

Department of Engineering

Lecture 11: Measuring S Parameters

Matthew Spencer Harvey Mudd College E157 – Radio Frequency Circuit Design



Most of the videos today talk about practical details of measuring S parameters, but in this video we're going to introduce one bit of useful theory that helps us understand measurement errors.



So let's imagine that decide to put two, two-port networks in a row, which might be the case if we decide to cascade some amplifiers. We'll call the first network S_X and the second S_Y, and we're calling them that because we are describing the two ports using S parameters. It is common to describe high frequency networks with S parameters because they're easy to measure, and most high frequency components will give you a set of S parameters in the datasheet.

I've drawn that situation on this slide, and it looks OK thus far. Each S network has its own set of a and b vectors, and the a and b vectors are related by the S matrices at each two port.

CLICK But there's an annoying sticking point if we want to combine these S parameter networks into one big matrix description (for instance, if we want to find the total gain). We have to do some footwork with a2x, b2x a1y and b1y. The waves are clearly related to one another, but not in a clean way. For instance, it looks like b2x becomes a2y. So we might need to some kind of rotation or transposition matrix betweek S_X and S_Y if we want to relate a1x and b1x to a2y and b2y. That's annoying, but the heart of the problem is that the S parameter matrices assume that waves are always going into them. Rearranging the matrices might make this much easier.

CLICK Which is where T parameters come in. T parameters, or transfer parameters, rearrange the a and be waves that are in the S parameter matrix, so that the T matrix describes the behavior at port two in terms of the waves at port 1. You can use a T parameter matrix to propagate waves from one side of a 2 port to the other, and we'll see how on the next slide. Quick note, this definition might look backwards to you – the matrix is transforming the b2/a2 signals on the right side of the picture to the b1/a1 signals on the left, and we're used to seeing signals or transformations flow from left-to-right. This convention makes some S-parameter to T-parameter conversion math a little easier, which is why we use it here. But it's important to remember that a T parameter matrix will propagate signals from port 2 to port 1, which is to the left in our drawings.



I've rewritten the matrix descriptions of our two-ports as T parameters. Let's assume we're still trying to find any linear matrix that relates a1x and b1x to a2y and b2y.

CLICK We might start with the sticky spot we identified on the previous slide. We can write the relationship between the a2X/b2X waves and the a1Y/b2Y waves on the middle transmission line as a vector equation.

CLICK Each of the vectors in this equation appears in our T parameter definitions, so we can plug the T parameter matrices times the outermost a1X/b1X and a2y/b2Y vectors to propagate our equation from the middle line to the outermost ports. This step shows how we can use T matrices to move wave relationships across a network, which we can't do with S parameters. We've taken the relationship in the middle of our network and propagated it outward across the T matrices. This is especially clear on the left side of the equation, where the T matrix times the port 2 signals replaces the port 1 signals. On the right side we had to use a matrix inverse because we want to imagine our port 1 signals going to port 2, but the principle is the same.

CLICK We can multiply both sides by the inverse of the T_X matrix to make this equation a bit cleaner

CLICK And in doing so we reveal how to achieve our goal of merging T_X and T_Y into a single matrix. The combined transfer matrix that takes b2y/a2y as an input and produces b1x/a1x is T_X times T_Y. Here we multiply the individual transfer matrices right-to-left, which is the opposite direction of the port2-to-port1 signal flow from a2y/b2y back to a1x/b1x. I call this out because my memory tool for this is noting that I need to apply TY to a2y/b2y first, then TX, and that matrices are applied to a vector from left-to-right.



Like all linear descriptions, there are transforms between T parameters and S parameters, and I've included those here. These could be used to turn our Tcascade from the previous page into a S parameter description of the combined amplifier network.

I've also got two quick asides to mention. The first is that there's a low frequency version of T parameters that plays well with our Z and Y parameters. These are called ABCD parameters, and their definition is given here. The second is to call out that there's an alternate definition for T parameters. We spent a lot of time noting that our T matrix definition propagated signals from port 2 to port 1, but there's an alternate definition out that there port 1 to port 2, and I've included it here.

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Summary

• T parameters relate port 1 to port 2 instead of describing reflections at each port like S parameters.

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$

• Multiplying by T parameters will move a signal from port 2 to port 1

• Cascaded T parameter networks multiply: $T_{cascade} = T_X T_Y$

• T-to-S and S-to-T conversion formulas exist.



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Fixturing and The Reference Plane

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In this video we're going to talk about practical considerations for measuring S parameters that arise from the wires used to measure a device under test, or DUT.



We first defined S parameters assuming that we'd need to measure some two-port network at high frequency, so our setup is that we need to measure some device under test, and that we have to attach transmission lines to port 1 and 2 because we want to model the effects our wires have on high frequency signals. Let's further assume that we have some fancy instrument that lets us control Vs1, Vs2, Zs1, and Zs2.

This setup puts us in a tough spot because we can only measure the S parameters of a combined system: our DUT plus the wires we used to connect it to our instrument. That's a shame, because we'd really like to know S_dut without the wires. So let's start figuring out how to model the wire effects by defining some terms. First, these wires, along with all the adapters and connectors that are attached to them, are referred to as fixturing.

CLICK Second, we're going to define separate sets of a/b waves at the instrument and at the DUT. I gave the waves into and out of the instrument the subscript VNA, and the waves at the DUT the subscript DUT. The subscript VNA stands for vector network analyzer, which is the name of the fancy instrument that I mentioned earlier.

CLICK If we care about these wires, then we need to define a few more properties for them, so we assign lengths S1 and S2 and velocities v1 and v2 to them. The VNA won't know these properties in advance, so we're going to have to figure out how they affect our

measurement. CLICK I'm also going to remind you that we define x=0 at the DUT and x=-S1 at the test instrument.

CLICK This might look hard to model at first glance, but our work on T parameters is a huge help here. We can imagine move our signals from dut port 1 left on the page to vna port 1 using the T parameters of the transmission line, and we can move our waves at DUT port 2 right on the page to VNA port 2 using variant T parameter.

CLICK To figure out what the variant T parameters are, we need to start by finding S parameters of a lossless line. Here I'm just going to appeal to your intuition: a matched, ideal transmission line won't have any reflections, so S11 and S22 are zero. S12 and S21 are just going to delay incident signals before they leave the other port, so the S12 and S21 values just add some phase.

CLICK We can use the S-to-T formulas from the last video to find the T parameters for an ideal transmission line. Because the S parameter matrix is symmetric, it turns out the variant T parameters are the same as the normal T parameters. Fortunately, both of these T parameter formulas are easy to deal with because they are a constant time the identity matrix.

CliCK So substituting these T parameter values into our original formula reveals that our Svna measurements are related to Sdut by multiplying in the phase of the fixturing.

CLICK This is pretty bad news. It means the S-parameters we measure will have an evolving phase on top of the behavior of the DUT, which I've pictured here. It's often difficult to back DUT phase behavior out of the phase of the fixturing, so we need to find a way to remove the effect of the cables. Fortunately, the math is simple: if we can find the value of kS, then we can cancel out the extra phase we've picked up from the transmission line. Don't forget that S1 and S2 are defined as negative numbers, so we should add positive phase to cancel out the delay of the transmission lines.



Fortunately, we can attach shorts to each wire to find the value of kS, a process that is called a short calibration. One algorithm for doing so, which was originally developed in the days of the slotted line, relies on easy-to-measure nulls in the standing wave pattern. By varying the drive frequency of Vs1 and Vs2, we can change k, which in turn changes the number of periods of the standing wave pattern that are standing on the transmission line. Whenever there's a null in the voltage at Vs1, we know that an integer number of half-wavelengths are on the line because we're effectively seeing a short as our driving point impedance. Using that fact, if we know the frequency of two adjacent nulls then we can find kS as a function of the frequency. That requires a few algebraic manipulations of the wave number, but I'll leave the process to the viewer to work out.

Phase is always measured relative to some reference signal, so when we cancel out the phase introduced by the fixturing, we're effectively saying that we consider x=0 as the point where our signal has zero phase. The point where your signal has zero phase is called the reference plane. By calibrating our system we've effectively moved the reference plane from x=-S to x=0.



The fixturing also usually has a frequency response due to attenuation and resonance in imperfectly matched connectors. However, you can cancel out the frequency response of fixturing by connecting the two wires attached to ports 1 and 2 directly to one another, which is called a through calibration. The VNA can measure the combined frequency response of the two wires and store that information to cancel it out later. Note that this type of calibration is very specific to the wires and connectors that you're using.





In this video we're going to learn about an important circuit called a directional coupler that's at the heart of vector network analyzers.



Directional couplers are matched 4 port networks, and the four ports are named Input, Through, Isolated and Coupled. The Isolated port isn't used much: it is usually just used to sink power. That might seem wasteful, but it turns out that 3 port networks can't be simultaneously matched, reciprocal and lossless because of some vagaries of matrix math, so we need that port for this device to work well.

The first function of the directional coupler is already illustrated on this slide. A wave incident on the input port will appear almost in full on the through port. That's not much more interesting than transmission lines, but if we have a wave incident on the through port, then something interesting happens. CLICK Most of the signal from the through port appears on the input port, but some fraction of the a2 wave appears on the coupled port. The coupling coefficient, or C, sets the fraction of the a2 wave that gets coupled.



That's weird behavior! And it takes accordingly weird circuits to accomplish it. At low frequencies, up to about 600 MHz, we can add and subtract voltages and currents using a pair of cross-coupled transformers to create voltages that are proportional to wave amplitude on the coupled and isolated ports. We create these voltages by making the transformers mimic the equations for wave amplitude, which we derived in a previous video. However, transformers are hard to build at high frequencies, so we need to rely on some tricky electromagnetic effects instead. In the directional coupler shown here, a voltage wave from the input to the through will result in some field lines landing on the wire between the isolated and coupled ports. That forces a reverse travelling wave to emerge on isolated-to-coupled wire in order to cancel out the wave induced by the forward travelling input-to-through wave.

These couplers can be designed for different coupling coefficients and relative phases on the ports. Also, its possible to make some interesting behaviors by swapping around what port is used the input. For example, if you set coupling coefficient to one half and drive an input on the through port, then half of that signal would come out on each of the input and coupled ports. Said another way: the through signal gets evenly split between two other ports, while keeping all the ports matched. A circuit that does that splitting is called a splitter, and many splitters and directional couplers are part of a broader class of circuits called hybrid couplers.



Finally, we need to discuss and important parasitic in directional couplers. Directional couplers aren't perfect devices, so they can leak signals between their ports. Directivity, which we give the symbol epsilon, refers to how much of the signal on the input port appears on the coupled port. In a perfect world epsilon is zero, but in the real world, some of the input signal appears directly on the coupled port. This can swamp out the signal coupled from a2 if a1 is a large signal.





In this video we're going to discuss the operation of a vector network analyzer.



This picture shows a simplified vector network analyzer. It features a signal source, a pair of directional couplers and a few termination resistors that are matched to the system impedance. Two sets of switches are turned on and off in sync to configure the behavior of the system. If the A switches are on, then the generator signal will propagate forward through the lower directional coupler. The generator signal will interact with our two-port DUT generating a transmitted and a reflected wave, and those waves will each hit a directional coupler, resulting in scaled versions of the transmitted and reflected signals appearing on the coupled ports of both the top and bottom directional couplers respectively. Flipping the switches to the B configuration will run a signal through the upper directional coupler, that will also result in transmitted and reflected signals appearing on the directional couplers, but those signals will be a result of the other half of the S-parameter matrix. Comparing the coupled signals to the amplitude to the incident wave allows you to calculate each of the four S parameters.



The components in the VNA can introduce errors into the VNA's measurements. We've already talked about how directivity in the directional couplers could be an issue, so I've left it off this slide for now, but we're going to discuss another potential issue called source mismatch.

CLICK this incident wave a1 will have some reflection off the DUT, and that signal will go back from the through port to the input port. That's fine, we expect that the signal will be terminated by Zs, which we assume is matched to ZO. CLICK However, if there is some mismatch, then some signal will get reflected back towards the directional coupler. We use the symbol delta to represent the fraction that gets reflected as a result of this source mismatch. CLICK The reflected mismatch signal passes through the directional coupler, creating an error as a result of finite directivity. CLICK The reflected mismatch signal also passes to the through port, and then gets reflected off of the DUT and coupled out as well. There are other terms from additional reflections off the source port, but we're going to neglect them by assuming delta squared is small.

Source mismatch matters most when the DUT is highly reflective, so that Gamma is close to one.

Department of Engineering HARVEY MUDD College Summary • Vector network analyzers use directional couplers to measure transmitted and reflected waves. • VNAs switch between driving and terminating ports to measure all Sparameters • Source mismatch adds directly to error, especially if Γ is high.



In this video we're going to discuss calibration methods for VNAs that fix the measurement errors we've discussed in previous videos. The fact that VNAs can be precisely calibrated is one of the most important features of the device, and good calibrations enables the RF measurements necessary to make sensitive receivers. Those receivers power applications from cell phones to satellites, so these calibrations are at the heart of much of the modern world.



The most common kind of calibration is called a short-open-load-through, or SOLT, calibration. Each of the SOLT calibration standards deals one of the issues that we've identified in the last few videos. As detailed previously, shorts let us correct for the delay of fixturing and throughs let us correct for the frequency response of fixturing. The remaining errors inside the VNA are all linear, so we only need two more measurements in order to calibrate them out. By convention, we choose an open circuit and a termination that matches the characteristic impedance of the system. We use the open first to calibrate out things that depend on high reflectivity like source mismatch, then we use a load to observe the remaining signal error when there is little to no reflection.

Often you'll achieve this calibration by screwing manual calibration standards onto your cabling, and I've included a picture of some of those calibration standards. However, there are instruments called ecals (which is short for electronic calibration), that automate the process by letting the VNA control switches between internal standards. They save a lot of time, but they are expensive and offer you the opportunity to destroy many standards and RF switches in one fell swoop if you are careless when using them. Even worse, when they fail they often fail silently, corrupting data rather than ceasing to work. So be careful with your ecal!



Calibration has the effect of moving the refence plane, so a SOLT calibration with the standards on the previous page will result in a reference plane at the end of your cables. However, devices on printed circuit boards will often have some microstrip transmission lines between the connectors and the device under test. That can be fixed: you can use other calibration techniques and carefully built test structures to move your reference plane onto a PCB or even an integrated circuit. For example, I've included a picture of a common set of calibration standards for devices on printed circuit boards. These are called through-reflect-line or TRL standards. As you can see from the Through and Line standards, this calibration relies on two through measurements of transmission lines with slightly different lengths. Line-reflect-match (or LRM) standards are popular for integrated circuits, but they rely on an expensive broadband match to operate correctly.

If you don't have access to on-board calibration structures, you can compensate for a misplaced reference plane by using the Port extension feature included with many modern VNAs. The port extension feature lets you specify a distance that you'd like to adjust the reference plane, and the VNA uses the existing calibration to add more phase to your measurement. This presumes that the additional line that you virtually add to your port looks like the transmission line you used for calibration, which isn't a great assumption when transitioning onto microstrip, so calibrate in hardware as much as possible.

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Practical Considerations Affect Calibration Calibration is specific to cables Cables are easy to break, and be especially careful of bad center pins Mechanical connections must be controlled, so use a torque wrench. Minimize disconnecting and reconnecting a system. Calibration is specific to a frequency range, we can achieve ~500MHz Let the VNA warm up, temperature changes can affect calibration accuracy.

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These are clever calibration methods, but they only work if we're mindful when we use them, because it's possible to introduce errors into your measurement that's swamp out your careful calibration. I'm going to list a few here.

CLICK The delay and frequency response of your fixturing depend on the specific cables used during calibration, so you need to recalibrate if you change cables.

CLICK That's a potential problem, because RF cables are fragile. Cables are sensitive to minor changes in characteristic impedance, so it's important not to bend them sharply and to relieve strain as often as possible. Metallic or rigid cables are tempting to reduce these cabling issues, but it's hard to prototype with them. Even worse, some cheap RF cables will have malformed center pins that damage other cables or ports they are plugged into. One bad cable can poison the data in a whole lab, so be mindful of your cables and don't be shy about paying for them.

CLICK VNAs are sensitive enough that the mechanical connection between cables will appear in your results. So use torque wrenches on every connection you make to ensure that they are all identical. This also minimizes damage to cables and connectors, which can be harmed by overtightening.

CLICK On that note, plugging and unplugging a connection can disturb a calibration, so minimize it.

CLICK Calibrations are only valid over the frequency range you used when calibrating, so be

careful about zooming in and out on your data. A closer look will require recalibration. We haven't had great luck doing very wide bandwidth calibrations in our lab, so keep in mind that we can calibrate ranges of 500MHz and smaller pretty reliably.

CLICK Finally, the components in the VNA are sensitive to temperature, so you need to let the VNA operate for a while before calibrating it. Often you need a full half hour of warmup time before a VNA calibration will be stable, so take your time after powering it up.

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Summary

- Short-open-load-through calibration is common for cabling.
- You can move the reference plane closer to the DUT with on-board or on-chip standards and other calibration techniques.
- Be mindful of practical considerations!