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Lecture 22: Noise in Receivers

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Noise Temperature of Lossy Passives and Antennas

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In this video we're going to learn about the noise temperature contributed by some common of components, specifically lossy passive systems and antennas.

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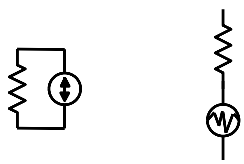
Reminder, we know noise temperature of R



$\sigma_i^2 = 4kT\Delta f/R$ $\sigma_v^2 = 4kTR\Delta f$

$$P_N = \frac{\sigma_v^2}{4R} = kT\Delta f = kT_n\Delta f$$

$T_n = T$ Noise temperature equal to physical temperature

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To start off, we should review the noise temperature of a resistor. We know from earlier videos that we can say resistors have a noise current variance or a noise voltage variance as shown on this slide. I’ve also introduced a new schematic notation here, which is how noise sources are usually indicated.

CLICK We also know from thermodynamics that the noise power is given by the noise voltage variance over 4R, and that noise power is equal to kT delta f. Finally, noise power is equal to k Tn delta f by the way we defined Tn, the noise temperature.

CLICK Looking at the last two of those equations it’s pretty easy to see that the noise temperature of an individual resistor is just equal to the physical temperature, T.

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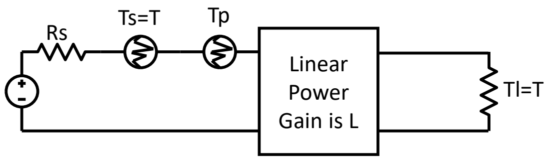
Temperature of Lossy Passives is (1/L-1)*T



$T_{out} = T_{load}$ because they're in equilibrium

$L(T + T_p) = T$

$T_p = \left(\frac{1}{L} - 1\right)T$

Works for any passive: attenuator, filter, splitter, couplers some mixers, etc.

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If we have a lossy, passive two-port network, we can try to calculate the contribution it makes to a noise temperature. This schematic shows a source with a source resistance Rs driving a lossy two-port network that is terminated with a load. By convention, we define noise temperature of a two-port at the input in order to explain the noise power at the output. We know the temperature of the source resistor and the load resistor, and we're hoping to use that fact to figure out temperature of the noise introduced by the lossy two port.

CLICK To do so, we can observe that we are assuming that we're in thermal equilibrium, which means the noise emitted by the two-port has to be equal to the noise emitted by the load. If that wasn't true, one resistor would be providing power to the other.

CLICK We can calculate the noise power at the output by multiplying the input powers by the power gain. As a reminder: The power gain is like the transfer function squared, so this is consistent with our previous finding that we refer noise to different places in the circuit using the transfer function squared. Also noise temperatures are linearly related to variance densities, so it's fine for us to add them. We're using the symbol L for power gain here to indicate that this system is actually going to have a loss, which means L is expected to be less than 1.

CLICK Rearranging a little lets us calculate the temperature of the passive in terms of L and the physical temperature, Tp is equal to 1 over L minus 1 times the physical temperature.

CLICK It’s worth noting that this is a very powerful equation. We can use it for any lossy passive, so this works for attenuators, filters, splitters, couplers and some mixers. If it has an insertion loss, this equation will tell you its noise temperature.

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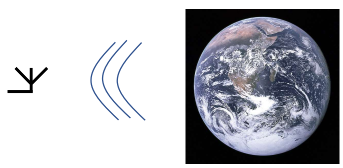
Antenna Temperature is Incident Radiation

$$T_a = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} R(\theta,\phi)T(\theta,\phi)\sin\theta\, d\theta d\phi$$

Noise temperature of night sky is 4-30K



Noise temperature of earth from space is 290K

https://commons.wikimedia.org/wiki/File:The_Earth_seen_from_Apollo_17.jpg
Apollo 17, Public domain, via Wikimedia Commons

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Antennas are a different case. They're passive structures, just pieces of metal, so you could expect them to behave like passives, but it turns out that their noise is much more dependent on the radiation that is incident on them. Various sources in the universe emit broadband radiation, and the antenna's radiation pattern will absorb that noise, so you calculate antenna temperature by integrating the radiation pattern, R, times the received radiation temperature, T, over spherical coordinates. In other words, you sum the total noise incident on the antenna to determine the antenna's temperature.

That temperature depends a lot on what the antenna is looking at, especially if the antenna is high gain so that it's whole field-of-view is occupied by one radiator. For example, the night sky, depending on where you look, will have a noise temperature of 4 to 30 Kelvins. The lower limit of 4K in that case is the cosmic microwave background radiation, which is a famous scientific result. On the other hand, looking at earth from space will give you a noise temperature of 290K.   It's just a coincidence that the 290K number is close to room temperature.

Summary

- Resistor noise temperature is equal to the physical temperature: $T_n=T$.
- In two-ports, noise temperature is input-referred.
- Lossy passives add noise with a temperature of $T_p=(1/L-1)T$.
- Antenna temperature depends on where the antenna is pointing.

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Signal-to-Noise Ratio, Noise Factor & Amp. Temperature

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In this video we're going to figure out the noise temperature of amplifiers. We're going to be doing so in terms of a commonly listed datasheet specification called the noise figure, which requires us to introduce a new metric called signal-to-noise ratio as well.

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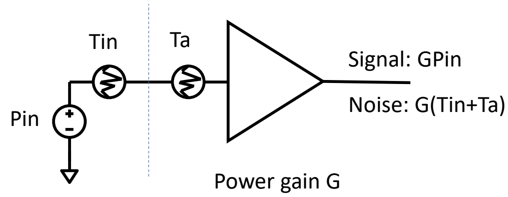
An Amplifier Adds Noise to Noisy Inputs



Signal-to-Noise Ratio defined as:

$$SNR = \frac{P_{signal}}{P_n} \qquad SNR_{in} = \frac{P_{in}}{kT_{in}B} \qquad SNR_{out} = \frac{GP_{in}}{kG(T_{in}+T_A)B} = \frac{P_{in}}{k(T_{in}+T_A)B}$$

Noise factor and noise figure defined as:

$$nf = \frac{SNR_{in}}{SNR_{out}} \qquad NF = 10\log nf \qquad nf = \frac{SNR_{in}}{SNR_{out}} = \frac{S_{in}}{S_{out}}\frac{N_{out}}{N_{in}} = \frac{1}{G}\frac{G(T_{in}+T_A)}{T_{in}} = 1 + \frac{T_A}{T_{in}}$$

Noise temperature of amplifier
(Let Tin=T as if it's a normal Rs)

$$T_A = (nf-1)T$$

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This is our picture of an amplifier, and it's worth noting that the amplifier's noise temperature adds to any noise temperature that's already present in our input signal. That's true of passives networks too, but we need a special formula for amplifiers because the active devices in them create noise in a different way from resistors. Accounting for those noise sources is tricky, but we're going to discover that we can skip that step using a number that shows up in most datasheets called the noise figure.

CLICK We can evaluate the impact this amplifier noise has on our signal by defining a measure called the signal-to-noise ratio. The name of signal-to-noise ratio is surprisingly expressive, it's literally just the ratio of signal power to noise power. Notice that the signal power is a total power, not a density, so we have to assume our system has some bandwidth B in order to calculate our total noise power. I've calculated the SNR of our input signal, which is comprised of everything left of the blue dashed line in the schematic, and the SNR of our system's output. The amplifier has clearly reduced our SNR.

CLICK That leads us to another important metric of performance, the noise factor, which is the ratio of SNR at an amplifier's input to SNR at the output. It measures how much worse an amplifier makes your SNR. The noise figure, which is denoted with the capital letters NF, is the noise factor expressed in dB.

CLICK We can find the noise factor for our example amplifier with this calculation. This derivation includes a fun step where we express the SNRin over SNRout as signal in over signal out times noise out over noise in, which we can do because SNR is just a ratio of powers. Because we're looking at a very generic amplifier model, our final expression for noise factor isn't terribly insightful. Our equations say that noise factor is equal to one plus the ratio of amplifier temperature over the input temperature. Sure, this is just another trite way of saying that the amplifier adds some noise. However, it is telling that the amplifier's noise is added to one; the one term in that equation comes from the fact that input temperature gets amplified and sent to the output, which means that noise factor always has to be greater than one. Every practical amplifier adds some noise, so it will always make your signal to noise ratio worse.

CLICK We can improve on our expression for noise factor by using it to find the noise temperature of the amplifier's noise. We do this the same way that we did with a passive network, by assuming that our input noise is produced by a source resistor, which has a temperature T. Substituting the physical temperature T into our expression for noise factor gets us an expression for antenna temperature, it's equal to noise factor minus 1 times the physical temperature.

Summary

- Signal-to-Noise Ratio is SNR=Psig/Pnoise
- Noise factor of a two-port, usually an amplifier, is nf=SNRin/SNRout
- Noise figure is dB version of noise factor
- Amplifiers add input-referred noise with temperature TA=(nf-1)T

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Quantization Noise

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In this video we're going to look at the other source of random noise we'll consider in our receivers: quantization noise.

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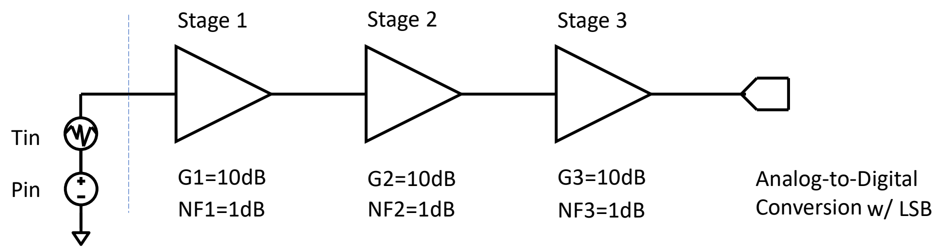
If Amplifiers Hurt SNR, Why Use Them?



SNRin = 10*log(kTinB/Pin)

SNRout = SNRin - 3dB

Pout = G1*G2*G3*Pin = Vout^2/2R

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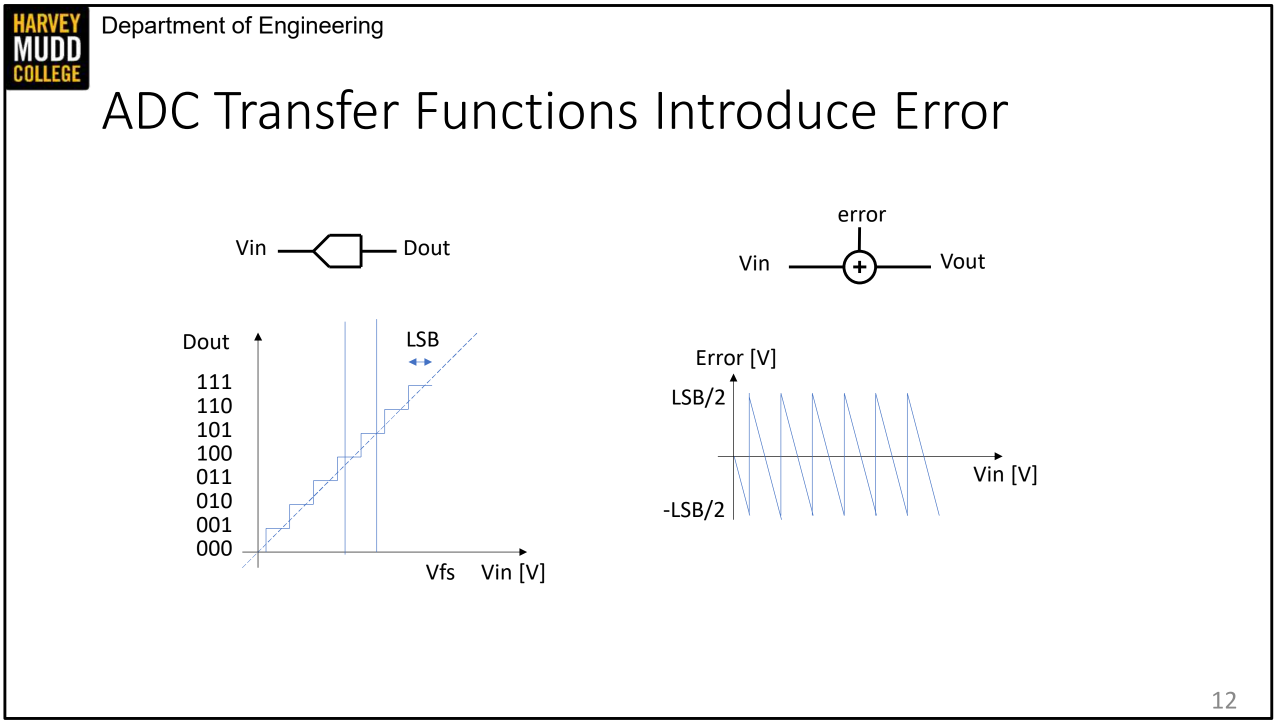
Our motivation for doing this is the observation that noise figure paints a disturbing picture of amplifiers: since noise figure is always going to be greater than 0dB (which is the same thing as saying noise factor is greater than 1), each amplifier we go through reduces our signal to noise ratio. Why should we use amplifiers if they make our signal quality worse? In the example I've drawn here, we have very low noise amplifiers with a noise figure of 1dB, and they still result in our SNR being halved at the output.

CLICK This analysis overlooks that our signal has been made much larger by this receive chain, which is very important when we're comparing our signal against the resolution of an analog-to-digital conversion. Analog-to-digital converters have a minimum signal that they can resolve called a least-significant-bit or LSB and that LSB can introduce significant noise into small signals.



We need to take a closer look at the operation of analog-to-digital converters to understand why the LSB is such a big deal. Quick aside, analog-to-digital converter is often abbreviated ADC, and I'm going to be using that abbreviation.

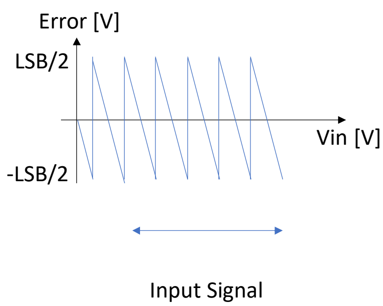
I've drawn a transfer function for an ADC on this slide. The x-axis is the input voltage and it has units of volts. The y-axis represents a digital number that the ADC produces, and it's referred to as a code. So for an input voltage of 0 volts, the ADC produces the digital code 000, which represents the number 0 in binary. If the input is at the full-scale voltage Vfs, the ADC will produce the code 111, which represents the number 7 in binary. Because the codes are discrete values, while the input is a continuous value, the transfer function has to include cut-points where the output jumps from one code to the next. That leads to the stair-step shape of the transfer function. The width of each stair-step is called one least-significant-bit, or LSB, because the code changes by 1 each time your input voltage changes by LSB volts.

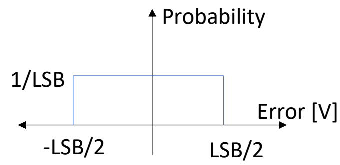
CLICK That stairstep is trying to approximate a straight line that would let the output codes linearly represent the input voltage. CLICK at some voltages, the output stairstep does that well. The voltage you get by multiplying Vfs by the ratio of this code to the max code is exactly the same as the input voltage. CLICK However, other input voltages pose a problem. This input voltage is going to be assigned to the 100 code, but it isn't at the sweet spot on the stairstep where the stairs line up exactly with the linear approximation. That means the conversion has introduced some error when it measured this input voltage.

CLICK We tend to capture this behavior in signal-processing or control models by imagining that ADCs look like transfer functions of 1, which assumes they achieve their perfect stair-step approximation and convert a continuous Vin in volts to a digital Vout in volts, but that the ADC also adds in some error to represent the process of quantization.

CLICK The amount of error you add depends on how far your input is away from a code transition. If your input falls right on a code transition, you're guaranteed to be plus/minus LSB/2 away from a perfect linear relationship, but when your input falls halfway between code transitions you introduce zero error because you're exactly on the linear relationship you were aiming for.

Error is Uniform → Quant Noise = LSB^2/12





$$\sigma_q^2 = LSB^2/12$$

Quantization Noise!

If we take this error vs. Vin plot and assume that our input spans a several LSBs and that it's spectrally rich, meaning that we don't just make the same pattern of codes every period of our input, then we can model the error as a uniformly distributed random variable. That's just saying that it's equally likely for our input to land anywhere between two code transitions.

The maximum and minimum values of the uniform distribution are plus/minus LSB over 2 to indicate the maximum and minimum values of error we could have. The height of the distribution has to be 1/LSB to ensure that our total probability is 1. The variance of this distribution is our quantization noise, and it has a value of LSB squared over 12.

Great! We've got an expression. Digging into it a little bit, we can notice that it has units of voltage squared because the LSB is a voltage. So that means that quantization noise is a voltage noise variance, which means we can add it to the voltage noise variance of other elements of our system. We'll see soon that increasing our gain reduces the effect of quantization noise, which is why we need amplifiers in our receivers.

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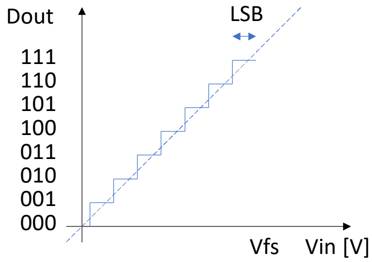
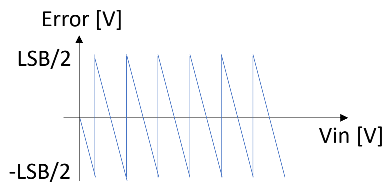
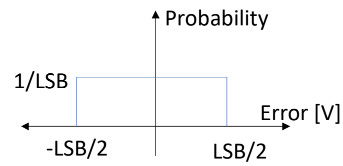
Summary

- Quantization noise comes from the difference between ADC transfer functions and a perfectly straight line.







- Quantization noise is $\sigma_q^2 = LSB^2/12$

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System Temperature and Cascade Formulas

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In this video we're going to talk about the noise performance of entire receiver chains.

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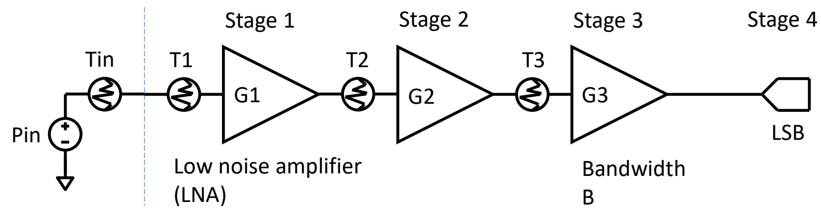
Total Noise at Output Dominated by 1st Stage



$T_{out1} = G_1(T_{in} + T_1)$ $T_{out2} = G_2(T_{out1} + T_2)$

$T_{out2} = G_2G_1T_{in} + G_2G_1T_1 + G_2T_2)$

$\sigma_{v4}^2 = 4BkR(G_1G_2G_3T_{in} + G_1G_2G_3T_1 + G_2G_3T_2 + G_3T_3) + LSB^2/12$

Can't control | T1 super important! | Care less about these | Gain reduces relative effect

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I've drawn a receiver chain here that shows the noise sources of each amplifier stage. Note that this is a pretty simplified receiver stage, most recievers would need transmit/receive switches, filters mixers and other components. More news on those soon.

CLICK We can calculate the noise temperature that the first amplifier emits by referring Tin and T1 through the power gain of the first stage.

CLICK That output noise becomes the input for the second stage, so the output noise of the second stage is given by referring the output of the first stage plus the temperature of the second stage through G2. This means Tin and T1 have an amplified impact on Tout2 because they have seen two gains while T2 has only seen G2.

CLICK If we follow this through every stage, then we can finally come up with an expression for the total noise voltage variance of our system. It consists of a thermal term and a quantization term. The thermal term is a big effective temperature that is multiplied by k, some assumed bandwidth B, and 4R to convert to a a voltage variance. We then add that voltage variance to our quantization term.

CLICK This expression shows us something really important about receiver design: the noise of the first stage matters much more than noise in the later stages. We can see in this effective noise temperature: the Tin and the T1 terms both see the full gain of the receiver, while T2 and T3 each see less gain. We can't do anything with the input noise temperature -- it's part of our input signal – but we can do our level best to make T1 small. CLICK This concept is so important that there are a special class of amplifiers, called low-noise amplifiers or LNAs, that are used as the inputs of the receivers to minimize the noise contribution of the first stage.

CLICK The quantization noise term in this expression is also interesting. It isn't multiplied by the gain of the receivers, so this noise term doesn't get bigger as gain increases. That means making a signal bigger will make the quantization noise a smaller fraction of the total signal. This answers the question of why we use amplifiers decisively: we make our signals bigger to overcome quantization noise.

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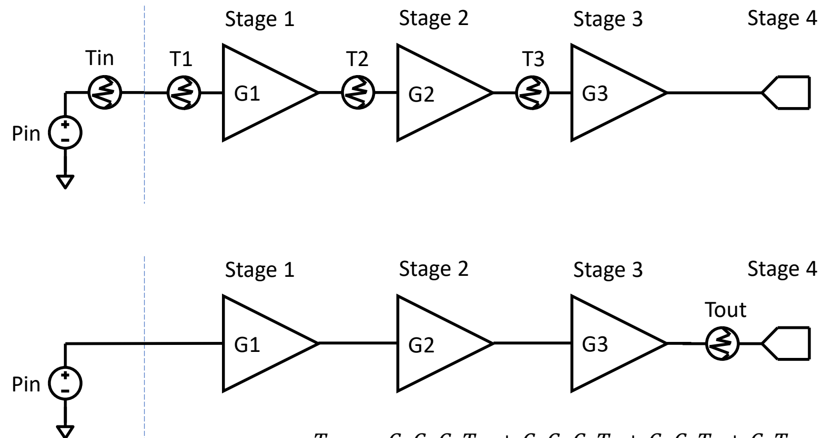
Referring Noise Lets us Make Simpler Models



$$T_{out} = G_1 G_2 G_3 T_{in} + G_1 G_2 G_3 T_1 + G_2 G_3 T_2 + G_3 T_3$$

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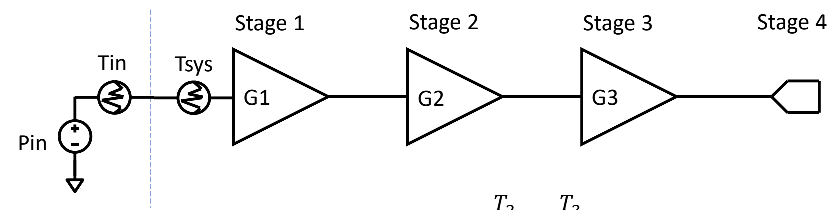
Our receiver model, pictured on the top of this slide, is kind of complicated. There are a lot of noise sources in it, which might be an unnecessary level of detail if we're not trying to optimize the design amplifier-by-amplifier. So we can make an equivalent model that just lumps all of the noise sources at the end of the receive chain. From the perspective of the ADC, these two chains are identical, but the designer can think less about the version with all the noise at the output. This model is called an output-referred noise model.

System Temp is Input-Referred Rx Chain Noise



$$T_{sys} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_2 G_1}$$

$$SNR \approx \frac{P_{in}}{k(T_{in} + T_{sys})B}$$ for high gain

Another popular, simplified noise model involves referring the noise of the amplifiers to the input. This noise source is called the system temperature. System temperature is popular because you can calculate your SNR very quickly if your gain is high enough to ignore quantization noise.

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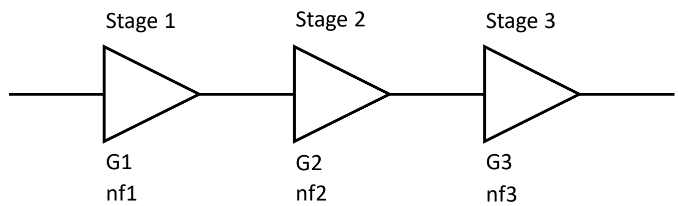
There are Cascade Formulas for Noise Factor



$$T_{out} = G_1G_2G_3T_{in} + G_1G_2G_3(nf_1 - 1)T + G_2G_3(nf_2 - 1)T + G_3(nf_3 - 1)T$$

$$nf_{total} = \frac{SNR_i}{SNR_o} = \frac{S_i}{S_o}\frac{N_o}{N_i} = \frac{1}{G_1G_2G_3}\frac{G_1G_2G_3T + G_1G_2G_3(nf_1 - 1)T + G_2G_3(nf_2 - 1)T + G_3(nf_3 - 1)T}{T}$$

$$= 1 + (nf_1 - 1) + \frac{nf_2 - 1}{G_1} + \frac{nf_3 - 1}{G_2G_3}$$

$$nf_{total} = nf_1 + \frac{nf_2 - 1}{G_1} + \frac{nf_3 - 1}{G_2G_3}$$ Confusingly, also the Friis Formula

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Finally, we can use output referred noise to derive a cascade formula for noise factors in amplifiers. I don’t love using cascade formulas and instead usually prefer keeping a spreadsheet of the noise level at each stage, but these are good formulas to know about anyway.

CLICK We can calculate the total noise factor of this amplifier chain using the output noise temperature and the total gain. We’re ignore k and the bandwidth in this equation because they’ll fall out of a ratio of noise temperatures, k and B would be on both the top and to bottom of the epression.

CLICK Cancelling gain terms simplifies this significantly.

CLICK And, finally, cleaning up leaves us with the cascade formula for noise factor. Confusingly, this is also called the Friis formula, so you’ll just have to know from context whether people are talking about path loss or noise factor of cascaded amplifiers. This formula also emphasizes how important the noise performance of the first stage is to the behavior of the whole amplifier. We see all of nf1, while nf2 and nf3 are discounted by the gain of the preceding amplifiers.

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Summary

- The first amplifier in the chain (the LNA) has the most impact on the total noise in the system
- Gain can reduce the relative effect of quantization noise on SNR
- Noise temperature can be input and output referred
- System temperature is input-referred noise from the amplifiers