Lecture 12: Power Flow

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Power Dissipated in Loads

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In this video we’re going to talk about how much power gets dissipated in loads, especially loads that have some reactance.

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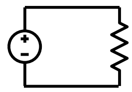
Average Power Dissipated in Load is Re{V*I}/2

$V_{zp}\cos(\omega t) = \mathrm{Re}\{V_{zp}e^{j\omega t}\}$  $Z = R + jX$

Real Valued Representation:

$$P(t) = V(t)I(t)$$

$$P(t) = V_{zp}\cos(\omega t)\,V_{zp}\cos(\omega t - \angle Z)/|Z|$$

$$P(t) = \frac{V_{zp}^2}{|Z|}\cos(\omega t)\cos(\omega t + \angle Z)$$

$$< P >= \frac{V_{zp}^2}{|Z|}\int_T \cos(\omega t)\cos(\omega t + \angle Z)\,dt$$

$$< P >= \frac{V_{zp}^2}{|Z|}\int_T (\cos(\omega t)\,(\cos(-\angle Z)\cos(\omega t) - \sin(-\angle Z)\sin(\omega t))dt$$

$$< P >= \frac{V_{zp}^2}{2|Z|}\cos(\angle Z)$$

Capacitors and inductors have angles of 90 → no power dissipated

Analytical Representation:

$$< P >= \frac{1}{2}\mathrm{Re}\{V^*(t)I(t)\}$$

$$< P >= \frac{1}{2}\mathrm{Re}\{V_{zp}e^{-j\omega t}V_{zp}e^{j\omega t}/|Z|e^{j\angle Z}\}$$

$$< P >= \frac{V_{zp}^2}{2|Z|}\mathrm{Re}\{e^{-j\angle Z}\}$$

$$< P >= \frac{V_{zp}^2}{2|Z|}\cos(\angle Z)$$

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I've drawn a very simple circuit here, just a supply driving a complex impedance, and we're going to find the power dissipated in the load in two ways to validate the analytical representation that I've referred to in earlier videos. Accordingly, I've represented the supply signal in two ways, as a real-valued quantity and as an analytic representation.

CLICK For our real-valued derivation, we start with the instantaneous power, which is the voltage across the load at time t multiplied by the current through the load at time t.
CLICK We can substitute in our real-valued supply voltage definition for V, and we can recognize that the impedance is a transfer function from voltage to current that will result in a phase shift and a scaling factor in the current value.
CLICK Pulling the constants out of the equation, we find that our instantaneous power varies as the product of two sinusoids with time. We also note that it has units of volts squared over some real number of ohms, which is encouraging.
CLICK However, we usually care about measures of average power more than we do instantaneous power, so we take an integral over one period of our sinusoids to find the average power.
CLICK That integral is simplified if we invoke the angle addition formula, which everyone definitely remembers from trigonometry.
CLICK The angle addition formula reveals that our second sinusoid can be expressed as a weighted sum of a sine and a cosine term, which is fortunate for our integral. The cosine

term will multiply with the other cosine to become cosine squared, and the integral of cosine squared over one period is one half.
CLICK The other term will be sine times cosine, and integrating that over one period has a value of zero.
CLICK That, and remembering that cosine is an even function, gives us a simple value for average power dissipated. This expression has some nice properties. Purely real loads simplify to V squared over 2R, which is consistent with results you've seen before. Further, purely imaginary results will result in the cosine term being equal to zero, indicating that no power is dissipated in ideal capacitors or inductors. One takeaway that's worth dwelling on is that power is dissipated when voltage and current are in phase. The cosine term here is showing how much the load moves current out of phase with the applied voltage.

Phew, that had a good result, but it was mathy and required me to invoke trigonometry black magic. I'm going to rederive the same result using our analytic representation to prove that we can get by with simpler computations, as long as we're willing to tolerate complex numbers.
CLICK We start with the expression I've asserted in the past, that the average power is equal to the complex conjugate of voltage multiplied by the current.
CLICK Then we substitute in our analytical representation of the supply voltage and note that we can find current by dividing the supply voltage by the load impedance. I've chose to represent the load impedance as a complex exponential here.
CLICK This simplifies nicely
CLICK and if we take the real part, we get the same result as before with less pain. We'll make a lot of use of this complex formulation of power, so make sure you're comfortable with it.

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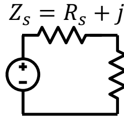
Max Power Transfer From Source if Zl=Zs*

$Z_s = R_s + jX_s$

$V_{zp}\cos(\omega t) = \mathrm{Re}\{V_{zp}e^{j\omega t}\}$  $Z_l = R_l + jX_l$

What's the average power in Zl?

$$< P_l >= \frac{1}{2}\mathrm{Re}\{V_l^*(t)I_l(t)\}$$

$$< P_l >= \frac{1}{2}\mathrm{Re}\left\{V_{zp}e^{-j\omega t}\frac{Z_l^*}{(Z_s+Z_l)^*}V_{zp}e^{j\omega t}\frac{1}{Z_s+Z_l}\right\}$$

$$< P_l >= \frac{V_{zp}^2}{2}\mathrm{Re}\left\{\frac{Z_l^*}{(Z_s+Z_l)^*}\frac{1}{(Z_s+Z_l)}\right\}$$

$$< P_l >= \frac{V_{zp}^2}{2}\frac{\mathrm{Re}\{Z_l^*\}}{|Z_s+Z_l|^2}$$

$$< P_l >= \frac{V_{zp}^2}{2}\frac{R_L}{|Z_s+Z_l|^2}$$

Maximizing average power in Zl

$$< P_l >= \frac{V_{zp}^2}{2}\frac{R_L}{(R_s+R_l)^2+(X_s+X_l)^2}$$

Control Zs?

Set Rs=0 and Xs=0, or as close as possible

Control Zl?

Set Xl=-Xs, and optimize Rl

$$\frac{d < P_l >}{dR_l} = \frac{V_{zp}^2}{2}\left(\frac{1}{(R_s+R_l)^2} - \frac{2R_L}{(R_s+R_l)^3}\right)$$

$$\frac{d < P_l >}{dR_l} = \frac{R_s - R_l}{(R_s+R_l)^3}$$ Set Rl=Rs

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One convenient application of this expression is proving a very useful theorem called the maximum power transfer theorem. This theorem tells us what values of Zs and Zl result in the most power being dissipated in Zl for the circuit pictured here.

CLICK We're going to start by finding the average power in Zl, then we're going to optimize it.
CLICK We have just proven the usefulness of analytic representations, so we're going to go ahead with an analytic representation of the load power in this problem.
CLICK We substitute in expressions for the load voltage and current in this step. The voltage on the load is set by the input driving the voltage divider between Zl and Zs, so we can see that divider ratio is conjugated in our expression here, and multipled by the conjugate of the input voltage here. We can also see that the current is set by the source voltage divided by the sum of Zl and Zs.
CLICK We factor and simplify a little bit, and we see that we have Zs+Zl multiplied by its complex conjugate in the denominator.
CLICK The product of a complex number with its conjugate is the magnitude squared, which we show in the denominator of this expression.
CLICK and finally, taking the real part of the load results in Rl in the numerator of this expression. This seems like a fine launching off point for optimization
CLICK so we copy the expression to the next column and expand out the magnitude in the

denominator to show both resistance and reactance of each impedance. This expression for load power has some obvious places to optimize, particularly in the denominator. Anything we can do to make the denominator smaller without affecting the numerator will get us closer to the maximum power.

CLICK If we control Zs then shrinking it will directly reduce the denominator without affecting the numerator. So making Rs and Xs as close to zero as possible helps.

CLICK If we control Zl then the first step is to cancel out the reactance of the source. The second is to optimize the value of Rl. Rl appears in both the numerator and denominator, which is why we need to treat it differently than Rs.

CLICK We differentiate the average power with respect to Rl here. We need to invoke the product rule, so we wind up with two terms.

CLICK Simplifying those terms, we find this expression goes to zero when Rl is equal to Rs. So we finish optimizing power by setting Rl equal to Rs. The overall load that we've designed here is referred to as a conjugate match: we want the same resistance as the source and the opposite reactance.

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Summary

- Analytical representations of V and I simplify power calculations
- Power is dissipated in loads by voltage and current that are in phase
- Maximum power is transferred from source to load if the load impedance is a conjugate match of the source impedance
  - If you control the source, minimize Zs
  - If you control the load, conjugate match Zl to Zs

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Power Transfer Through Transmission Lines

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In this video we're going to increase the complexity of our modeling of power flow by adding transmission lines into our model. We'll also talk about representing power on a log scale.

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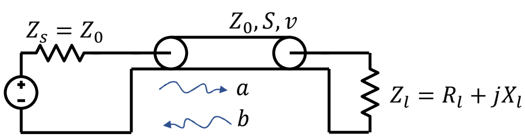
Load Power Can Be Found From Wave Powers

$V_{zp}\cos(\omega t + kS) = \mathrm{Re}\{V_{zp}e^{j(\omega t + \quad)}\}$



Calculate load power from load voltage & current

$$< P_l >= \frac{1}{2}\mathrm{Re}\{V_l^*(t)I_l(t)\}$$

$$< P_l > = \frac{1}{2}\mathrm{Re}\left\{\frac{V_{zp}}{2}e^{-j\omega t}(1+\Gamma)^*\frac{V_{zp}}{2Z_0}e^{j\omega t}(1-\Gamma)\right\}$$

$$< P_l > = \frac{V_{zp}^2}{8Z_0}\mathrm{Re}\{(1+|\Gamma|e^{-j\angle\Gamma})(1-|\Gamma|e^{j\angle\Gamma})\}$$

$$< P_l > = \frac{V_{zp}^2}{8Z_0}(1-|\Gamma|^2)$$

Calculate load power from wave conservation

$$< P_{wave} > = \frac{1}{2}|a|^2 = \frac{V_+^2}{2Z_0} = \frac{V_{zp}^2}{8Z_0}$$

$$\frac{1}{2}|a|^2 = \frac{1}{2}\mathrm{Re}\{V^*I\} + \frac{1}{2}|b|^2$$

$b = \Gamma a \rightarrow |b|^2 = |\Gamma|^2|a|^2$ ← Reflected power

$$\frac{1}{2}\mathrm{Re}\{V^*I\} = \frac{1}{2}|a|^2(1-|\Gamma|^2)$$

$$< P_l > = \frac{V_{zp}^2}{8Z_0}(1-|\Gamma|^2)$$

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I've drawn the circuit we'll be considering here, it's similar to the circuit we used to prove the Maximum Power Transfer Theorem, but we've added a transmission line between the source and the load and replaced Zs with a purely real impedance matched to the line. We'll take a look at what happens when Zs isn't matched in a later video, but for now it adds complexity without adding much understanding.

CLICK We're going to calculate power delivered to the load in two different ways to show that (1) they're the same and (2) the second way is pretty quick.
CLICK We know power in the load is going to be given by the complex conjugate of V times I.
CLICK And we can represent those values in terms Gamma and the wave amplitude, which is 1/2 of the generator amplitude because our source is matched. Recall that the voltage at the load is given by the sum of waves, so it's proportional to (1+Gamma), and the current is given by the difference of the waves, so its proportional to (1-Gamma). Also, note that we find the current of the forward going wave by dividing the amplitude of the voltage wave by Z0.
CLICK We can factor out Vzp and the factors of two to get a distinctive coefficient of Vzp squared over 8Z0. This coefficient is all over power flow calculations, so it's comforting when you find it. Notice that the factor of 8 comes from three factors of two: one from finding average power when driven by a sinusoid, and two from the amplitude of the

forward going wave being half of the generator voltage. We've also expressed Gamma in its complex exponential form, which has let us find the complex conjugate where needed. CLICK The multiplication of these two Gamma terms leaves us with one minus the magnitude of Gamma^2. This is an intuitive result: high reflection coefficients should reduce the amount of power in the load. We're going to hone that intuition further by deriving this from conservation of power in the system.

CLICK Leaning on power conservation requires us to remember how much power is in a wave. Fortunately, we defined a to have an easy relation to power. Specifically power is one half of the magnitude of a squared. a was defined as the right travelling wave amplitude squared over two Z0, which is equal to Vzp squared over 8Z0 in this case.
CLICK Power conservation states that any power carried into the load by wave a needs to either be dissipated or reflected out on wave b
CLICK And we can express how much power is reflected in terms of reflection coefficient. We know b is Gamma times a because Gamma is defined as V- over V+, and if you multiply both sides of that equation by their complex conjugate you can find an expression for the magnitude of b squared. That value is the reflected power, and it's directly dependent on gamma.
CLICK Rearranging our conservation equation, we find that the power in the load has to be the incident power multiplied by 1 minus the magnitude of Gamma squared
CLICK and substituting for incident power, we find the same result as we had in the left column. However, by using the magnitude of Gamma squared as a kind of power gain, we were able to skip all the complex math. That's handy here, and it's really handy when working with measured quantities in lab, as we'll see on the next slide.

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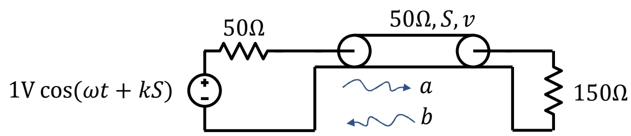
It’s Natural to Talk About Power Flow in dBm



Linear calculation of reflected power

$$< P_a > = \frac{1}{2}|a|^2 = \frac{1}{2}\left(\frac{1V}{2}\right)^2 \frac{1}{50} = 2.5\text{mW}$$

$$\Gamma = \frac{150-50}{150+50} = 0.5$$

$$< P_b > = \frac{1}{2}|a|^2|\Gamma|^2 = 2.5\text{mW} \cdot 0.25 = 0.625\text{mW}$$

$$< P_l > = \frac{1}{2}|a|^2(1-|\Gamma|^2) = 2.5\text{mW} \cdot 0.75 = 1.875\text{mW}$$

dB & power gain calculation of reflected power

$< P_a > = 4$ dBm 10*log(P/1mW)

$RL = 20\log(\Gamma) = -6\text{dB}$ Power ratio is Gamma^2

10^(-2/10)=0.63

$< P_b > = < P_a > + RL = -2$ dBm

Logs don’t add, so dissipation is tough to find

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In the spirit of this video set, we’re going to try calculating the power dissipated in the 150 ohm load in this example in two different ways.

CLICK we’ll try doing normal multiplying and adding first, and then we’re going to see if we can go even faster using logarithms.
CLICK The power in our incident wave is one half of the magnitude of a squared, which we find to be 1/4 of a milliWatt. As an aside, I find it handy to remember that the leading 1/2 and the 1/Z0 factor combine to a factor of 1/100 in 50 ohm systems.
CLICK Gamma is 0.5 for this example
CLICK So the reflected power is going to be one quarter of a milliwatt times 0.5 squared, which is 1 sixteenth of a milliwatt
CLICK The rest of the power goes into the load, so it receives 3 sixteenths of a milliwatt.

Fine, that seemed easy enough, but it can get kind of tough to carry power levels that are minute fractions of a milliwatt around in our heads. So we’re going to try this again with logarithms.
CLICK The power in this incident wave is -6 dBm. That’s a unit you might not have used a lot, but it means decibels relative to a milliwatt, and because decibels are defined by 10 times the log of a ratio of powers, you can think of it using the expression I’ve written on the right. I didn’t actually use that expression to calculate this value though: I just

remembered that -3dB corresponds to a factor of 1/2 in power, and we had two of those from our expression for wave power on the left.
CLICK Rather than finding the reflection coefficient for this approach, we're going to find a quantity called the return loss. This is the ratio of reflected power to incident power expressed in deciBels, and note that because power is proportional to Gamma squared, we're using 20 log in this expression rather than the 10 log we use for power quantities.
CLICK Logs are great because we can turn tricky multiplications into addition. We know the incident power is -6dBm, and we add our return loss to that to find that reflected power is -12dBm. That was super easy to do in our head! decibels and power gains remain popular in practice because you can add these log quantities in your head rather than having to multiply.
CLICK In case you can't do the conversion right off hand, 10 raised to the -12 over 10 is 0.063, so this answer gives us the same result as the left column.
CLICK However, logs have a downfall. We can't just find the difference between -6dBm and -12dBm to determine how much power is in the load because logs don't add that way. You have to convert your results back to linear quantities to use conservation relationships.

One final note, definitions of return loss are inconsistent, and it's sometimes defined as a positive quantity that you have to subtract from your incident power. This is fine, and it's easy to convert between definitions by remembering that your reflected signal should probably be smaller than your incident signal. Just make sure to read context clues when you're dealing with return loss

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Summary

- Wave power conservation is an easy way to calculate load power.

$$\frac{1}{2}|a|^2 = \frac{1}{2}\mathrm{Re}\{V^* I\} + \frac{1}{2}|b|^2$$

- Return loss is the reflection coefficient expressed in dB
- Power gains in dB make it easy to find power levels in your head

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Power Gain

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E157 – Radio Frequency Circuit Design

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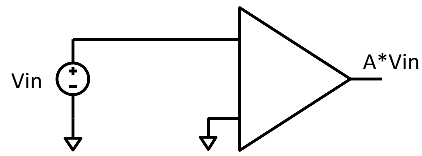
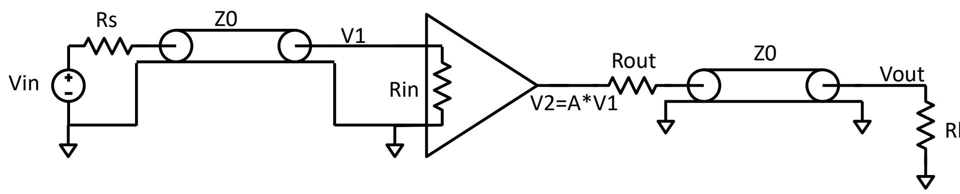
In this video we're going to introduce a new measure of how effective an amplifier is at making a signal bigger called power gain, which is relevant because we care about how power flows through S parameter networks, especially amplifiers. However, we're going to skip the S-parameters in this video so that we can get a sense of power flows without worrying about complex math. We'll add S-parameters to our power gain analysis in a future video.

Power Gain Limits Performance w/ Small Load



Voltage gain is A



$$\frac{V_{out}}{V_{in}} = \frac{R_{in}}{R_s + R_{in}} A \frac{R_l}{R_{out} + R_l}$$

One of the first things to discuss is the need for power gains at all. You've used amplifiers before, and we've mostly thought of a voltage source driving them, and the amplifier providing some voltage gain to that voltage source. For instance, this is probably how your learned to think about op-amps. Voltage gains seemed to work great up until now.

One argument against voltage gains is that we've been examining how convenient it is to do math with power. That math lets us avoid thinking about complex exponentials and it lets us compare signals of very different power levels using logs. That argument still holds water.

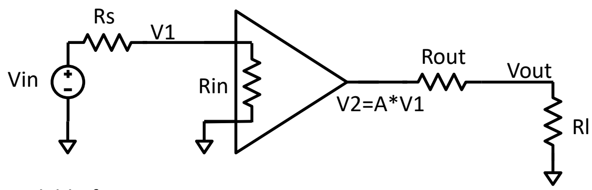
CLICK But this first model hides some details, so we need a more detailed model. Our first model presumes that the input impedance of the amplifier is really high, so that the amplifier doesn't load the voltage source. That doesn't work at RF because terminating a transmission line on an open will result in reflections, and reflections make designs complicated in a lot of ways. For instance, you have to care about the length of your cable. Instead, we'll usually want our source impedance and our input impedance to both match our line, which means dealing with the relatively low impedance level of 50 Ohms.

CLICK We have the same problem at our output. Rout and Rl are probably pretty close to Z0.

CLICK: Since we've matched our cables, we know that eventually all our voltages will settle to levels that are consistent with a circuit where the cables didn't exist, so we can find a loaded Vout/Vin using voltage dividers. This loaded voltage gain is smaller than the amplifier's voltage gain of A. So we'd like to make sure that our metric for gain takes source and load impedances into account somehow.

The next question might be: "Why isn't loaded voltage gain a good enough metric? Why go to power gain?" There are several reasons. As I mentioned above, power gain is convenient for many analyses because we can simplify our analysis by thinking about gain magnitudes and discarding phase information. Power gain is also useful when the input and output have very different impedance levels. Finally, power gain lets you capture information about gain even when your signals are currents and your gains are current gains, so it's a universal language for lots of types of circuits. This is particularly important because, voltages and voltage gains are often small at high frequencies, which makes them hard to measure.

Power Gain is Slippery Because of Power Defn



Power from the source

$$P_s = P_{RIN} = \frac{V_1^2}{R_{in}} = \frac{V_{in}^2}{R_{in}}\left(\frac{R_{in}}{R_s + R_{in}}\right)^2$$

Power available from the source if matched

$$P_{avs} = P_{RIN}|(R_S = R_{IN}) = \frac{\left(\frac{V_{in}}{2}\right)^2}{R_{in}} = \frac{V_{in}^2}{R_{in}} \cdot \frac{1}{4}$$

Power to the load

$$P_l = P_{RL} = \frac{V_{out}^2}{R_L} = \frac{V_2^2}{R_L}\left(\frac{R_l}{R_{out} + R_l}\right)^2 = \frac{A^2 V_1^2}{R_L}\left(\frac{R_l}{R_{out} + R_l}\right)^2 = \frac{A^2 V_{in}^2}{R_L}\left(\frac{R_{in}}{R_s + R_{in}}\right)^2 \left(\frac{R_l}{R_{out} + R_l}\right)^2$$

Power available to the load if matched

$$P_{avl} = P_{RL}|(R_L = R_{out}) = \frac{\left(\frac{V_2}{2}\right)^2}{R_l} = \frac{A^2 V_1^2}{4R_l} = \frac{A^2 V_{in}^2}{R_L} \cdot \frac{1}{4}\left(\frac{R_{in}}{R_s + R_{in}}\right)^2$$

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However, we're going to hit a sticking point almost immediately as we try to calculate power gain: there are lots of ways that we can define power. I've shown four options on this slide: the power pulled from the source, the power available from the source if the amplifier input is source matched, the power delivered to the load, and the power delivered to the load if the output was load matched. You may notice that I've also omitted the transmission lines from the schematic on this slide. That's fine because we're just using this as a simplified example, so we can imagine that we're calculating power gains at DC where the transmission lines don't matter. We'll include transmission lines when we think about power gains with S parameters later. Let's calculate these powers!

CLICK The first is the power from the source. This is the power that came out of the source and makes it into our amplifier. It's given by the voltage on Rin squared over Rin. The voltage across Rin is set by a voltage divider attached to Vin, so we get a big squared ratio in this expression. Note that you could measure this value by measuring V1, even if you didn't know Rin exactly.
CLICK We'll also consider the power available from the source, which is the power that would come out of the source and into the amplifier in the best-case scenario where the amplifier is source matched. We know that a matching impedance is the best case scenario because of the max power transfer theorem. Since we know Rs and Rin are the same in this case, we know the voltage on Rin is Vin/2, so squaring appropriately shows this is

Vin^2/4Rin. This is exactly half of the power delivered from Vs, again by the max power transfer theorem.
CLICK We need to compare source power to power dissipated in the load, which is given by the voltage across the load squared divided by Rl. We can relate this power back to Vin by tracing back through the dividers and the amplifier. The load voltage is set by a divider between the amplifier output, V2, and Vout. V2 is A*V1. And V1 is given by a divider between Vin and V1. Squaring all these gains and dividers gives a sprawling expression for the load power.
CLICK Finally, the power available to the load is the best-case power delivered to the load when Rout is matched to Rl. As with the power available from the source, half of the power that comes out of the amplifier winds up in the load in this case. The equations I show here track some dividers back through the circuit to show a relation between the quantity and Vin. That quantity depends on the Rs to Rin divider because we only assume the load is matched when we calculate this power.

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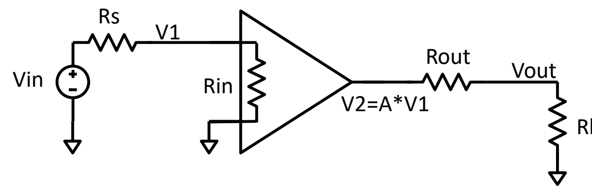
Power Gains Differ in Accounting for Matches



$$P_S = \frac{V_{in}^2}{R_{in}}\left(\frac{R_{in}}{R_s + R_{in}}\right)^2 \qquad P_{avS} = \frac{V_{in}^2}{4R_{in}} \qquad P_L = \frac{A^2 V_{in}^2}{R_L}\left(\frac{R_{in}}{R_s + R_{in}}\right)^2\left(\frac{R_l}{R_{out} + R_l}\right)^2 \qquad P_{avL} = \frac{A^2 V_{in}^2}{4R_L}\left(\frac{R_{in}}{R_s + R_{in}}\right)^2$$

Power gain

$$G_P = \frac{P_L}{P_S} = A^2 \frac{R_{in}}{R_L}\left(\frac{R_l}{R_{out} + R_l}\right)^2$$

"Power into load over power into the amp"

Forgives input mismatch
Easy to measure

Transducer gain

$$G_T = \frac{P_L}{P_{avS}} = 4A^2 \frac{R_{in}}{R_L}\left(\frac{R_{in}}{R_s + R_{in}}\right)^2\left(\frac{R_l}{R_{out} + R_l}\right)^2$$

"Power into load assuming no matching over the input power assuming a source match"

Most pessimistic, affected by source and load match
Most commonly used

Available gain

$$G_A = \frac{P_{avL}}{P_{avS}} = A^2 \frac{R_{in}}{R_L}\left(\frac{R_{in}}{R_s + R_{in}}\right)^2$$

"Max power into the load for a given source power"

Forgives load mismatch, "what can I get out of my network?"

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Now that we have some interesting powers, which I've copied over from the last page, we can think about taking ratios of them to make gains.

CLICK The first power gain is probably the most intuitive: power into the load divided by power from the source, called power gain or GP. The expression for this is easy to find by comparing the PL and PS expressions and looking for terms that knock out: the Vin^2 terms cancel, Rin and RL become a ratio, and the RS to Rin divider cancels from both expressions.
CLICK I've included a brief sentence describing power gain in words to try to make it easier to apply in practice. This gain is the ratio of power into the load to power that makes it into the amplifier.
CLICK Consequently, this gain doesn't account for output power we are leaving on the table by not matching the input. On the upside, it's relatively easy to measure by finding V1 and Vout.

CLICK The second power gain we care about is called the transducer gain, GT, and it attempts to rectify some of the issues with GP. The expression again comes from a ratio of the powers on the page, and because PavS assumes an ideal match while PL does not, none of the divider terms in PL cancel.
CLICK Describing this gain in words, it's the power into the load assuming the input is source matched. Or you could say that it's the power into the load divided by the best

power that we could get from the source.
CLICK Comparing against a power that assumes a perfect input match makes GT more pessimistic than GP. It is affected by the quality of both the input match, to see if the real system input power lives up to PavS, and by the output match because that is baked into PL. This definition is the most common gain reported for RF systems because it cares about both input and output matching.

CLICK Finally, we sometimes would like to know the best power gain we could achieve, which is the Available Power Gain or GA. This expression cares about the Rin to RS divider because that affects PavL, and isn't cancelled by PavS.
CLICK So, said in words, this is the power into the load assuming the load (but not the source) is matched, divided by power into the input assuming the source is matched.
CLICK This is mostly nice as an intermediate gain that captures how well the input network is doing.

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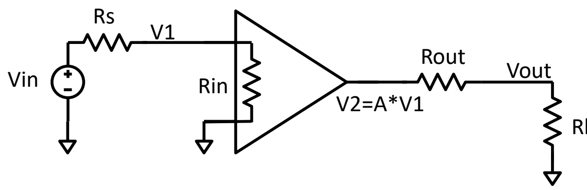
Power Gains All Equal if In and Out Match



$$P_S = \frac{V_{in}^2}{R_{in}}\left(\frac{R_{in}}{R_s + R_{in}}\right)^2 \qquad P_{avS} = \frac{V_{in}^2}{4R_{in}} \qquad P_L = \frac{A^2V_{in}^2}{R_L}\left(\frac{R_{in}}{R_s + R_{in}}\right)^2\left(\frac{R_l}{R_{out} + R_l}\right)^2 \qquad P_{avL} = \frac{A^2V_{in}^2}{4R_L}\left(\frac{R_{in}}{R_s + R_{in}}\right)^2$$

Power gain

$$G_P = \frac{P_L}{P_S} = A^2\frac{R_{in}}{R_L}\left(\frac{R_l}{R_{out} + R_l}\right)^2$$

Transducer gain

$$G_T = \frac{P_L}{P_{avS}} = 4A^2\frac{R_{in}}{R_L}\left(\frac{R_{in}}{R_s + R_{in}}\right)^2\left(\frac{R_l}{R_{out} + R_l}\right)^2$$

Available gain

$$G_A = \frac{P_{avL}}{P_{avS}} = A^2\frac{R_{in}}{R_L}\left(\frac{R_{in}}{R_s + R_{in}}\right)^2$$

Let $R_s = R_{in},\ R_{out} = R_l$

$$= \frac{1}{4}A^2\frac{R_{in}}{R_L} \qquad = \frac{1}{4}A^2\frac{R_{in}}{R_L} \qquad = \frac{1}{4}A^2\frac{R_{in}}{R_L}$$

Max unilateral transducer gain: $G_{TUmax} = P_{avl}/P_S$

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Last slide my thesis was that different types of power gains accounted for different types of matching. So, perhaps it's unsurprising that all of these gains assume the same value if your system is matched at the input at the output.

CLICK So if we assume Rs is equal to Rin and Rout is equal to RL

CLICK We find that both GP, GT and GA assume the same value. Each of the divider ratios becomes one half, and squaring that yields one fourth. The four in GT cancels out with one of those factors.

CLICK This quantity is called the maximum unilateral transducer gain, and you might note that you can also find it by dividing Pavl by PS. This is a handy upper bound when you set out to design matching networks.

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Summary

- Power gain is important because it accounts for impedances of sources and loads.

- We care about four powers:
  - Power from the source, P_S
  - Power available from the source if input is matched, P_{avS}
  - Power to the load, P_L
  - Power available to the load if output is matched, P_{avL}

- We care about three types of power gains
  - P_L/P_S – Power gain, neglects input mismatch
  - P_L/P_{avS} – Transducer gain
  - P_{avL}/P_{avS} – Available power gain, neglects output mismatch
  - P_{avL}/P_S – Max unilateral transducer gain, neglects both matches ,handy upper bound

- All power gains are the same if the input and output are matched.