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Lecture 17: Near and Far, Radiation Patterns, Links

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Near and Far Field

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In this video we're going to discuss how field strength changes as we move away from a radiating antenna.

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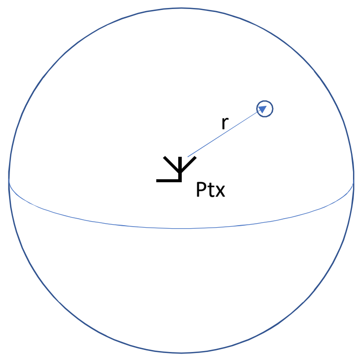
Radiated Power Spreads Over a Sphere



Note S is an intensity: units of [W/m^2]

$$S(r,\theta,\phi) \propto \frac{P_{tx}}{r^2}$$

$$S(r,\theta,\phi) = \frac{1}{2}\mathrm{Re}\{E^* \times H\} = \frac{|E|^2}{2\eta_0}$$

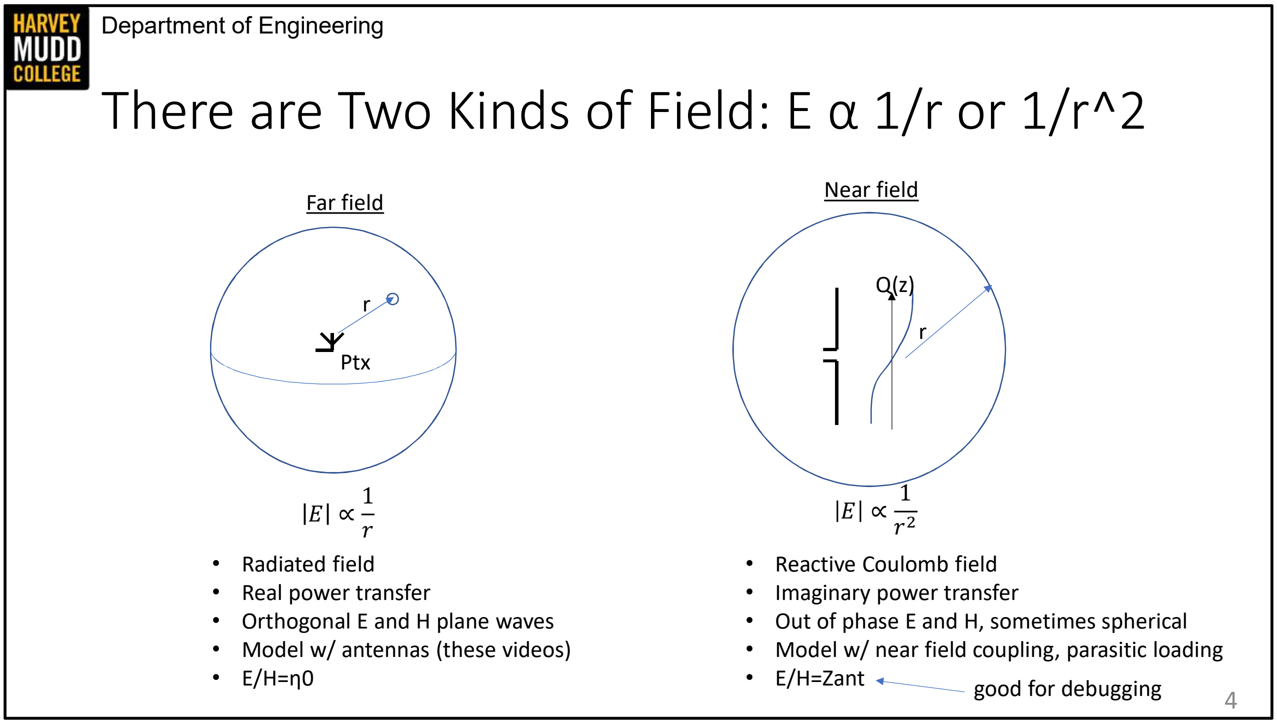
$$\frac{P_{tx}}{r^2} \propto \frac{|E|^2}{2\eta_0} \rightarrow |E| \propto \frac{\sqrt{P_{tx}}}{r}$$

3

First, I want to figure out what happens when we're far away from the antenna. The easiest way to find that is to picture power radiated by an antenna spreading over the surface of a sphere of radius r. That means we're going to be considering the power per unit area, or intensity, on the surface of the sphere.

We know that if an antenna is radiating power, then the power radiating from the antenna has to spread over the surface area of a sphere. That means the power in any direction is going to be proportional to the total power thrown out of the antenna, and inversely proportional to the distance from the antenna squared. We're leaving this as a proportionality instead of an equality for now because of some subtleties about how antennas radiate that we'll get to soon.

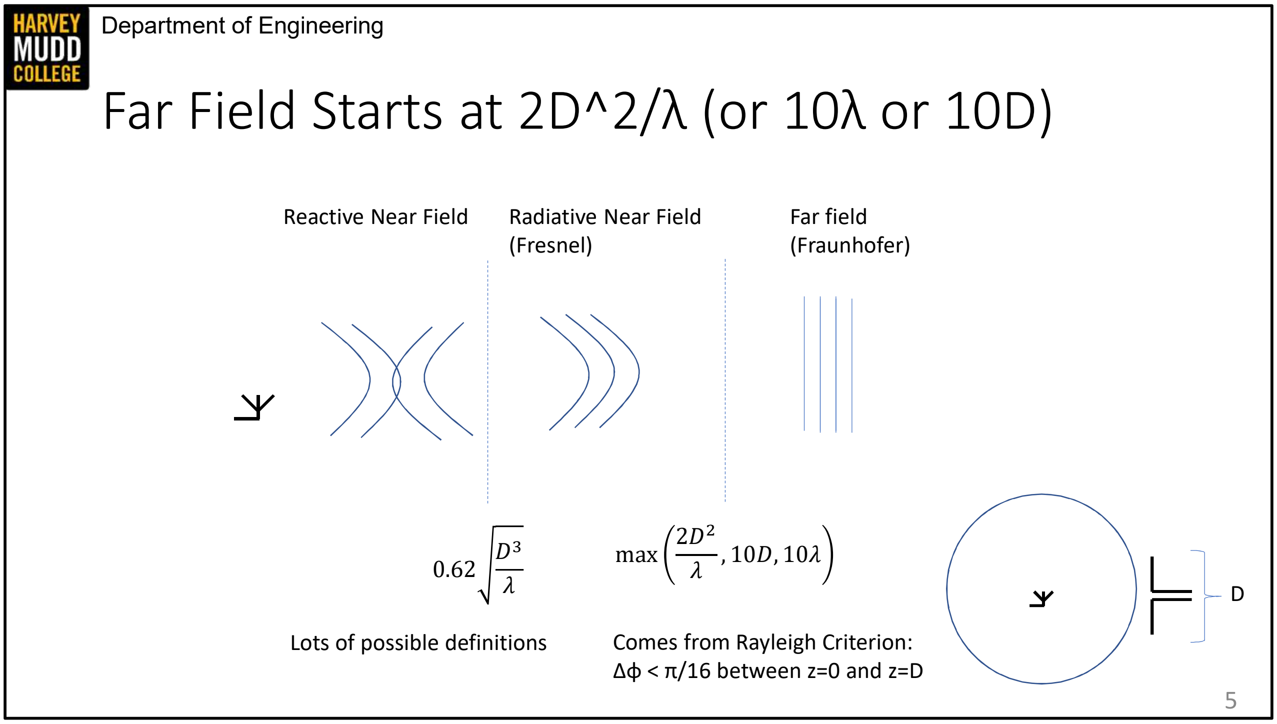
We know from the Poynting vector that the intensity in a direction has to be the cross product of E and H in that direction, and we know that E and H are related by the impedance of free space in the absence of other materials, charges or currents. Combinign these facts, we can find that the electric field has to fall off as 1/r as we move away from an antenna.



But that result leads to an apparent contradiction. We know that if we're far away from an antenna that field falls off as 1/r. But if we look close to an antenna, then there's some charge along it at any moment in time. We could approximate the antenna charge as two point charges, one positive and the other negative, and the field around a point charge falls off as 1/r^2. So which of these indicates how field changes around an antenna?

CLICK The answer is that both of these are true and, unintuitively, so is 1/r^3. These two types of radiation are referred to as near and far field radiation, and they are both characteristic of antennas. Far field radiation represents real power transfer away from the antenna, so it's called the radiative field. Near field radiation is characterized by imaginary power transfer, which means power flows out of the antenna for part of a cycle and then back into it. That means near field makes the antenna behave like a big weird capacitor or inductor. The real power transfer in the far field requires that E and H are in phase, while the near field requires them to be out of phase to prevent power transfer. We model the far field using links between antennas, which we're talking about in just a minute, while we model the effect of fields in the near field using the near field coupling models we used earlier. That means that capacitances or inductances that get into the field of antenna will change the field distribution around it, which can change Xrad, cause mismatch with the feed line and, as a result, shift the resonant frequency. Finally, it's worth noting that the ratio of E to H in the far field is set by the impedance of free space, while E and H close to an antenna are set by the V and I in the radiator, so their impedance is given by the impedance of the antenna.

CLICK I want to call some special attention to this last point. This is really useful when you're trying to identify a source of radiation using near field probes, because the ratio of E to H fields will tell you the impedance of your radiator. You can then go look for that value on a schematic or at least identify if you're looking for a high or low impedance node.



This picture summarizes some of the difference between radiation regions in a different way and introduces a new field region. In the far field we see the wave fronts represented as straight lines because waves are plane waves. In the reactive near field, we see that waves are going both forward and backward to indicate reactive power flow. The new field region is at the outer edge of the reactive near field, and it consists of radiating energy that isn't necessarily settled into the shape of a plane wave, so E will very with phi and theta in the radiative near field. The names Fraunhofer and Fresnel are names given to these field regions in optics, and they're sometimes reused here.

The most commonly accepted boundary between the near and far fields is 2 D squared over lambda, where D is the largest dimension of an antenna. However, that expression is insufficient in some corner cases, and it presumes that we're already far away from the antenna in terms of both antenna dimensions and field wavelengths. So I've given an expression that provides alternate far field boundaries at 10D or 10lambda if either of those happen to be bigger than 2D^2/lambda. The 2D^2/lambda expression comes from a condition called the Rayleigh Criterion, which we also borrow from optics. It's a measure of when a spherical wave starts to look like a plane wave to an observer with dimension D, and I've put an illustration that helps visualize that condition in the lower right. Because the spherical wave will arrive at the tallest part of the observing antenna a little later than it arrives at the middle of the antenna, there's going to be a phase difference in the incident waves. The Rayleigh Criterion says that phase difference should be less than pi/16 for plane waves.

I've also included a boundary condition between the reactive near field and the radiative near field on this schematic, but the near field is complicated and lots of people have argued about where to draw lines inside of it. That's one of many possible conditions. In fact, some disciplines have other ways of defining the far field boundary too.
Radiative field … E and H in phase, but vary w/ theta and phi

Lots of definitions, see linked PDF

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Summary

- The division between near and far field is given by $2D^2/\lambda$
- Near/Far field differences:
  - Impedance of waves in near field is set by circuit, far field set by η_0.
  - Parasitic loading effects and near field coupling in near field
  - Plane waves w/ E&H in phase in the far field
  - Spherical (or weirder) waves w/ E&H in quadrature in the near field
  - Radiated E field falls off as $1/r$, reactive E field falls off as $1/r^2$ (or $1/r^3$)

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Radiation Patterns

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7

In this video we're going to discuss the shape of radiation that comes out of practical antennas, and we'll discover that they inevitably have interesting blind spots.

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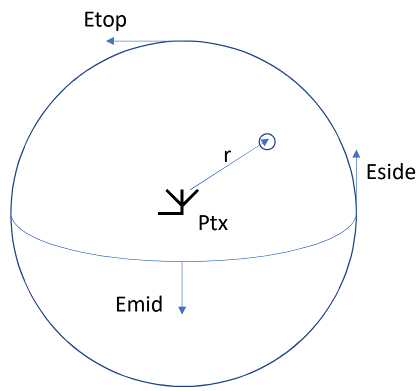
Antennas Can't Be Isotropic Radiators



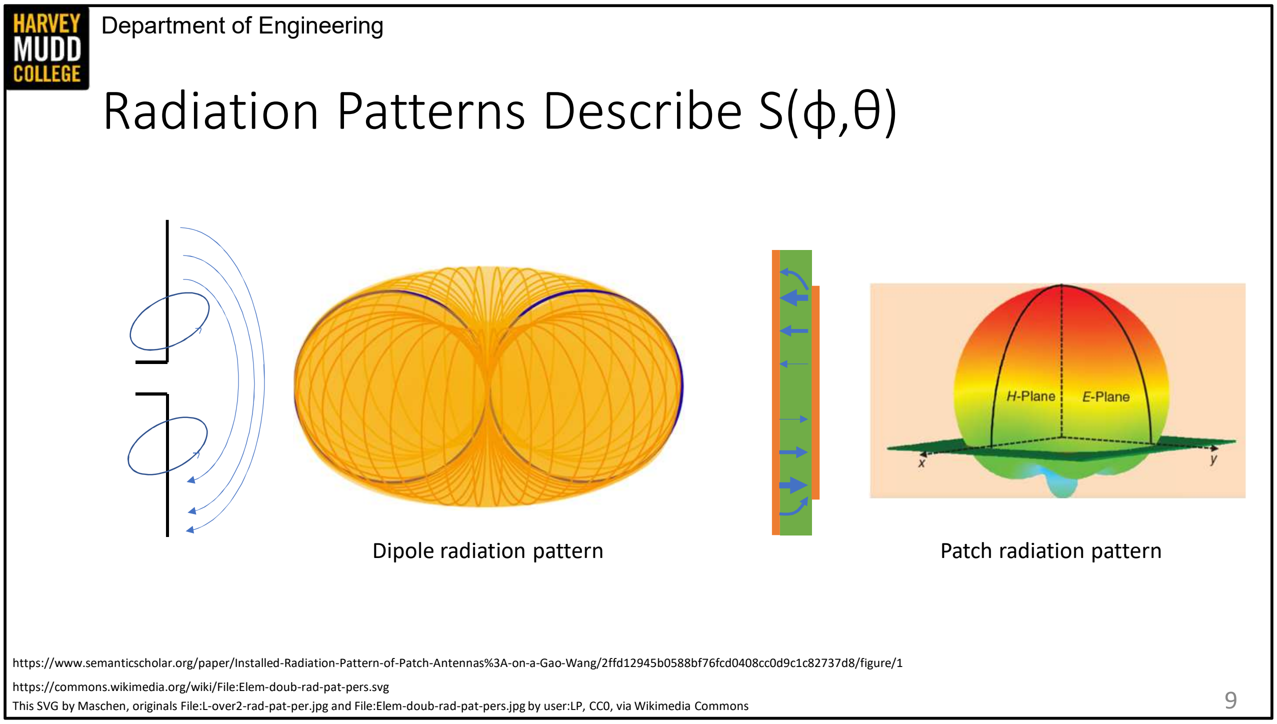
$$EIPD = \frac{P_{tx}}{4\pi r^2}$$

Equivalent Isotropic Power Density

Can't achieve it b/c of "Fuzzy ball problem"

8

We can see that pretty clearly by considering our radiation model. We could imagine that our energy radiated equally in every direction, or isotropically, from a transmit antenna. The power on the spherical shell around our antenna would be Ptx/4pi r^2 in that case, and we refer to that quantity as the equivalent istotropic power density. However, this spherical shell would need S to be normal to the surface everywhere, which means E would need to be tangent to the surface everywhere. This is problematic because there's no way to define a vector field that's tangent to the surface everywhere without it going to zero somewhere. This is adorably called the fuzzy ball problem: you can't comb a fuzzy ball flat without leaving a few cowlicks. That means it's impossible to make isotropic radiators in the real world.



We'd already gotten some warning of this because the antennas we have looked at had a preferred direction of radiation. I've included a dipole and a patch on this page, and we found that there was an S vector pointing out of both of these antennas towards the right of the slide. You can see that clearly for the E and H fields I've drawn on the dipole. However, if you looked at these antennas from the top of the slide, you wouldn't see any radiation coming out of them: the E field points in that direction for the patch, and cross product of E and H goes to zero in the dipole as E bends perpendicular to the antenna. That means the S vector around these antennas has some shape

CLICK We call that shape the radiation pattern, and we often capture that shape using fancy 3D visualizations like the ones I've copied onto this page for a dipole and a patch. You can see the patch antenna doesn't radiate out of its tips, it's donut shaped, and the patch doesn't radiate backwards or sideways.

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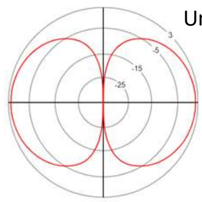
Radiation Patterns Often on Plane Plots in dBi

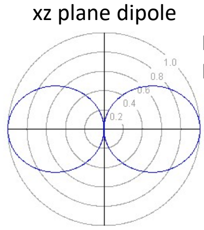
xy plane dipole

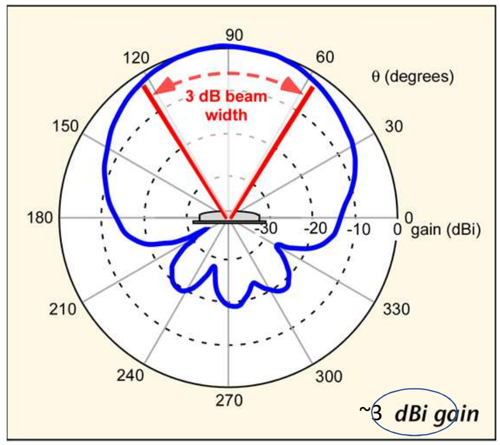
Units of dBi



xz plane dipole

Normalized?
Linear?





$$dBi = 10\log\left(\frac{P_{rad}(r,\theta,\phi)}{EIPD(r)}\right)$$

- Beam Width is -3dBi angle
- Max theory dBi is Directivity
- Max measured dBi is Gain
- Dipole directivity is 1.76dBi
- Patch directivity is ~3dBi

10

It’s hard to interpret fancy 3D graphs, so it’s very common to represent radiation patterns as polar plots that slice through various planes of an antenna’s radiation pattern. I’ve included these polar plots for a side view and a top view of a dipole on the left, and for a patch antenna on the right. Because these antennas are very symmetric, we don’t have all three plane slices through the patterns measured, but in datasheets you’ll often see radiation patterns come in sets of three, one for the xy plane, one for the xz plane and one for the yz plane.

CLICK the units on these plots, dBi, are probably unfamiliar to you.
CLICK deciBels are always a ratio of powers, and in this case they are a ratio of the power an antenna throws in a direction to the power an isotropic antenna would throw in the same direction. That means we’re comparing the radiation power out of an antenna against a sphere, and radiation patterns in dBi show you where your power has been sqeezed to beat a sphere.
CLICK This suggests a bunch of interesting design specs. One of them is the beam width, which measures the angle at which the radiation pattern has fallen off by 3dB from it’s peak value.
CLICK That peak value is called directivity if the radiation pattern is an ideal, lossless patterns, CLICK or it’s called gain if we’re measuring the radiation pattern and thereby including sources of loss and non-ideality. Saying that again, the gain is a measure of the

biggest field an antenna will spit out compared to a sphere, so it's a measure of how the power radiating from the antenna is squeeze into a narrower beam.
CLICK For reference, the directivity of a dipole is extremely low. Dipole antennas are almost omnidirectional, which is another reason they caught on early.
CLICK Patch antennas are more directional since they mostly throw power forward and not to the sides or the back. 3dBi of gain is a low estimate for a patch, and they peak at ~10dBi.

CLICK Finally, a clarifying note, the units on the lower plot are less clear because Wikipedia is a crazy wild west. This shape is consistent with the radiation pattern seen from the top of a dipole antenna, but the units are unclear. They look like normalized linear units, which is an unusual though technically valid way to plot radiation patterns.

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Summary

- Antennas can't radiate uniformly in all directions
- The ways they do radiate are captured by radiation patterns
- Radiation patterns are often measured in dBi=10*log(P(r,ϕ,θ)/EIPD(r))
- The peak of the radiation pattern is directivity, gain is directivity + loss

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Reciprocity and Receive Gain

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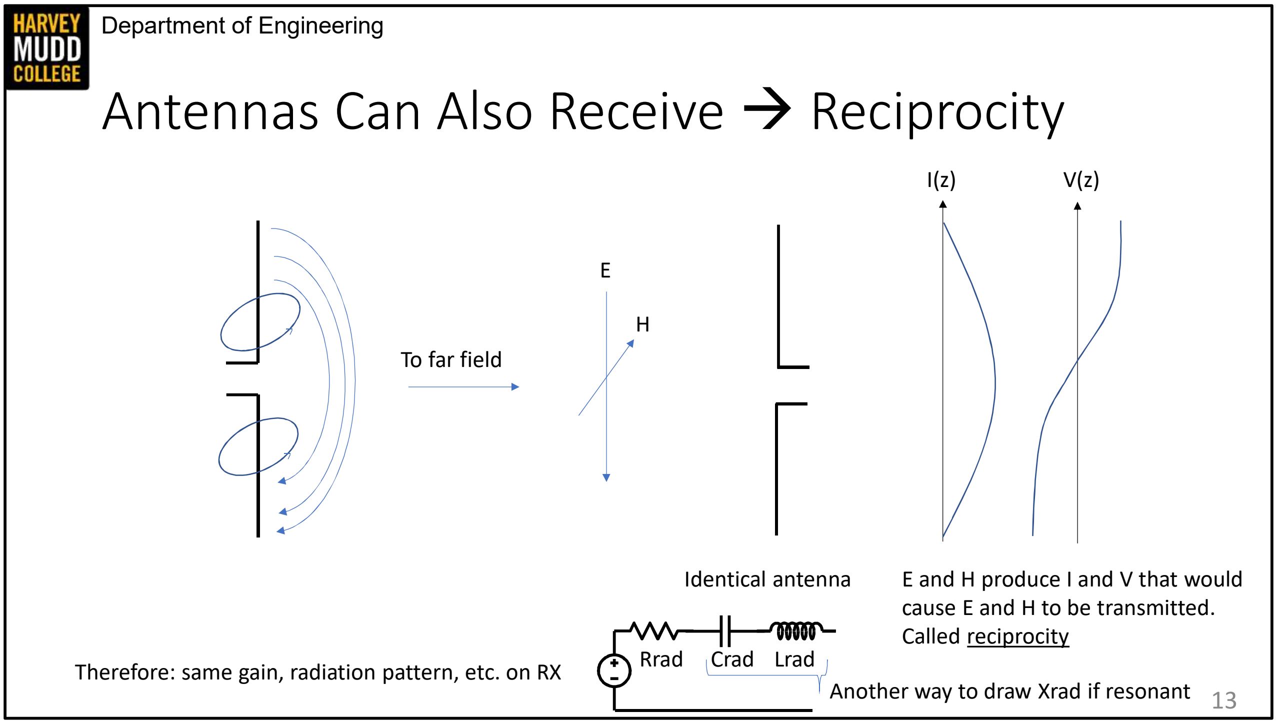
12

In this video we're going to start considering the use of antennas as receivers, which is a change because we've thought of them as transmitters up to this point.



The key to understanding why and how antennas act as receivers is an idea called reciprocity. To understand it, I've drawn a radiating antenna throwing some E and H into its far field. If this E and H hits an identical antenna, it will induce some voltage and current by wiggling charges around in the metal that makes up the antenna. The principle of reciprocity says that the V and I that are induced by an incident field have to be the same as the V and I that would produce that radiating field. This comes from a symmetry argument: the antenna is just a piece of metal, so the E and H fields and voltages and currents just have to be instantaneous functions of one another. If you perturb V and I, then E and H will change, including possibly causing some power to flow. If you change E and H, then V and I will change, possibly causing some power to flow.

CLICK This is a really powerful principle, because it lets us say that all of our antenna properties – the radiation pattern, the gain, the input impedance, etc. – are the same for receive antennas as transmit antennas. We can talk about these things as properties of antennas rather than of receiving or transmitting. This also lets us draw a more complete antenna circuit model. Here I've added a voltage source to represent the voltage induced by incident fields. That voltage source is present in an antenna regardless of whether it's being used as a transmitter or a receiver.

CLICK These inductors and capacitors are new in our antenna model, but they're just a compact way to represent how Xrad changes around resonant peaks. It’s capacitive below a peak an inductive above it.

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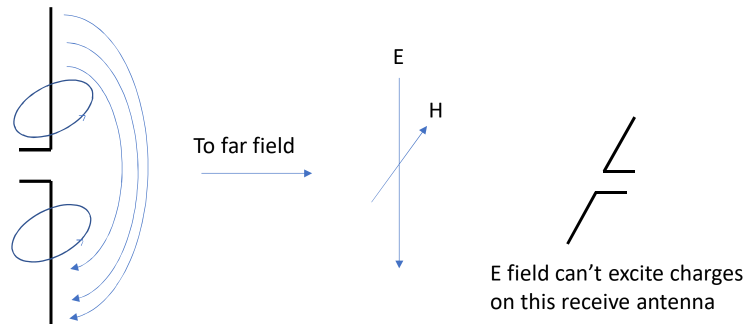
Antennas Emit Polarized Waves



Antennas have an associated polarization, and reciprocity includes spatial orientation.

14

One interesting quirk of reciprocity is that it depends on the spatial orientation of antennas. It turns out that most antennas emit polarized waves, so we often describe antennas as having a polarization. You can see that in the dipole on the left, the E field is definitely pointing up and down the slide and not in and out of it. If your receive antenna isn’t matched with the polarization of the transmitter, then incident fields won’t necessarily induce voltage on it.

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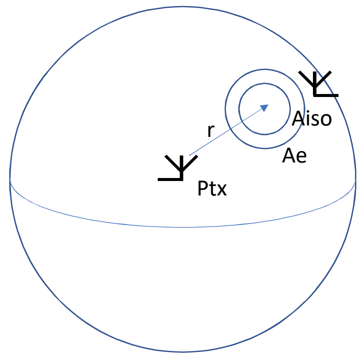
Derivation of The Receive Aperture



$$G = \frac{P_{rx,ant}}{P_{iso,ant}} = \frac{A_e}{A_{iso}}$$ Must be same as TX gain by reciprocity

$$A_{iso} = \frac{\lambda^2}{4\pi}$$ By some thermodynamics

$$A_e = G\frac{\lambda^2}{4\pi}$$ Appears to vary w/ ω

15

The reciprocity property of antennas is great, and it promises that we'll be able to send waves from one antenna to another. However, gain is currently defined as power transmitted over EIPD, which doesn't make a lot of sense for a receive antenna.

We need a new definition, which ideally talks about how much more power a receive antenna captures than an isotropic antenna. To do that, we imagine that we're putting an isotropic antenna at the edge of our EIPD sphere, and we're asking how much power it will capture. We specify how much power the antenna captures using an effective area. We do that because the EIPD is a power density, so we can talk about how much power an antenna absorbs by multiplying the power density by the antenna's capture area. This capture area is called the antenna's aperture. I've drawn the aperture for an isotropic antenna on this sphere. CLICK the hope is that our receive antenna will have a bigger aperture than the isotropic antenna, and that we can turn that into a definition of gain.

CLICK We define the receive gain as the ratio of the power received by an antenna divided by the power received by a hypothetical isotropic antenna. This definition doesn't depend on what's getting transmitted, so we can imagine the transmitter is isotropic for simplicity. That means the same power density will be incident on both the RX antenna and the isotropic antenna we're considering, so the only difference in the power they capture will be the ratio of their apertures. We can note that this definition is identical to the definition of transmit gain, comparing the power on a sphere between an antenna and an isotropic radiator, and it has to be that way to obey reciprocity.

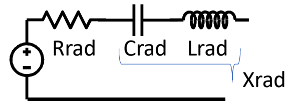
CLICK The aperture of an isotropic radiator is given by lambda^2 over 4 pi by a somewhat involved thermodynamic derivation. It's readily available on the internet if you're curious.

CLICK That means we can define the aperture of a receive antenna as the gain times lambda^2 over 4 pi. This is going to prove useful for calculating the power that makes it into a receive antenna, but you might notice a few quirks. First, aperture scales with gain, so antennas that are very high gain will be sensitive to signals in a very large area. That's counterintuitive because we think of high antenna gains corresponding to tight beams in the radiation pattern. The resolution is to remember that this equation is for the far field, where waves are assumed to be plane waves, so the large aperture of high gain antennas is just a way of representing how sensitive the antenna is in a particular direction. (Though, some antennas do indeed become physically limited in their aperture, which is why satellite dishes are so big: they have a huge aperture to increase their gain.)

Another quirk is that the effective aperture appears to change with the frequency of the wave you're measuring because lambda changes with frequency. This is weird because it's not like the shape of the antenna changes with frequency. That shrinking of the aperture comes from the fact that the isotropic antenna's aperture is frequency dependent, which goes back to the frequency behavior of Black Body radiation. The frequency dependence of our aperture comes from the comparison to an isotropic antenna.

Summary

- Reciprocity means antennas are symmetric in transmit and receive
- That gives us a new, more complete antenna circuit model
- The waves that come off of antennas are polarized



- Receive gain is based on comparing effective receive area (antenna aperture) to isotropic radiator

$$A_e = G\frac{\lambda^2}{4\pi}$$

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Friis Equation and Link Budgets

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17

In this video we're going to put our pictures of antennas together to calculate how much power gets across a link between antennas.

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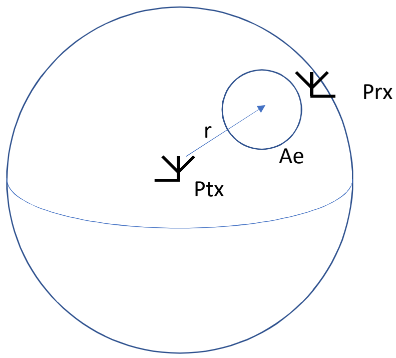
Find the Power at a Receiver w/ the Friis Eqn



$$P_{RX} = \underbrace{P_{TX}\frac{1}{4\pi r^2}}_{\text{EIPD}} G_{TX} G_{RX} \underbrace{\frac{\lambda^2}{4\pi}}_{\text{Ae}}$$

$$P_{RX} = P_{TX} G_{TX} G_{RX} \underbrace{\left(\frac{\lambda}{4\pi r}\right)^2}_{\text{Path Loss – frequency dependence?}}$$

Log version:

$$P_{RX} = \underbrace{P_{TX} + G_{TX}}_{\text{EIRP – equivalent isotropic radiated power}} + G_{RX} + 20\log\left(\frac{\lambda}{4\pi r}\right)$$

18

We're going back to our picture of a spherical shell around a transmitting antenna to think about power transfer. We're also imagining that our receive antenna is at the perimeter of the shell capturing an amount of power proportional to its aperture. That picture can be directly translated to the equation on the right. We take the power density on the surface of the sphere, which is given by EIPD, multiply it by the capture area of the receive antenna, which is given by it's effective aperture, and then finally multiply the amount of power incident on the receiver by the transmit gain, which is the ratio of power in a direction to EIPD.

CLICK We can rearrange this equation to the form shown in this line. This equation is referred to as the Friis Equation or sometimes as the Link Equation. It is fundamental to understanding communication links, and we're going to be considering this equation in conjunction with models of receivers to build up something called a link budget, which will represent a whole communication system.

The squared term in the Friis Equation is referred to as the path loss, and it can be thought of as the loss due to the signal spreading around all of free space. This is different than attenuation, which is the power absorbed by the medium your wave is travelling through. We're not modeling attenuation here. Path loss appears to vary with frequency, which is a bit weird because the surface area of a sphere doesn't change as the frequency increases or the wavelength decreases. Again, this frequency dependence can be traced back to the aperture of the receive antenna, and in turn to the aperture of an isotropic antenna, which is intrinsic to our definition of receive gain.

CLICK It's common to write the Friis equation in logarithm format, and various texts will fight holy wars about whether path loss is specified as a positive or negative number of decibels. In our version of this equation, it's defined positive, then subtracted in the equation. Just remember that path loss makes your signal smaller and you'll be able to navigate differences in the definition. One final note is that the combination of the transmit power and the can is often called the EIRP, or the equivalent isotropic radiated power. It's the amount of power you'd have to put into an isotropic antenna to match the power delivered by an actual antenna.

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Summary

- Received power is given by the Friis equation, featuring path loss

$$P_{RX} = P_{TX} G_{TX} G_{RX} \underbrace{\left(\frac{\lambda}{4\pi r}\right)}_{\text{Path Loss}}^2$$

- Path loss varies with frequency because of isotropic RX aperture

- Combining this with models of the receiver gives you a link budget