Lecture 15:
Electromagnetic Radiation

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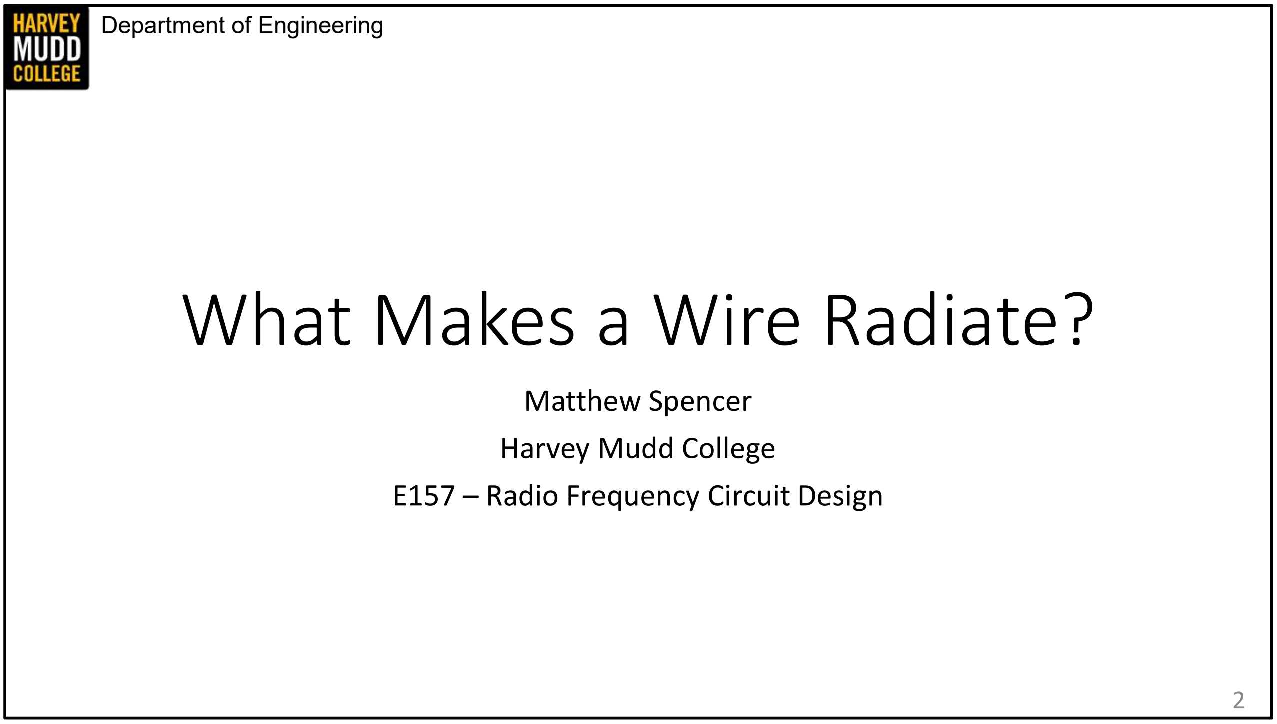
E157 – Radio Frequency Circuit Design



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What Makes a Wire Radiate?

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In this video we’re going to talk about the origin of radiation from antennas.

Poynting Vector is Zero for Quadrature Fields

$$< S > = \frac{1}{2}\mathrm{Re}\{E^{*} \times H)$$

$$E(x,t) = \hat{z}E_{+}e^{j(kx-\omega t)}$$

$$H(x,t) = \hat{y}E_{+}e^{j(kx-\omega t\quad)}$$

Assume a phase shift on H field

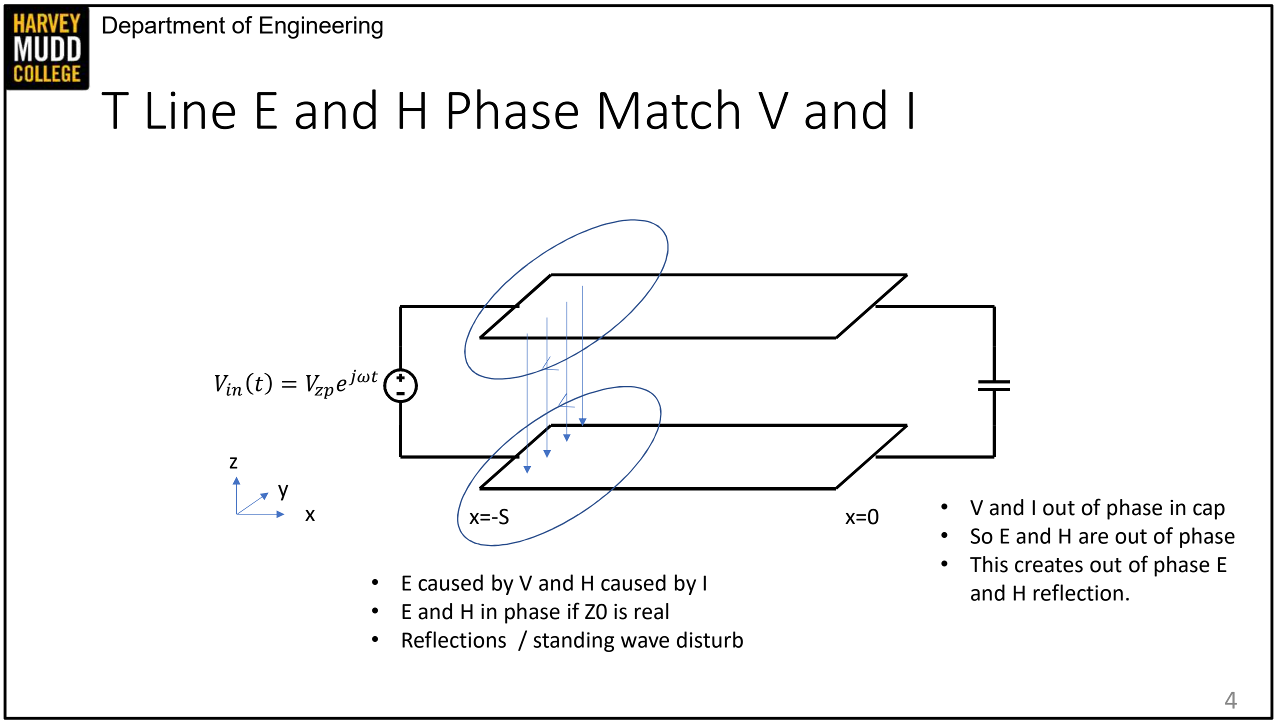
$$< S > = \frac{1}{2}E_{+}H_{+}\cos\phi$$

Need fields in-phase to move power / radiate

Radiation is power flowing away from wires in the form of electromagnetic waves, so it’s worth digging into the Poynting vector, our measure of electromagnetic power flow a bit more.

Specifically, we’re going to assume candidate E and H fields that have a phase difference phi between them. When we evaluate the Poynting vector, we can see that this phase shift significantly affects power transfer: the time averaged power is multiplied by the cosine of phi. We can use this observation to identify one of the most important conditions for fields radiating, which is that E and H fields need to be pointed at 90 degrees to one another in space, and they need to be in-phase in time. We can see that in this equation: If phi is zero, then we maximize the Poynting vector, and if phi is 90 degrees then no average power is radiated.

This condition tells us some important details to look for in antenna design: any time we see fields adding up in phase then we can presume that we are going to get radiation.



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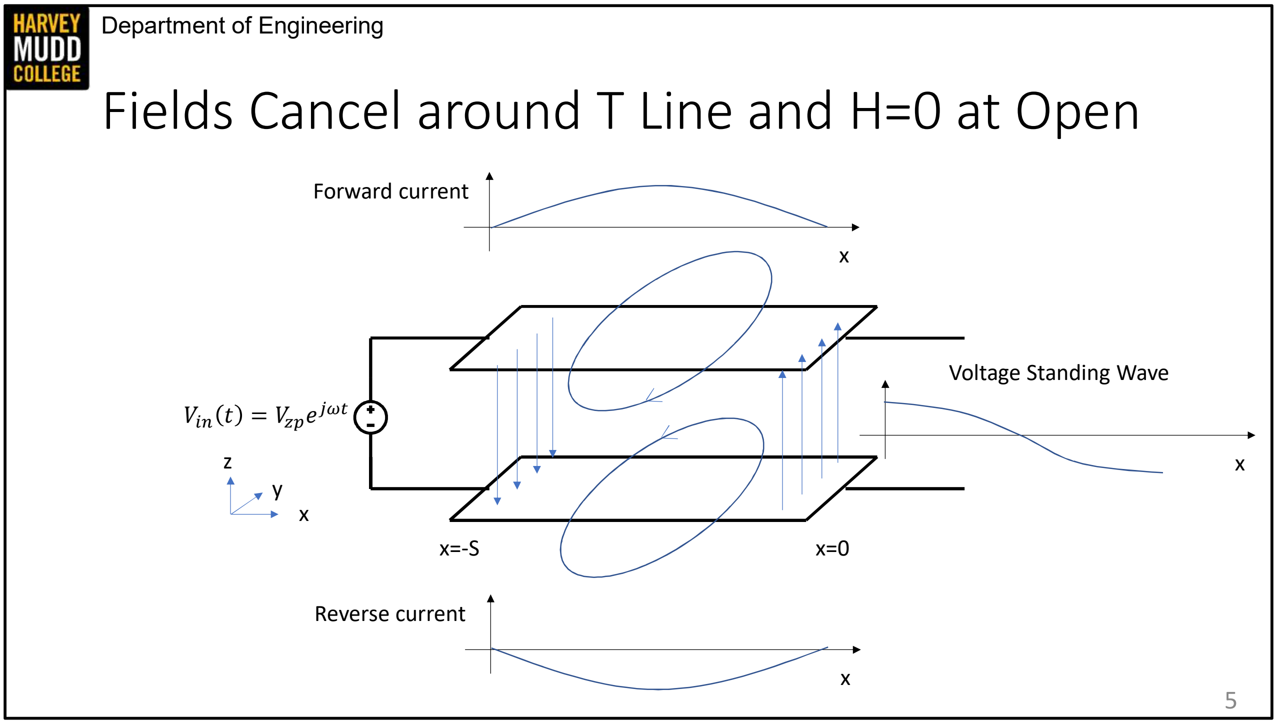
T Line E and H Phase Match V and I

- E caused by V and H caused by I
- E and H in phase if Z0 is real
- Reflections / standing wave disturb

- V and I out of phase in cap
- So E and H are out of phase
- This creates out of phase E and H reflection.

To start thinking about power flow in these terms, let’s look at a single travelling wave as it’s launched down a transmission line. Right as Vin(t) starts up, we’ll see E fields pointing from the conductor of the transmission line, which is attached to high voltage, towards the return path, which is attached to ground. Electrons will move around the circuit in response to these E fields, creating a current and resulting loops of H field. The E and H fields correspond directly to voltage and current on the transmission line and, in fact, E is proportional to V and H is proportional to I. That means E and H will be in phase because Z0 is real, which makes V and I in phase. This tells us that real power is transmitted down the transmission line until a reflection makes it back to the source to change something.

CLICK When the wave reaches this capacitor, the voltage and the current will be out of phase in the load, which implies two things. First, if we opened up the capacitor and looked at the fields inside of it, E and H would be out of phase. Second, the capacitor generates a reflected wave that has E and H out of phase, which will eventually result in no power being transmitted down the line.



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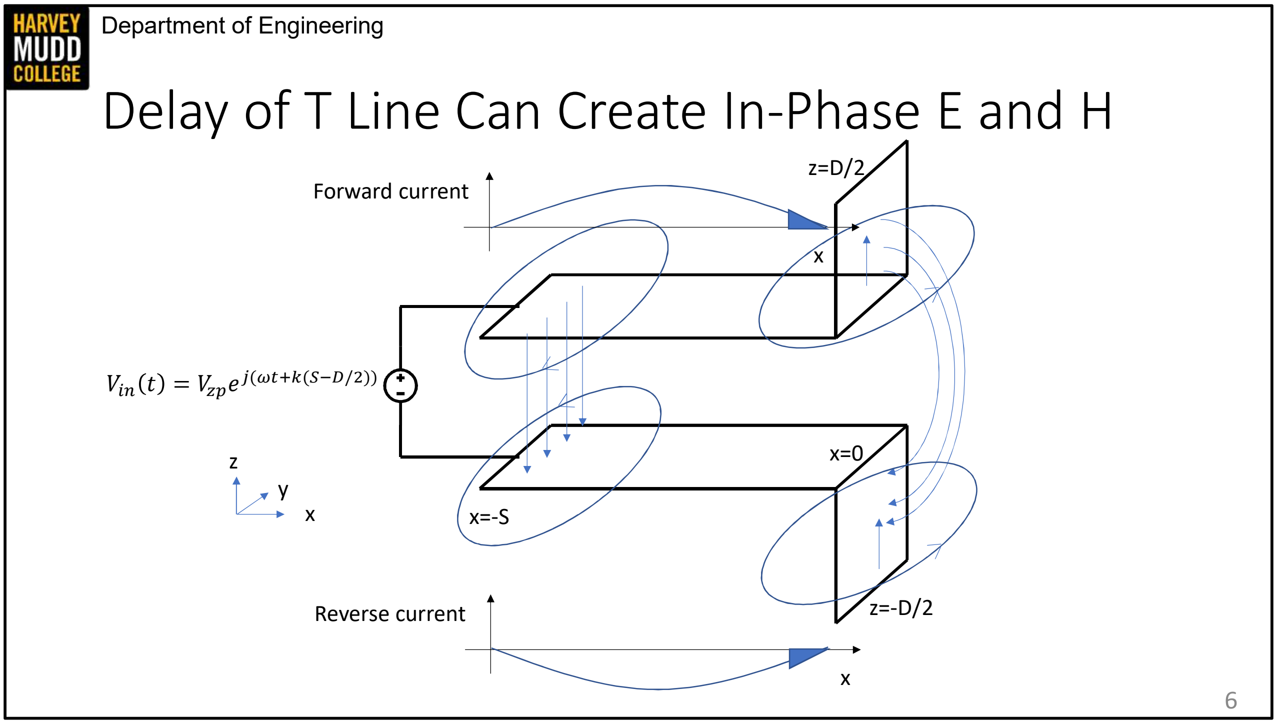
Fields Cancel around T Line and H=0 at Open

We’re going to review a similar situation on this slide, which depicts the E and H fields on a line after we’ve been driving a wave into an open circuit for a while. We’re presuming that our transmission line is a half-wavelength long in this example. This standing wave pattern from the open circuit results in a positive half-wavelength current distribution on the top conductor, a negative half-wavelength distribution the bottom conductor, and a voltage that varies from positive to negative over a half-wavelength between the conductors. Note that these distributions are the envelopes of the standing waves, and every point of the line will see voltage and current oscillate up and down at the drive frequency.

A few things in this diagram prevent radiation. First, the E and H fields are totally out of phase, just like the V and I waveforms. You can see this most clearly at the position –S/2, where we have a peak in current and an accordingly strong H field, but V is zero resulting in no E field. This makes sense because there is no net power flow into an open circuit.

Second, you’ll notice that some field exists outside the space between the transmission lines because of the H field loops. However, if you freeze time, and calculate the fields surrounding the top and bottom conductor separately, then superpose them, you’ll find that their fields cancel almost completely except in between the two conductors. That happens because the top and bottom conductors have exactly opposite charge and current densities. In order for radiation to happen, the charge and current densities on the top and bottom conductors need to be rearranged so they don’t exactly cancel.



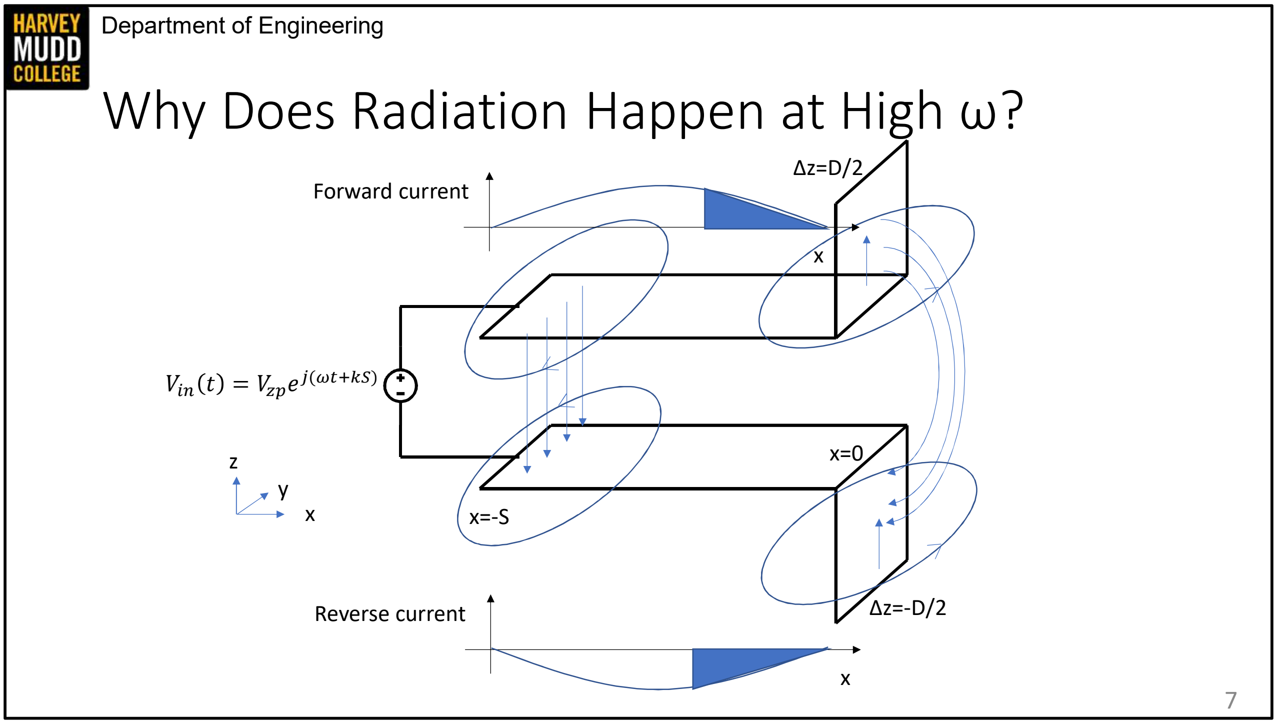
Delay of T Line Can Create In-Phase E and H

We can arrange for a radiative condition by bending the very tips of our transmission line in opposite directions. We assume these bent tips are each of a length D/2, resulting in a total length D oriented normal to the standard transmission line. We further assume D is much much less than lambda/2. This results in the shaded parts of the current distributions falling on the bent regions.

Because we bent the transmission line segments in opposite directions, the current on both segments point in the z direction, which means that the H fields on the top and the bottom electrodes both orbit in the same direction, and therefor add together instead of canceling. There is also an E field normal to these H fields that fringes from the top bent segment down to the bottom segment. This field is normal to the H field, and that means we could have a non-zero Poynting vector aimed in the positive x direction. Great!

Normal E and H fields means we have a good chance of making a propagating wave, but we also need the phase of E and H to not be completely in quadrature. Fortunately, the E field generated by the charge at z=D/2 and z=-D/2 is slightly delayed (by a phase angle of kD/2) compared to the H field generated by the current at z=0. That means the E and H fields aren't completely in quadrature even though the V and I waveforms are. This phase delay is small compared to the total phase of the wave, so the phase angle between E and H phasors, phi in our earlier example, is still close to ninety degrees. That means there will be some real power flow, which represents radiation, and some imaginary power flow that represents fields being charged up and discharged as reverse travelling E and H waves on the transmission line.

This structure is our first antenna, and it’s called a short dipole not because it’s a short circuit but because D is a small fraction of lambda.



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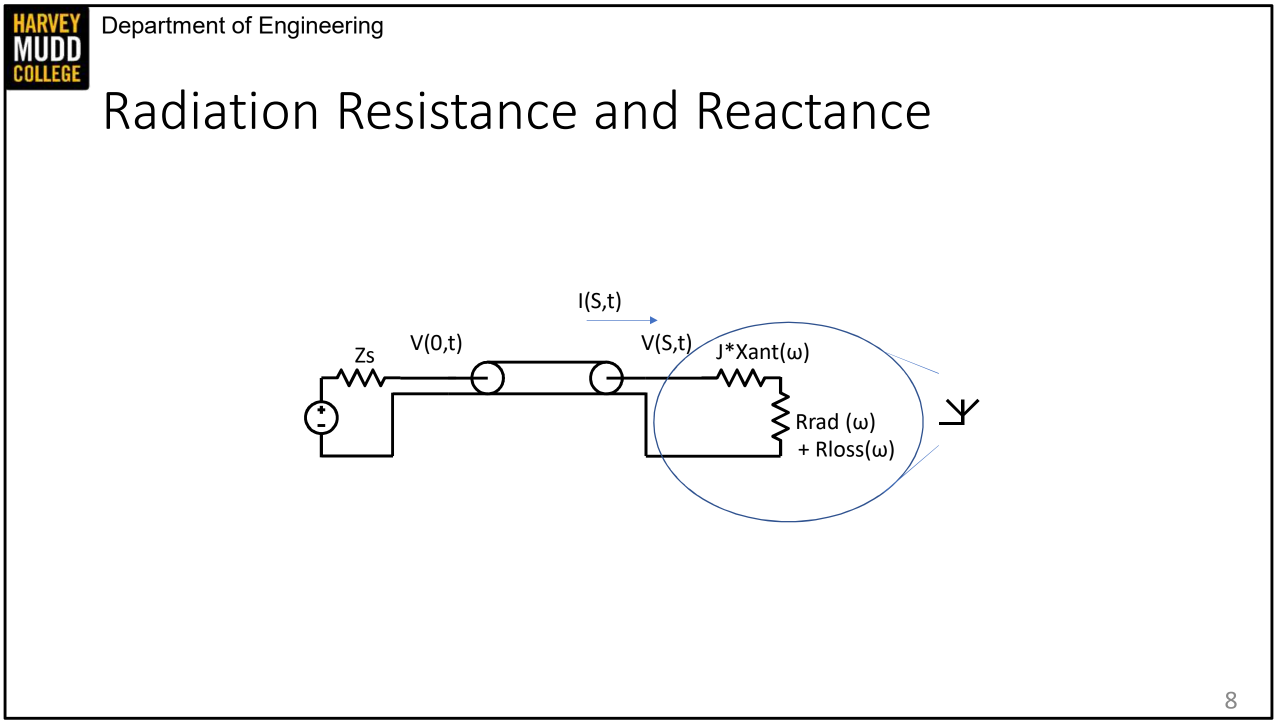
Why Does Radiation Happen at High ω?

One lingering question about this radiation discussion is why circuits you've seen before weren't radiating all the time. After all, we've just said that a slightly bent wire will radiate, and I'm sure many of your first circuits had bends in the wires used in their construction.

The answer is that power transfer depends on how much of the standing wave falls onto the antenna. In this slide I've suggested a differently sized antenna, possibly with a higher frequency wave, where more of the current distribution will fall onto the antenna. That is indicated by the larger shaded region of the current distribution. This antenna would radiate more than the antenna on the previous page because there is more current to make the H fields that cause radiation.

By way of contrast, If your waves are very low frequency,, then your waveform will be huge and very little of your waveform will fit onto the antenna. That means the current in the antenna will be close to zero, so no power will be radiated. One way I remember this is by keeping a reference in my head: a 300MHz wave has a wavelength of 1m in free space. If your early classes were working at tens of kiloHertz, then the wavelengths would be even longer than 1m because the frequencies were lower, that means your wires would have to be tens of meters long before they radiated meaningfully.



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Radiation Resistance and Reactance

Finally, it’s worth asking what the driving transmission line, which is often referred to as the feed line in antenna parlance, perceives when an antenna is radiating. As indicated on this slide, it’s going to see two components, a reactance and a resistance. The resistance is referred to as a radiation resistance, and it exists because the driving circuit can’t differentiate between power being radiated and power dissipated by heat. From the driver’s perspective, energy that is radiated is gone forever, so we represent the antenna with a lossy resistor, the radiation resistance, to show that energy flowing away.

The antenna reactance is related to the fact that fields in the antenna aren’t completely in quadrature. That makes the antenna look either capacitive or inductive. As we saw earlier, E and H fields in short dipoles are only a little bit in phase, so Xrad is very big for short dipoles. It also turns out that it’s negative, indicating a capacitance. More on that soon.

Both Rrad and Xrad depend on the wavelength of the signal on the line because the fraction of the wave that fits on the antenna affects the relative phase of E and H. That means both Xrad and Rrad change with frequency.

CLICK One modification to this model is the addition of loss resistance. Antennas are still made of metal, which has a finite resistance, so some of the current passing through the structure experiences I^2*R losses. The loss resistance is in series with the radiation resistance, and it also varies with frequency because of a phenomenon called the skin effect. The ratio of radiation resistance to loss resistance can be used to measure the efficiency of the antenna, which is defined as the ratio of power radiated to power dissipated as heat. The antenna efficiency shouldn't be confused with the efficiency of power transfer into the antenna from the feedline, which is characterized by the reflection coefficient defined by Z0 and Rrad+Rloss. Arranging for Rrad+Rloss to be close to Z0, or building a matching network, helps a lot with getting power out of the antenna.

CLICK Finally, this is all often summarized using the antenna symbol in a circuit diagram.

Summary

- Power is radiated when E and H are in phase and don't cancel.
- You can set that up by bending a transmission line 90 degrees to make a short dipole.
- Power being radiated is lost from the circuit, so it appears as a radiation resistance.

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The Short Dipole and Antenna Bandwidth

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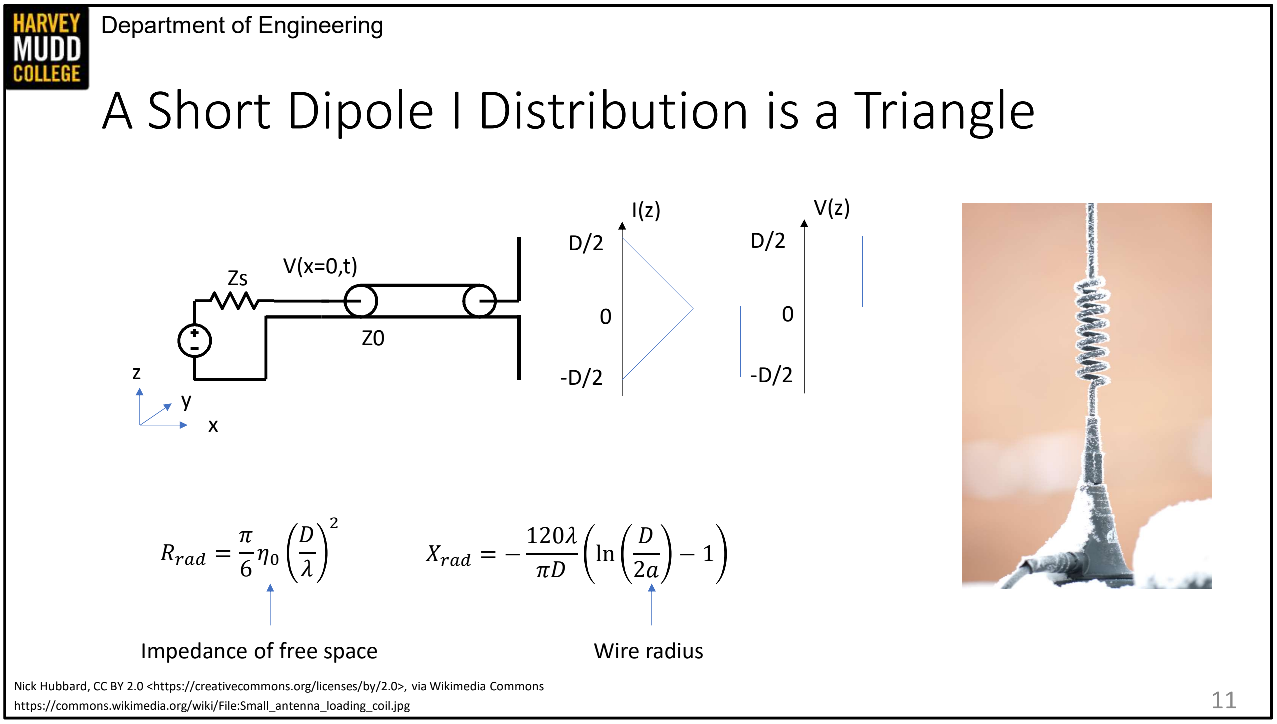
In this video we're going to examine the properties of the Short Dipole in greater detail. We'll also observe that antennas only work properly over a narrow frequency range, so their bandwidth is an important design specification.



A Short Dipole I Distribution is a Triangle

$$R_{rad} = \frac{\pi}{6}\eta_0\left(\frac{D}{\lambda}\right)^2$$

$$X_{rad} = -\frac{120\lambda}{\pi D}\left(\ln\left(\frac{D}{2a}\right)-1\right)$$

Impedance of free space

Wire radius

Nick Hubbard, CC BY 2.0 <https://creativecommons.org/licenses/by/2.0>, via Wikimedia Commons
https://commons.wikimedia.org/wiki/File:Small_antenna_loading_coil.jpg

I've drawn a short dipole on this slide, and I've included the current and voltage distributions on the short dipole axes next to it. We define distance along the dipole antenna in terms of the coordinate z, and we specify that the total length of the dipole is D, split into D/2 above the terminals at z=0 and D/2 below it. We can approximate the very tips of a sinusoid as straight lines using a small angle approximation, which is why the current distribution looks like a triangle. The voltage is being approximated the same way, so voltage looks high and constant at the end of the antenna. As usual, these profiles represent the envelopes of standing waves, so they vary up and down at a frequency of omega.

I've included the radiation resistance and reactance below the antenna without justifying where I got those formulas. If you want some vocabulary words for your own research, you find these equations by calculating the magnetic vector potential of a Hertzian dipole, taking geometric derivatives of it, and computing the resulting Poynting Vector. I've linked a website that does this on the class page.

The first takeaway from these equations is that the radiation resistance drops as the short dipole becomes longer, but when lambda is small compared to D then short dipoles have high radiation resistance. This means they can be efficient because Rrad is larger than Rloss, but also that power transfer into them is tough. The second is that Xrad is negative and quite large, so short dipoles look like big capacitors. Looking at the voltage is one way to see the large capacitive behavior of the short dipole: lots of charge is being stored in the antenna to make a high voltage out of the small capacitance at the end of the short dipole, while the current flowing is rather low, which means little of that charge is generating radiative E field.

You can compensate for the capacitance of a short dipole by adding a loading coil to your antenna, which looks like spiraling the antenna around to add some series inductance. You can see an example of a loading coil in the figure on the right.

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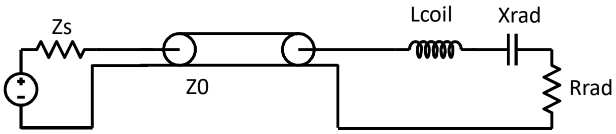
Bandwidth of a Matched Short Dipole



$$Q(\omega) = \frac{\sqrt{L_{coil}/C_{rad}(\omega)}}{R_{rad}(\omega)}$$

Lcoil only matches at one frequency because Crad changes

If we look at the circuit model of a short dipole with a loading coil, we see that it forms a series RLC circuit. That means we can define a Q for the system, which suggests that the resonant peak where power is delivered to Rrad has a finite bandwidth. Defining that bandwidth is complicated by the fact that both Crad and Rrad change with omega, so the whole system winds up being non-linear, which means Q is poorly defined and it varies with frequency. However, even if this Q is poorly defined, it serves to highlight that antennas only work at certain frequencies. The bandwidth of an antenna, usually specified as a fractional bandwidth, is one of their major design specifications.

Summary

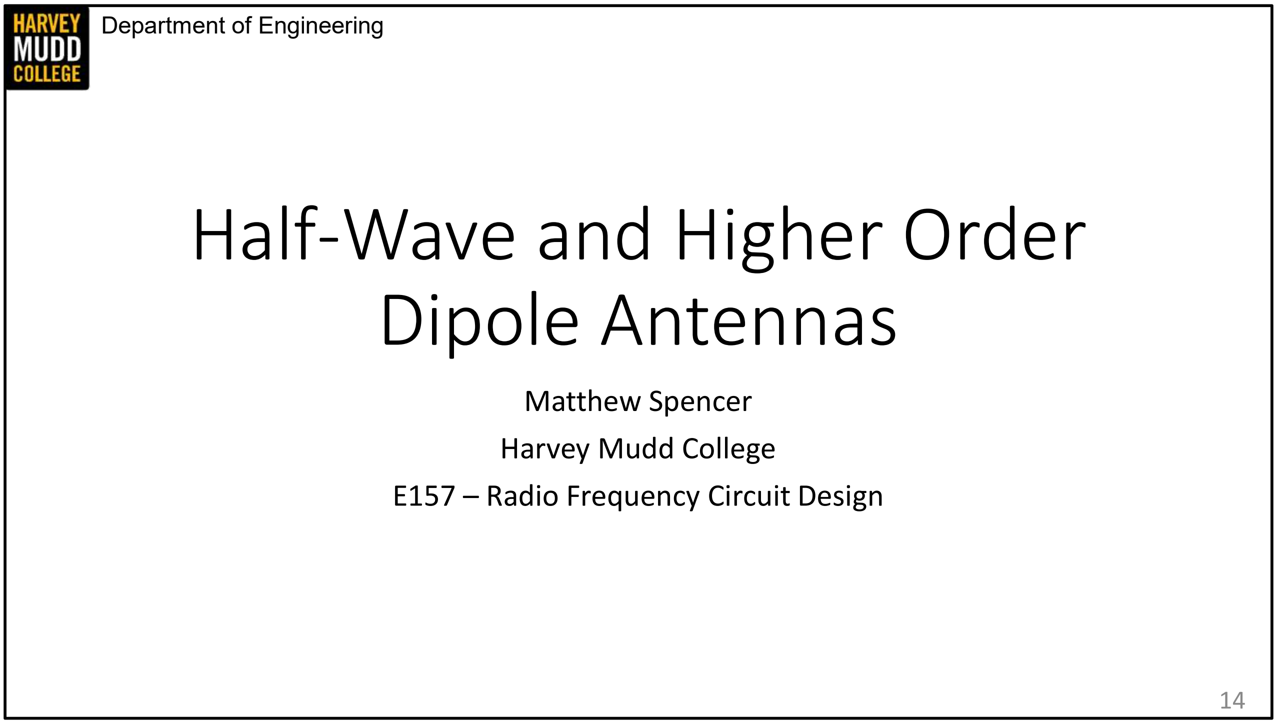
- Short dipoles have high Rrad (pretty efficient) and capacitive Xrad

$$R_{rad} = \frac{\pi}{6}\eta_0\left(\frac{D}{\lambda}\right)^2$$

$$X_{rad} = -\frac{120\lambda}{\pi D}\left(\ln\left(\frac{D}{2a}\right) - 1\right)$$

- Loading coils can help match short dipoles

- Antennas have bandwidth, hard to define for short dipole



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Half-Wave and Higher Order Dipole Antennas

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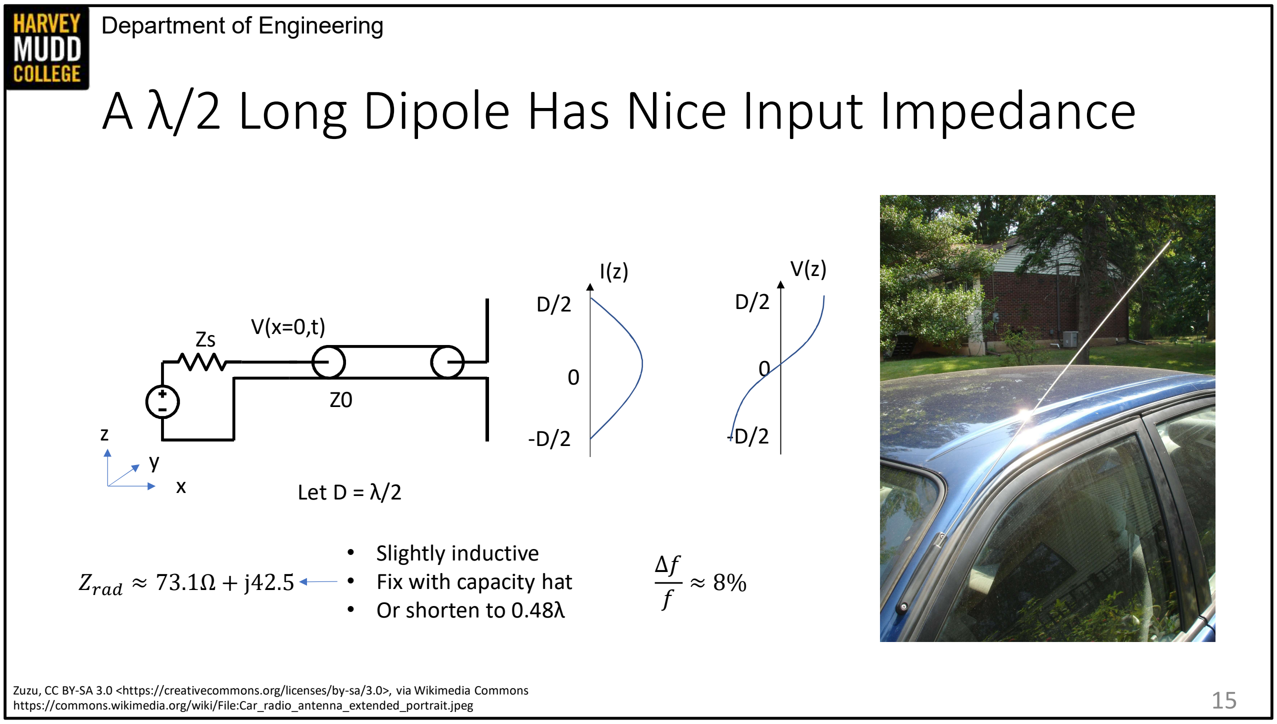
In this video we’re going to discuss the behavior of a dipole when more of a wave fits onto it. This discussion will include some of the most common dipole configurations, including the popular half-wavelength dipole.



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A λ/2 Long Dipole Has Nice Input Impedance

Let D = λ/2

$Z_{rad} \approx 73.1\Omega + j42.5$ ←

- Slightly inductive
- Fix with capacity hat
- Or shorten to 0.48λ

$\frac{\Delta f}{f} \approx 8\%$

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https://commons.wikimedia.org/wiki/File:Car_radio_antenna_extended_portrait.jpeg

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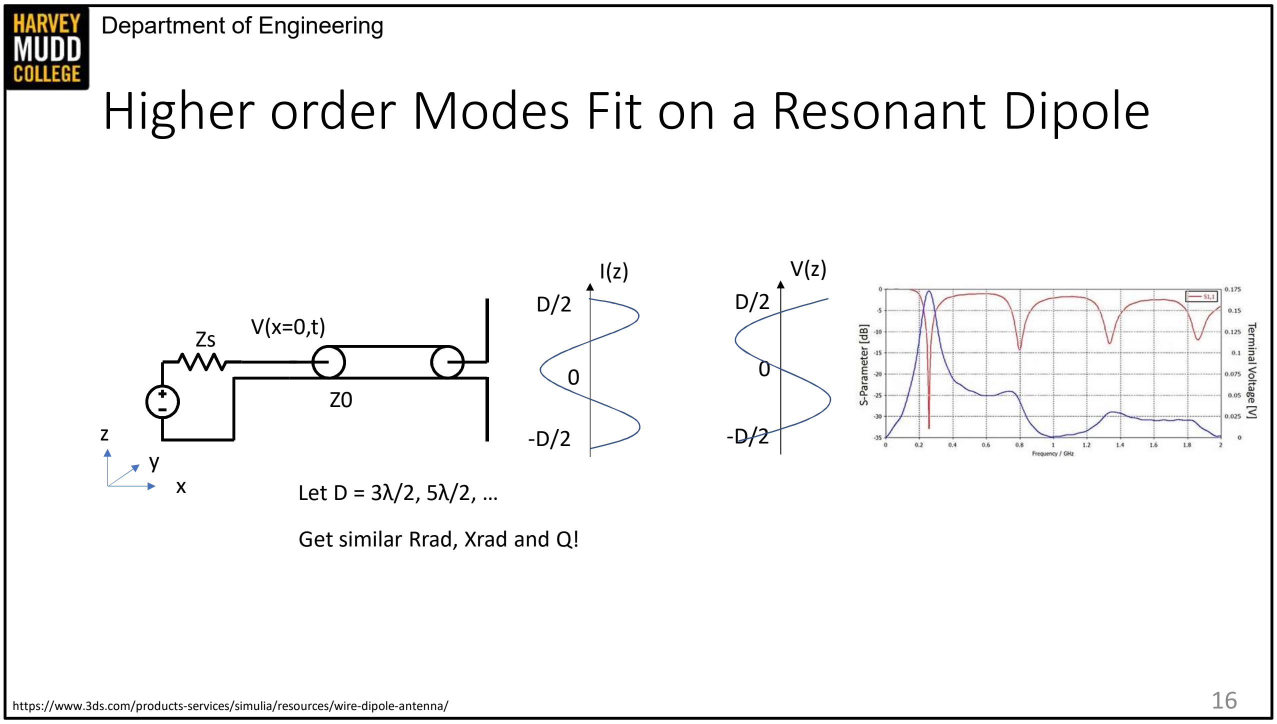
As the name implies, a half-wavelength dipole is a dipole antenna where half of a wavelength fits on the dipole. We still specify the coordinate along the antennas as z, and note that the whole antenna has a length of D. We have made this dipole into a half-wavelength dipole by picking a drive frequency such that lambda over 2 is equal to D.

If we do the same math we did with the short dipole, we can discern that a lambda over 2 dipole has an input impedance of 73 ohms with a slight inductive component. That's good news! We can imagine matching to that impedance. The news gets even better, because adding a little ball to the tip of an antenna, called a capacity hat, will add a little capacitance to cancel the inductance. You can see an example of a capacity hat in the figure on the right if you look closely. It's also possible to get a similar effect by making the antenna a little bit shorter, 0.48 lambda antennas have purely real impedances.

The half-wavelength dipole's driving point impedance is quite low compared to a short dipole, and one justification for that behavior can be found in the current distribution on the antenna. The ratio of I to V at the feed point specifies the impedance seen by the feed line. In this antenna, V is close to zero near the feed point, while I is quite high. That suggests a low drive impedances. Short dipoles had the opposite: the very tips of the voltage and current distribution had very large voltages and relatively low currents at the feed point.

The fractional bandwidth of a dipole is surprisingly high at eight percent. That may be particularly surprising because lambda/2 is only equal to D at one frequency, but the inductance, capacitance and radiation resistance of a half-wavelength dipole are such that the antenna has a relatively low Q.

Xrad is increasingly inductive when lambda is longer than D, and Xrad is increasingly capacitive when lambda is shorter than D. This is the same behavior as a second order RLC circuit.



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Higher order Modes Fit on a Resonant Dipole

Let D = 3λ/2, 5λ/2, …

Get similar Rrad, Xrad and Q!

https://www.3ds.com/products-services/simulia/resources/wire-dipole-antenna/

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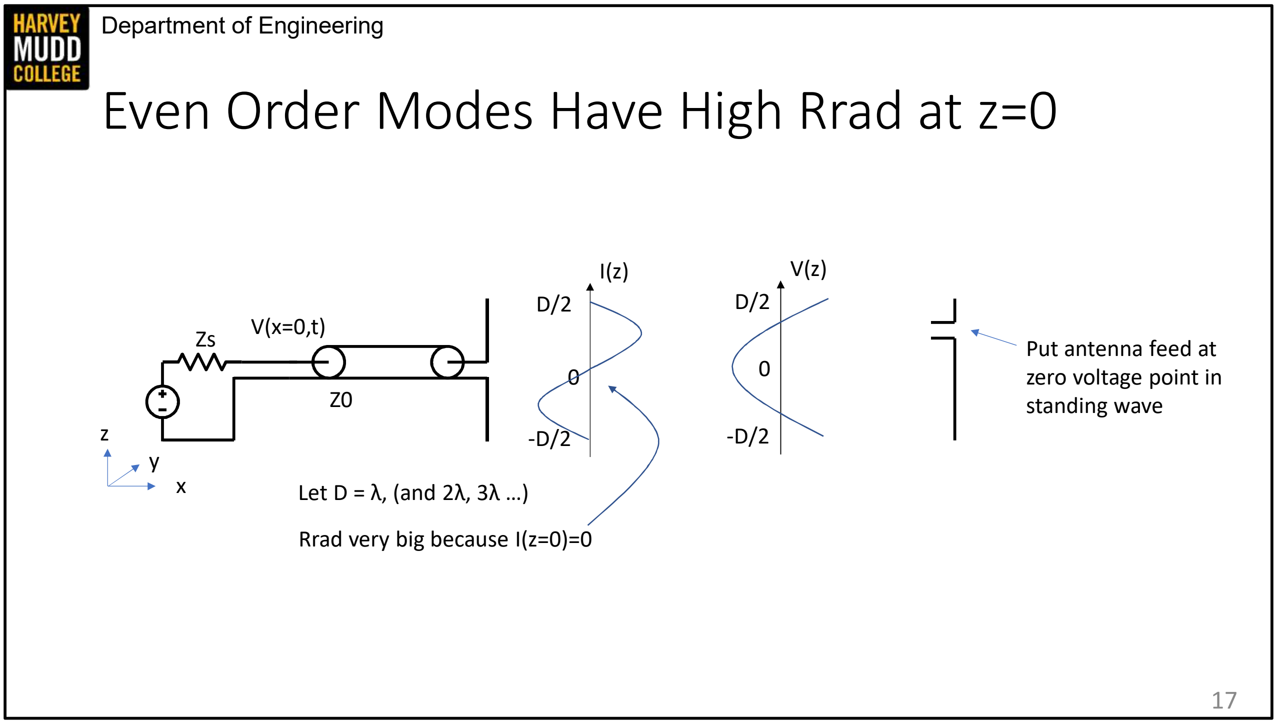
There are many standing wave patterns that will fit on a dipole antenna and that have the same behavior as a half-wavelength dipole. For instance, I’ve drawn a 3 lambda over 2 length dipole in this plot. It meets the same boundary conditions of zero current and high voltage at the end of the antenna, and it has the similar low voltage and high current at the feed point. That means the Rrad, Xrad and Q are similar even at the higher frequencies that produce the 3*lambda over 2 and 5*lambda over 2 wavelengths.

This means the antenna is multiresonant, and the plot of S11 vs. frequency I’ve included on the right shows that behavior. If you measure a dipole over many frequencies, You’ll see multiple frequencies where the antenna absorbs (and radiates) power at odd multiples of a fundamental frequency.



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Even Order Modes Have High Rrad at z=0

Let D = λ, (and 2λ, 3λ …)

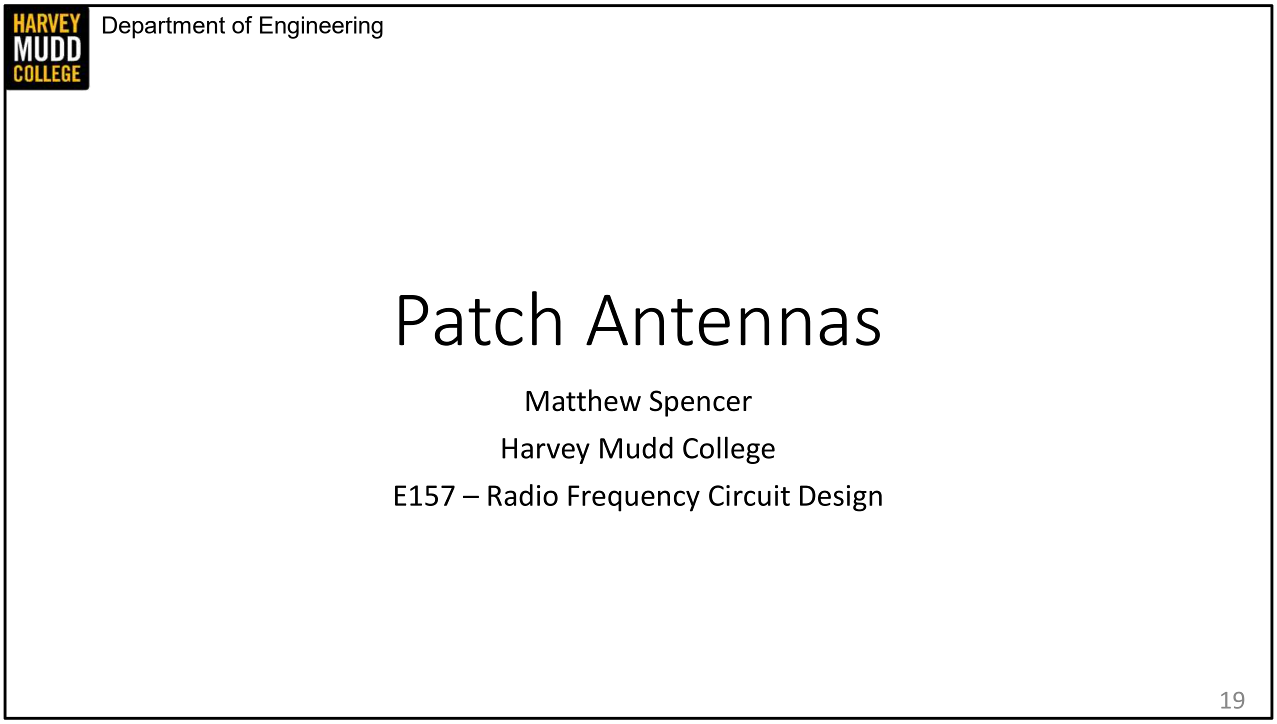
Rrad very big because I(z=0)=0

Put antenna feed at zero voltage point in standing wave

Dipoles containing even multiples of lambda over 2, like the one lambda dipole sketched on this slide, have a very high input impedance at z=0 because the current waveform is close to zero at the feed point. This makes one lambda antennas somewhat unpopular, though if you're bent on using them there's an easy fix. You can move your feedline to another point in the standing waves on the antenna by making one arm of the antenna longer than the other. It sometimes seems surprising that the same current/voltage distribution evolves on the antenna when you move the feed location, but I'm able to square the idea in my head by remembering that the dipole is just an extension of the transmission line, so we're just seeing some chunk of the transmission line standing wave pattern living on it. I also think of the dipole as a resonant filter that we're just choosing to excite in a different location.

Summary

- Resonant dipoles have nice input impedance, lousy bandwidth.
- Antennas are multiresonant whenever λ fits onto the structure
- Changing where you drive the standing wave on the antenna changes the antenna's input impedance.



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Patch Antennas

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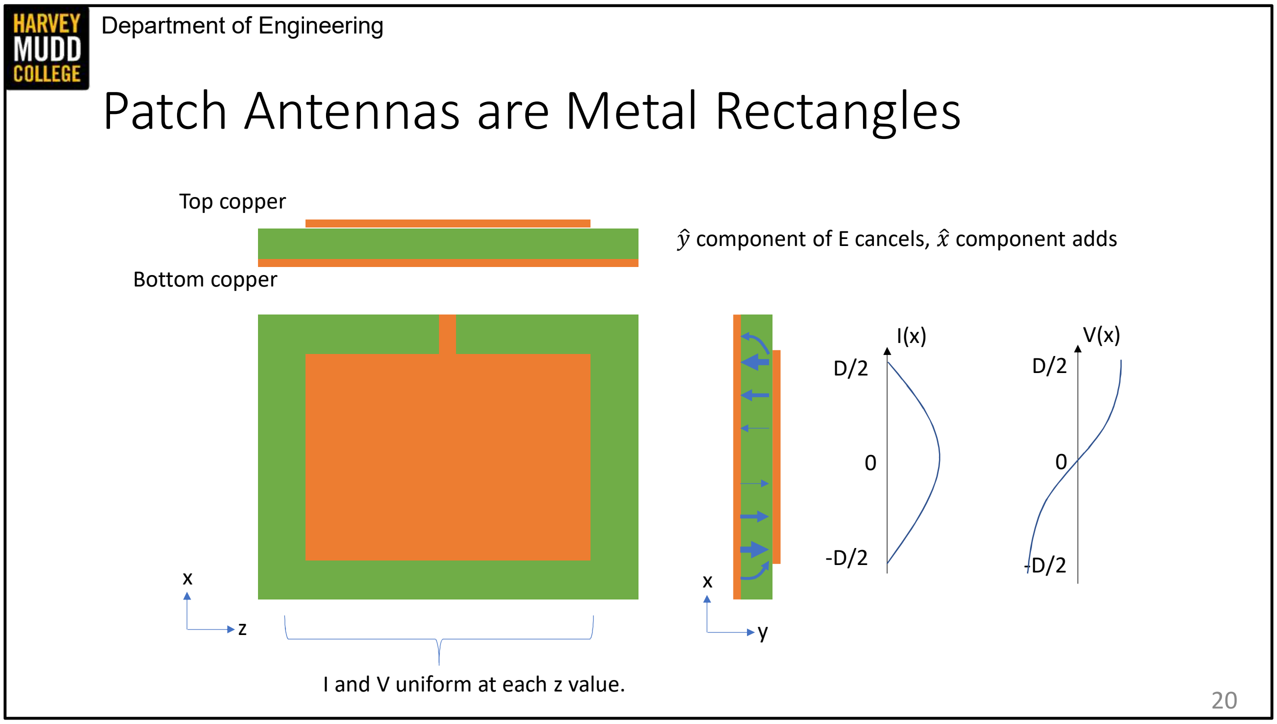
E157 – Radio Frequency Circuit Design

In this video we're going to learn about patch antennas. I'm choosing to discuss patches instead of the many other types of antennas because they are easy to build on PCBs.



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Patch Antennas are Metal Rectangles

$\hat{y}$ component of E cancels, $\hat{x}$ component adds

I and V uniform at each z value.

You can see that in this diagram of a patch antenna, which reveals in glorious plan view that a patch is just a rectangular piece of metal sitting on a PCB over a ground plane.

I've drawn a feed line at the top of my patch antenna, but it's actually a pretty poorly designed feed. Feeds at the edge of a patch antenna see zero current because the current distribution goes to zero at the edge of the patch. That means the patch edge has a very high input impedance. We'll talk about better ways to feed the antenna in the next few slides. However, the feed is important here because it helps establish the direction of current and voltage standing waves on the antenna.

Those current and voltage standing waves align entirely along the x direction when the patch antenna is working properly. In fact, it's common to design the width of patch antenna to be longer than the length, which we're still referring to as D, in order to prevent standing waves from evolving along the width at the same frequencies as the length.

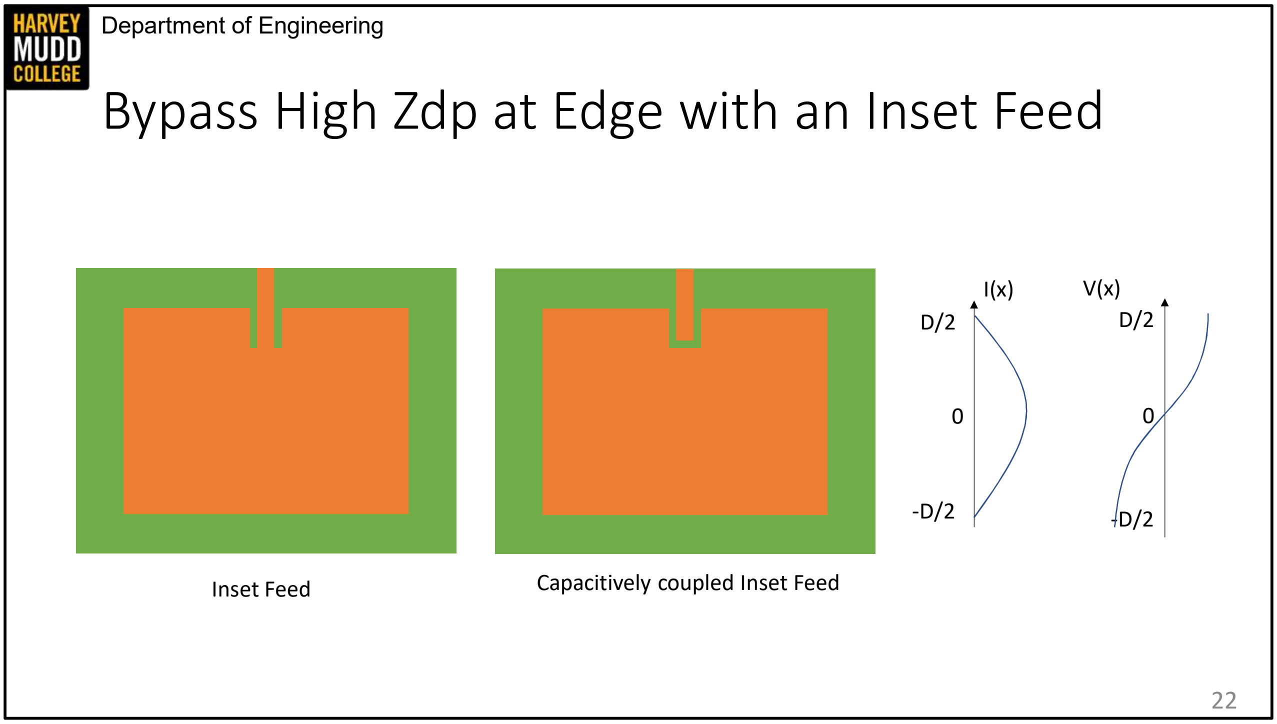
Just like the dipole, the patch antenna works by arranging for some fields to add up in phase. Understanding that starts with the voltage, current and electric field distributions I've drawn on the right. In this case, it's the x component of fringing electric fields that adds in phase. The electric field at the very end of the patch bends as it spreads from the patch to the ground plane, which means the field has a component pointing out of the patch and a component pointing along the patch. The components pointing out of the patch cancel, because the top and bottom of the patch have opposite voltages on them. However, the components along the patch both point towards to top of this slide, which means they add together. H field does something similar in the z direction, so a Poynting vector points out of the surface of the patch in the y direction.

Patches have Narrow Bandwidth

- Center frequency given by λ/2=D, need a wave to fit on the patch
- $\frac{\Delta f}{f} \approx 3\%$
- “Looks like many dipoles in parallel, so losses low and L, C are high”
- Try to suppress modes in z direction by making width about ~1.5x D

The center frequency of a patch is easy to calculate, we just need a half-wavelength to fit on the face of the patch antenna. The bandwidth is quite narrow, which is often justified using a folk derivation that a patch antenna looks like lots of dipoles in parallel, so the losses are reduced. Feel free to remember that phrase if it helps remember that Q is very high in this antenna.

Finally, I’ve included one more reminder that D needs to be smaller than the width in order to prevent a standing wave from showing up on the z dimension of the antenna and messing up our desired operation. An aspect ratio of 1.5 is usually about right to suppress standing waves in the z direction.



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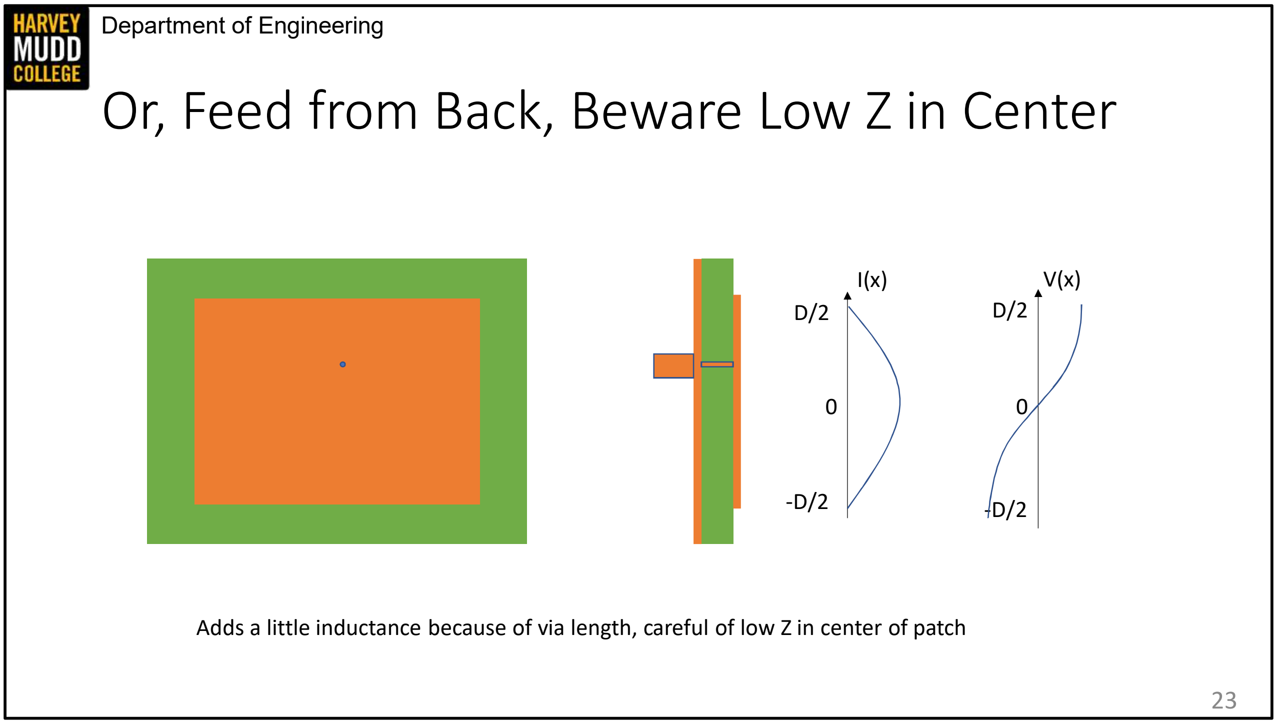
Bypass High Zdp at Edge with an Inset Feed

Inset Feed

Capacitively coupled Inset Feed

I mentioned earlier that the impedance at the edge of a patch is very high because the current distribution goes to zero at the edge of a patch. You can get around this by feeding the patch antenna in the middle of a patch, which lets you tap into a part of the standing wave that has a more favorable current to voltage ratio. It’s easy to do that without disturbing the standing wave pattern by making an inset feed line, which extends the feed transmission line into the middle of the patch with small cuts surrounding it. You can also capacitively couple the inset feed into the patch by creating a small gap between the feedline and the patch antenna. That’s a handy way to build a capacitance into a matching network.



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Or, Feed from Back, Beware Low Z in Center

Adds a little inductance because of via length, careful of low Z in center of patch

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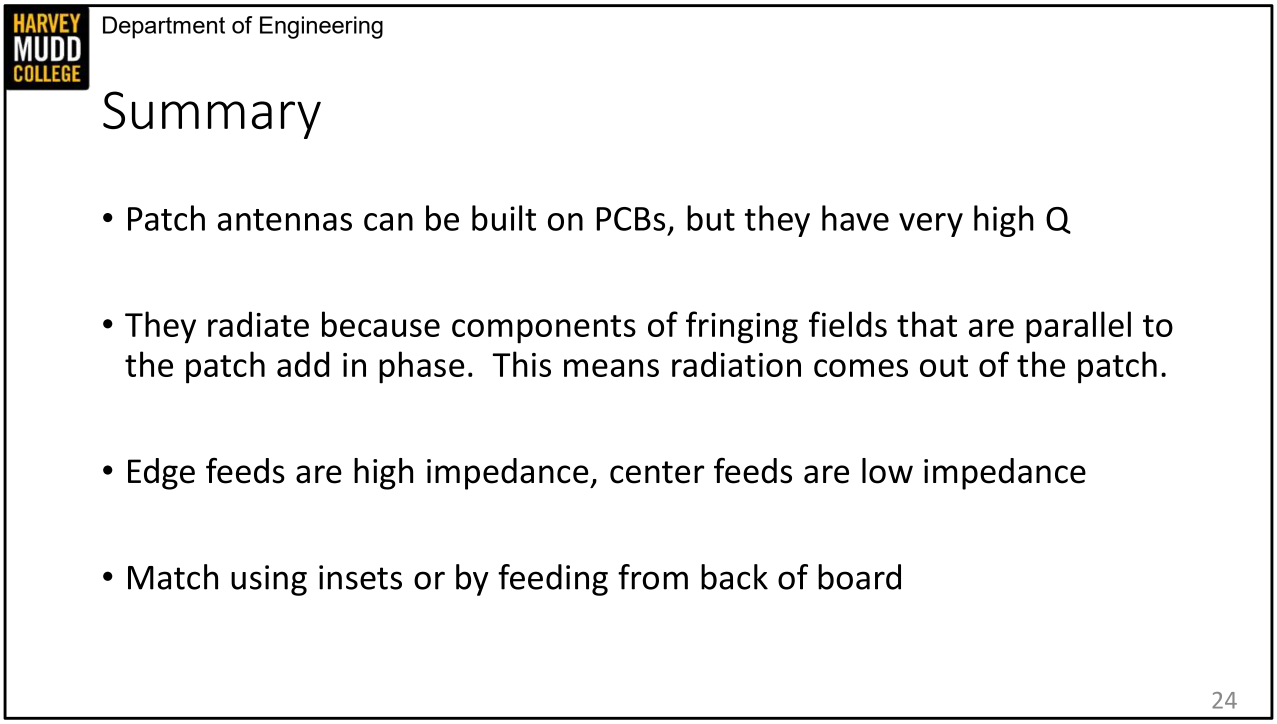
Another common technique for feeding the antenna is putting a feed connector, often an SMA connector, on back copper and extending a pin through a via to the front of the antenna. This lets you place a feed anywhere in the voltage and current standing waves that you like, including the sweet spot where the ratio of V to I is exactly 50 ohms.

However, there's a trap here for the uninitiated. It's super tempted to say "symmetry is always good" and drill your feed line right into the center of your patch. However, patches have very low impedance right in the middle because the current distribution is high and the voltage distribution is close to zero. You need to offset your feed from the center to achieve good input impedance.

However, this works well if you calculate the right offset. I have personally achieved ~51 ohm matches by picking the right spot for these back copper feeds.



Summary

- Patch antennas can be built on PCBs, but they have very high Q
- They radiate because components of fringing fields that are parallel to the patch add in phase. This means radiation comes out of the patch.
- Edge feeds are high impedance, center feeds are low impedance
- Match using insets or by feeding from back of board