## Visual Perception of Luminance

- Spectral energy distribution of light source:

$$L(\lambda), \quad 350\,nm \;\leq\; \lambda \;\leq\; 780\,nm$$

- Light reflected by (or transmitted through) an object:

$$I(\lambda) = \rho(\lambda)\, L(\lambda)$$

  where $\rho(\lambda)$ is the reflectivity (or transmissivity) of the object.

- Luminance (intensity) of a spatially distributed object:

$$f(x,y) = \int_0^\infty I(x,y,\lambda)\, V(\lambda)\, d\lambda$$

  where $V(\lambda)$ is the *relative luminous efficiency function* of the visual system. The function of human eye is a bell-shaped curve.

- *Apparent brightness (brightness)* is the *perceived* luminance which depends on the luminance of the object and its surroundings.

  Two objects with different surroundings could have identical luminance but different brightnesses (see example).

- *Simultaneous contrast*

  Assuming the luminance of an object is $f$ and the luminance difference between the object and its surrounding is $df$, then according to Webber's law, we perceive the same contrast between the object and its surrounding if

$$\frac{df}{f} = d(ln\, f) = constant$$

  which implies equal increment in $ln\, f$ will be perceived to be equally different (equal contrast). So we define contrast as a log function of f:

$$c = a + b\, ln\, f$$

  where $a$ and $b$ are two parameters.

- *Mach bands* See examples.

# Visual Perception of Colors

- **Color Representation**
  - **What Determines the Color?**

    Along the visible wave length (350 nm – 780 nm), there are only about 128 fully saturated colors that can be distinguished.

    It is the energy spectral distribution $P(\lambda)$ of the signal that determines the colors we perceive.

  - **Three Components of Color**
    * *Hue:* the dominant wavelength, the redness of red, greenness of green, etc.
    * *Saturation:* how pure the color is, or how much white is contained in the color. For example, red and royal blue are more saturated than pink and sky blue, respectively.
    * *Luminance:* the amount or intensity of light.

      Brightness: for measuring self-luminous objects that emits light (CRT, etc.)

      Lightness: for measuring reflected light.

  - **Tristimulus Theory**

    There exist 3 types of cells (cones) in human retina of different response functions (luminous efficiency functions): $S_i(\lambda)$, $(i = 1, 2, 3)$. They overlap with each other and peak in the yellow-green, green and blue regions, respectively.

    The responses of these cells to a signal of intensity $C(\lambda)$ (a "color") are therefore

    $$r_i(C) = \int S_i(\lambda) C(\lambda) d\lambda \quad (i = 1, 2, 3)$$

    The perceived color is determined by the combination of these 3 responses $r_i$, $(i = 1, 2, 3)$. In other words, if two colors $C_1(\lambda)$ and $C_2(\lambda)$ produce the same responses:

    $$r_i(C_1) = r_i(C_2) \quad (i = 1, 2, 3)$$

    then they are perceived as the same color.

- **Color Models**
  * RGB model: using Red, Green, and Blue as three primaries to represent a color.
  * HSV model: using Hue, Saturation, and Value to represent a color
  * HLS model: using Hue, Lightness, and Saturation to represent a color.

- **Color Matching and Reproduction**
  - **Color Matching**

    It is possible for different colors, energy distributions, to produce exactly the same visual perception in the human visual system. These colors are said to be matched and are called *metamers*. Two matching colors $C_1(\lambda)$ and $C_2(\lambda)$ can be represented by

    $$C_1(\lambda) \equiv C_2(\lambda)$$

    Note that in general matching colors do not necessarily have identical energy distributions,

    $$C_1(\lambda) \neq C_2(\lambda)$$

  - **Three-Color Theory**

    Any color can be reproduced by mixing an appropriate set of three primary colors (e.g., CIE X, Y, Z, or red, green, and blue, not unique) with energy distributions $P_k(\lambda), \ \ (k = 1, 2, 3)$.

  - **Matching Colors with Primaries**

    Suppose in order to match a given color $C(\lambda)$ the three primaries need to be mixed in proportions of $\beta_k, \ \ (k = 1, 2, 3)$:

    $$C'(\lambda) = \sum_{k=1}^{3} \beta_k P_k(\lambda)$$

    For the mixed color $C'(\lambda)$ to be perceived the same as the given color $C(\lambda)$, the responses of the three types of cone cells to $C'(\lambda)$ should be the same as those to $C(\lambda)$:

    $$r_i(C) = r_i(C') \quad (i = 1, 2, 3)$$

    The cone cells' responses to $C(\lambda)$ are

    $$r_i(C) = \int S_i(\lambda) C(\lambda) d\lambda \quad (i = 1, 2, 3)$$

    and their responses to the matching color $C'(\lambda)$ are

    $$\begin{aligned} r_i(C') &= \int S_i(\lambda) C'(\lambda) d\lambda = \int [\sum_{k=1}^{3} \beta_k P_k(\lambda)] S_i(\lambda) d\lambda \\ &= \sum_{k=1}^{3} \beta_k \int S_i(\lambda) P_k(\lambda) d\lambda = \sum_{k=1}^{3} \beta_k a_{ik} \quad (i = 1, 2, 3) \end{aligned}$$

where $a_{ik}$ is defined as the response of ith cells to the kth primary:

$$a_{ik} \stackrel{\triangle}{=} r_i(P_k) = \int S_i(\lambda) P_k(\lambda) d\lambda \ \ (i, k = 1, 2, 3)$$

which can be found given the cone cells' sensitivities $S_i(\lambda)$ and the three primary colors $P_k(\lambda)$.
For $C'(\lambda)$ to be perceived the same as $C(\lambda)$, we require

$$r_i(C') = \sum_{k=1}^{3} \beta_k a_{ik} = r_i(C) = \int S_i(\lambda) C(\lambda) d\lambda \quad (i = 1, 2, 3)$$

These three equations are called the *color matching equations*. As both $a_{ik}$ and the right-hand side of the equations (available from the given $C(\lambda)$ and $S_i(\lambda)$) are known, the 3 coefficients $\beta_k$ $(i = 1, 2, 3)$ can be obtained by solving the 3 color matching equations, and the matching color is produced by mixing the three primaries:

$$C'(\lambda) = \sum_{k=1}^{3} \beta_k P_k(\lambda) \equiv C(\lambda)$$

- **CIE Primaries**

  The Commission Internationale de l'Eclairage (CIE) defined three standard primaries called **X**, **Y**, and **Z**. Any color $C(\lambda)$ can be matched using these primaries with positive weights $X(C)$, $Y(C)$ and $Z(C)$.

  The *chromaticity* values of a color is defined by its weights for the three primaries normalized by the total energy $X + Y + Z$:

$$x = \frac{X}{X+Y+Z}, \quad y = \frac{Y}{X+Y+Z}, \quad z = \frac{Z}{X+Y+Z}$$

  so that $x + y + z = 1$. Chromaticity values depend on the hue and saturation of the color, but are independent of the intensity.

  All visible colors are represented by the points inside an enclosed area in the $X + Y + Z = 1$ plane. And the chromaticity diagram is the projection of this enclosed area on $(X, Y)$ plane.