the hierarchical nature of the patterns becomes more obvious, with as many as four visible hierarchical levels of similar patterns.

C. The Self-organized Patterns affected by the weighting function

The width σ of the Gaussian weighting function affects the number of homogeneious regions in the SOM. Fig. 7 shows images B of two SOMs trained with $\sigma = \sqrt{m}/3$ and $\sigma = \sqrt{m}/8$, respectively. Comparing these two images we see that when σ is reduced, the SOM becomes less homogeneous, as it is partitioned into a larger number of regions each composed of fewer nodes responsive to a set of similar colors. Moreover, the web-like patterns become less regular but richer in shape variations.

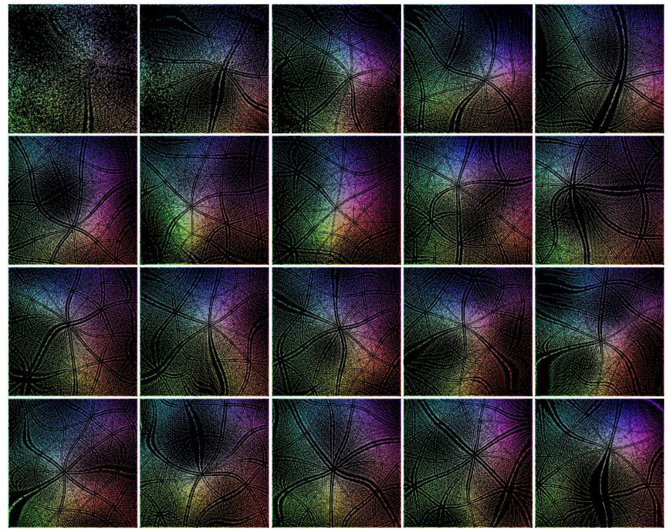

Fig. 6. The B images after every 10 subsequent iterations (30, 40, $\cdots$, 220)

D. The Self-organized Patterns in 3-D SOM

When the dimension of the input vectors is increased from $N = 3$ to $N = 4$, the patterns in the 2-D image B of the SOM become less obvious than those trained by 3-D samples, as shown in Fig. 8 (only the first three principal components extracted from the four components are color coded as R, G, and B). However, when the m output nodes are rearranged as a cubic grid, a 3-D SOM, instead of a 2-D square grid, some intersections formed by three linear features previously observed in 2-D image B appear again, as shown in Fig. 9.

A few more examples for these self-organized patterns are shown in the appendix.

In summary, whether or not the self-organized patterns will form in image B of the SOM, and the specific natures of the patterns when they do form, depend on the following parameters used in the training process:

- The number of training samples L^3 used in training (not too low or too high).
- The number of output nodes m in the SOM (not too low).
- The number of training iterations.
- The width σ of the Gaussian weighting function.

III. Mechanisms of the formation of SOM Patterns

To discover the mechanisms of the formation of the web-like patterns, a sequence of the B images are obtained after each iteration of the training process to see how the pattern gradually emerge and evolve. The size of the SOM and the number of the training samples are deliberately kept small, so that the patterns in the B image are relatively simple and therefore easy to study. As expected, initially both images A and B are composed of nodes of random colors, due to the random initialization of their weights. As the number of training iterations increases, neighboring nodes gradually learn by modifying their weights to selectively respond to similar colors and form homogeneous regions represented by the colors they are most responsive. Moreover, as the training

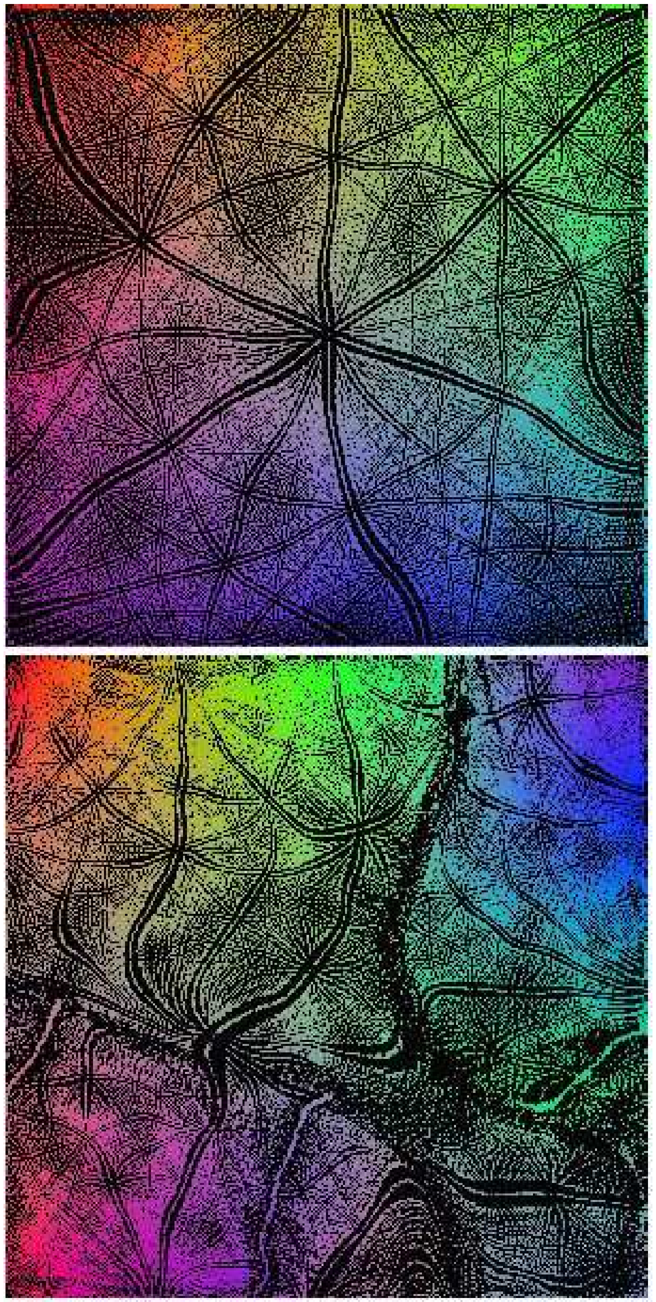

Fig. 7. SOMs trained with $\sigma = \sqrt{m}/3$ (left) and $\sigma = \sqrt{m}/8$ (right)

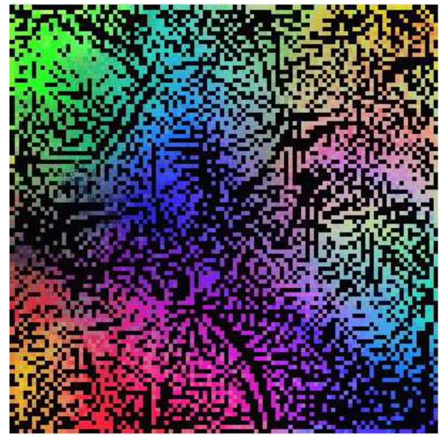

Fig. 8. Image B of 2-D SOM of the output nodes trained on 4-d samples

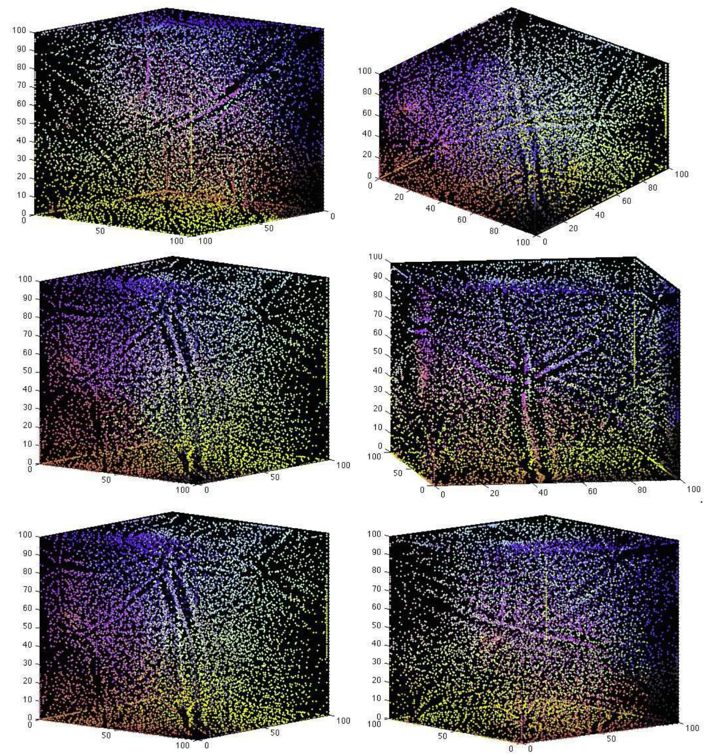

Fig. 9. Image B of 3-D SOM of the output nodes trained on 4-d samples. The cubic image B is viewed from six different vantage ponts to reveal the self-organized patterns in 3-D space.

process further progresses, some line features start to emerge in image B, and they grow longer and more regular to gradually form an intersection around the center of the image, as shown in the four images in Fig. 10. A much longer sequence of such images can be viewed on the author's web page at: http://fourier.eng.hmc.edu/e161/lectures/SOM/node4.html

To explore the mechanisms of the formation of the web-like patterns in the SOM, efforts are made to gain some insights regarding how the patterns in the images gradually emerge, develop, and become progressively more organized, and some preliminary and qualitative explanations can be made as shown below.

- During training, when node n_0 becomes the winner with respect to a specific input of color C_0, the weights of n_0 and its neighbors are modified in such a way that they all get closer to the input, i.e., in the local region around node n_0, the color component C_0 becomes elevated. Consequently, due to the elevated C_0 of all nodes, another color C_1 previously represented by node n_1 in the neighborhood around n_0 will now be represented by a different node n_1' farther away from n_0 than n_1, whose weights match most closely C_1. In other words, the effect

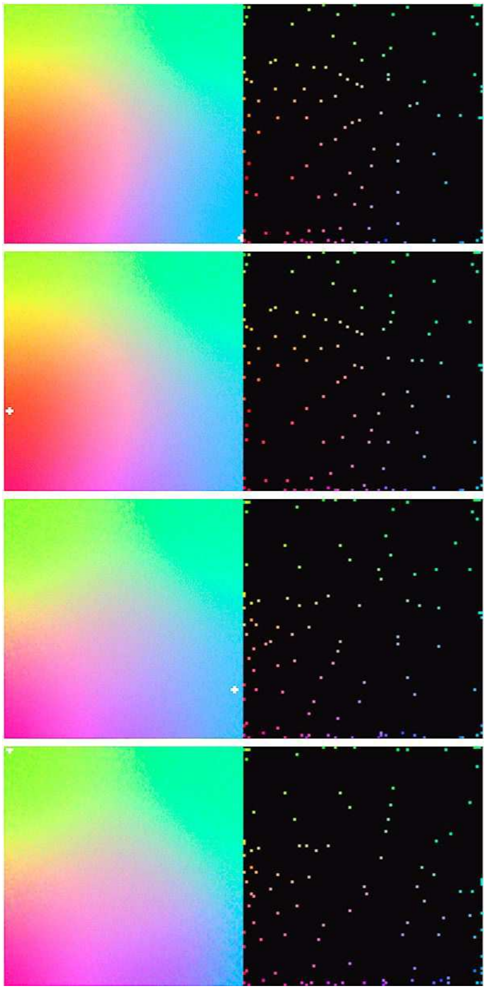

Fig. 10. SOM patterns of four consecutive iterations in training. The white dots in the A images (left) indicate the winner locations. In the B images (right) some line features gradually emerge and develop.

of the elevation of certain color component in image A as the result of the learning is that in image B those nodes in the neighborhood representing some colors similar to C_0 are pushed away from n_0.

- Assume two colors C_1 and C_2 were represented respectively by nodes n_1 and n_2 in image B before the current iteration, and n_1 is closer to the winner n_0 than n_2. After weight modification, C_1 and C_2 will now be best matched and represented by two different nodes n'_1 and n'_2, both farther away from n_0, due to the elevation of the color component in the updated SOM. However, as n_1 is closer to n_0 and therefore elevated more than n_2, n'_1 best matching C_1 has to be farther away from n_0 than n'_2 best matching C_2. In general, the nodes in image B will be pushed farther away from the winner if they used to be close to it, but less far away if they used to be farther away from it. Because of this effect, all nodes around the winner in the B image are pushed away in such a way that their distances to the winner tend to become more similar than before the iteration, i.e., they tend to be aligned to form a curve along a contour line of the Bell shape elevation around the winner. This effect is illustrated in Fig. 11 (left).

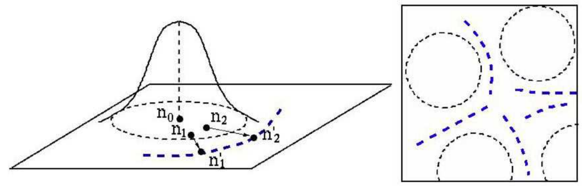

Fig. 11. Formation of linear features (left) and their intersections (right)

- As most colors similar to the current input are represented by the nodes in the neighborhood of n_0, which are pushed to different degrees to form a line, there will be no other nodes in the immediate vicinity of the line, consequently a black strip will appear along either side of the line.
- The linear features discussed above gradually form and grow in length, while being pushed around repeatedly in various directions depending on the winners' locations, which are randomly distributed in the SOM. As it is more probable for the winners to be located away from the central region of the SOM, the linear features tend to be oriented along the radial directions of the center. In other words, they gradually form longer curves all intersecting at some point in the central region, as illustrated in Fig. 11 (right).
- As training progresses, the patterns formed by intersecting linear features will be in progressively finer scales, as the width σ of the Gaussian function decays exponentially. When training is complete after a large number of iterations, the overall patterns in the final SOM appear to be hierarchically structured.

IV. Summary and Further Work

In summary, this paper reveals an interesting property of the classical self-organizing map (SOM) previously unknown to the artificial intelligence and neural network community. Specifically some highly structured hierarchical patterns in two or higher dimensional space may be spontaneously developed during the regular competitive learning process. However, due to the random and iterative nature of this process, to pinpoint the exact mechanism of the development of such patterns may be difficult, before more in-depth investigation. To further explore the phenomenon and understand the underneath mechanism, more experiments need to be carried out with varying parameters, larger scales, and different dimensionalities. The author welcomes joint efforts from all interested researchers. Although it is not clear what potential applications, if any, this property may have, it still has certain significance to discover and understand such a property of this most popular and widely used SOM neural network algorithm.

References

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[4] *http://www.pymvpa.org/examples/som.html*

V. Appendix: large scale patterns

Here the B images of a few large size SOM are shown to show the detailed structures. For better viewing effect, see the author's webpage at: http://fourier.eng.hmc.edu/e161/lectures/nn/node15.html

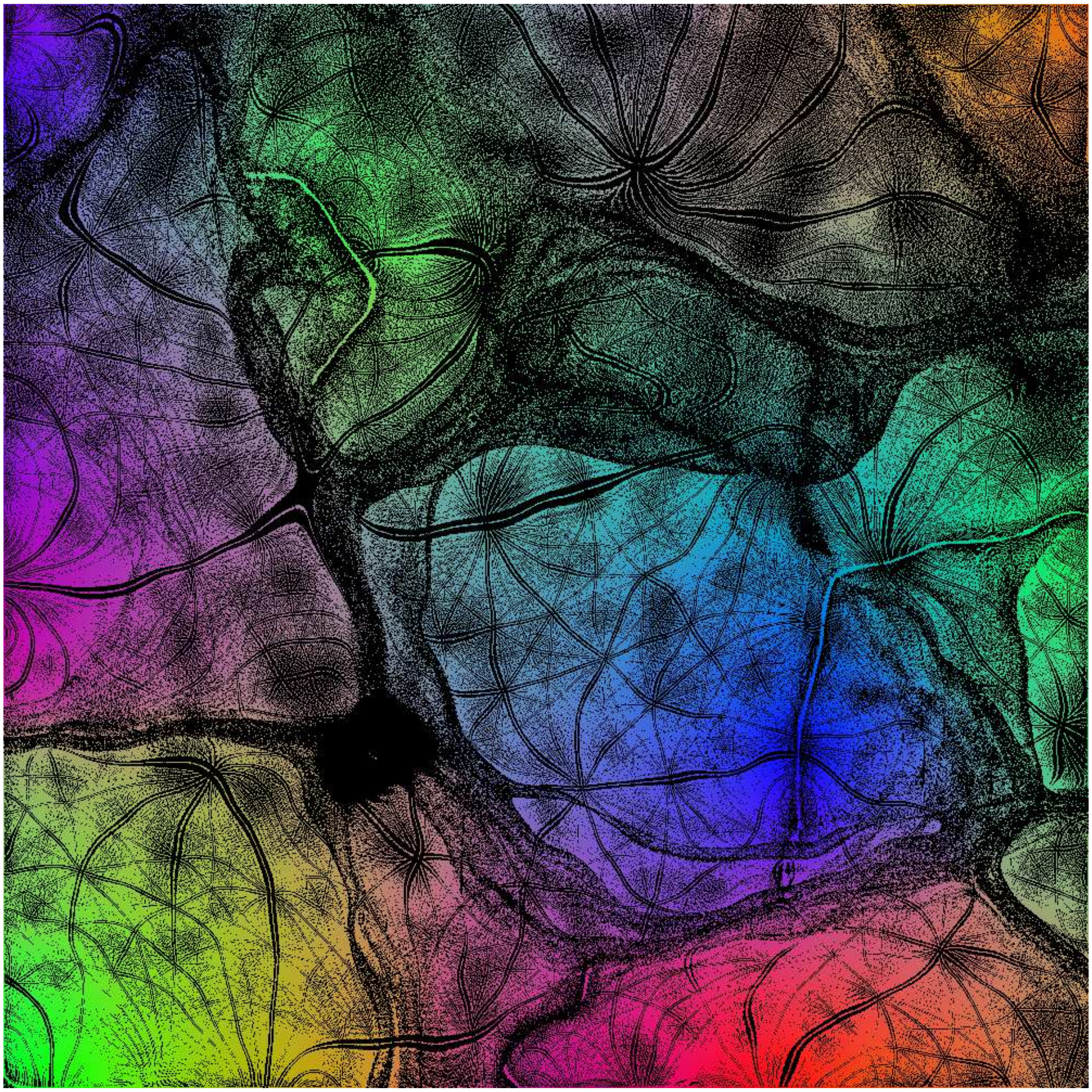

Fig. 12. Images A and B of a SOM with the width of the Gaussian weighting function σ reduced to one tenth of the SOM size. The resulting image A looks less homogeneous as now it is partitioned into a larger number of smaller regions each composed of fewer nodes. In the corresponding B image the web-like features become less regular and richer in shape variations. Also note that there exist some line features that are across the boundaries between some of the color regions.

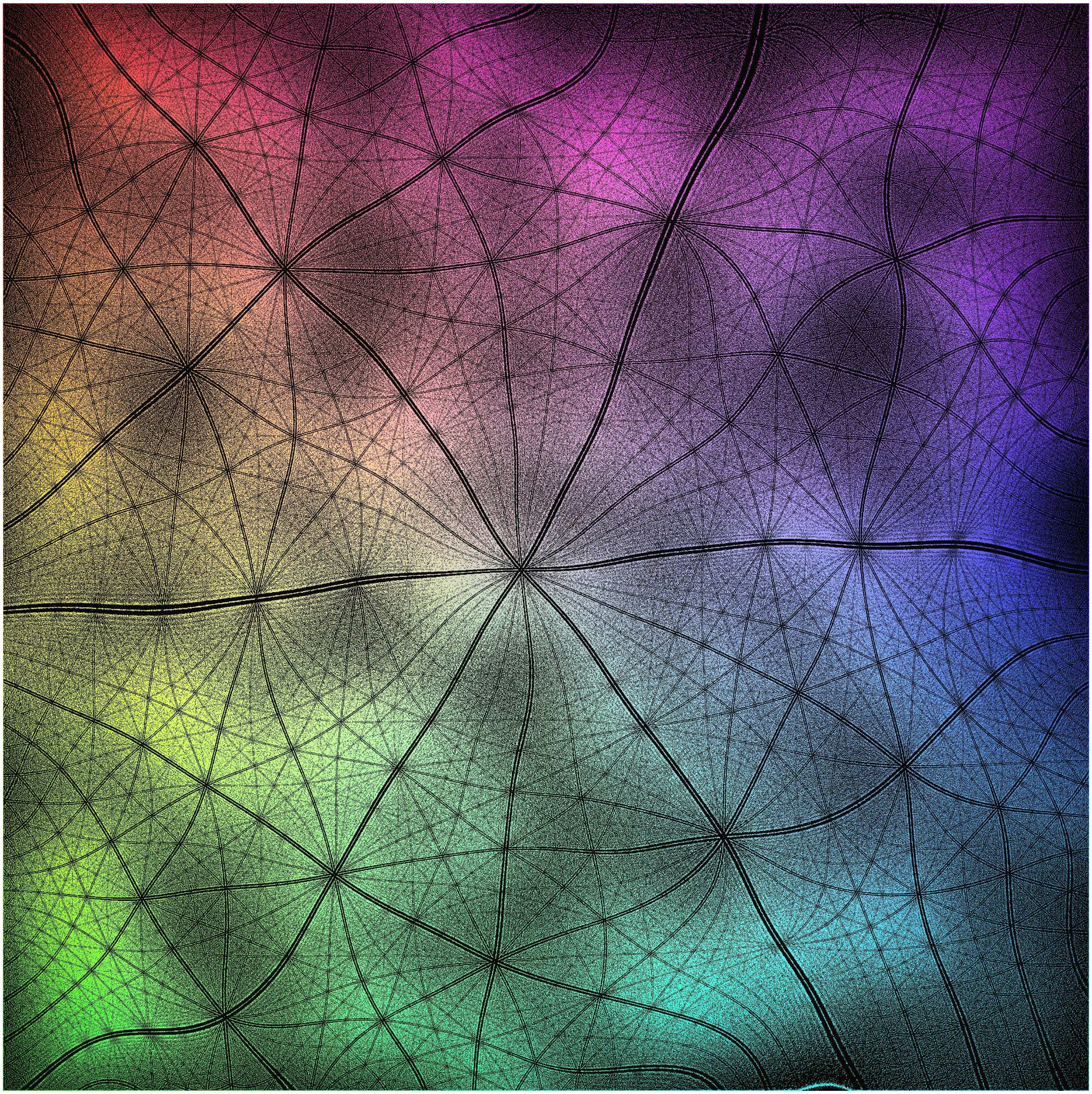

Fig. 13. A SOM of size 2000×2000, trained on $160^3 = 4,096,000$ colors. It can be seen that there exist many intersections formed by curves of different scales, and they form some hierarchical web-like structures.

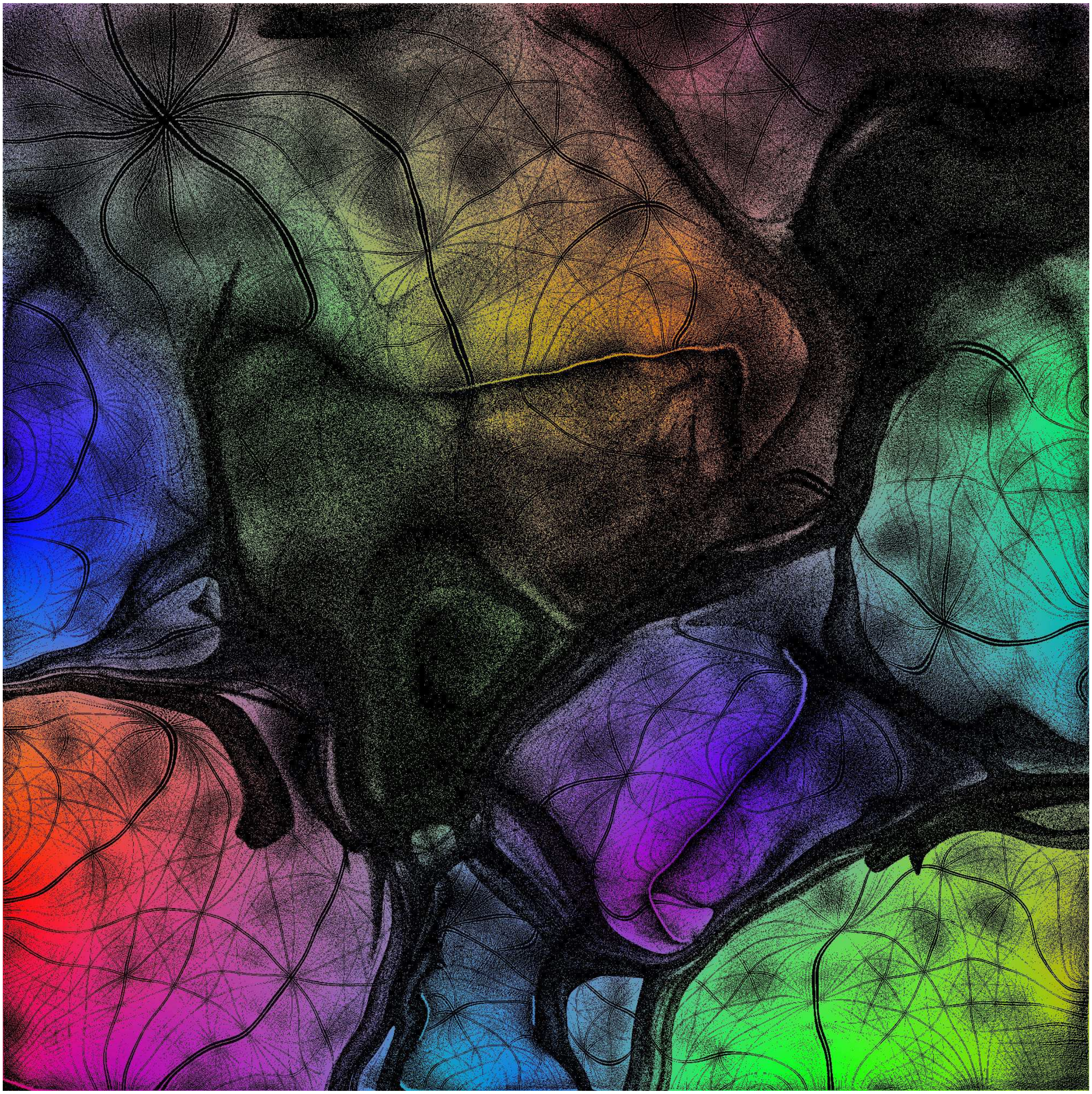

Fig. 14. A SOM of size 2000×2000, trained on $160^3 = 4,096,000$ colors. It can be seen that there exist many intersections formed by curves of different scales, and they form some hierarchical web-like structures.