Lecture 08: Impedance Matching 2

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# The L Match

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In this video we're going to return to matching networks, and we're going to do some better work on them now that we have all this knowledge of second order systems. Specifically, we're going to tackle the challenge of building a matching network for Rn values that aren't equal to 1. We're splitting up that task into two parts: we're going to get a conceptual idea of how our matching networks work in this video, and then we'll come up with design equations in the next video.

New Matching Challenge: Rn Not Equal to 1

See Zm, I'm looking here
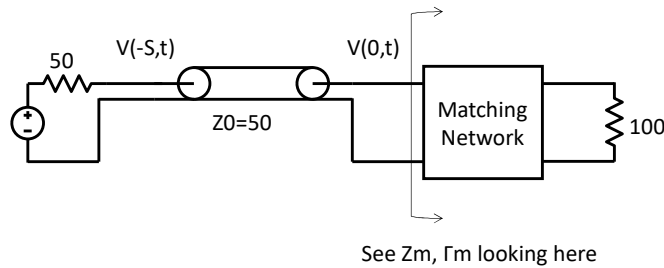
- Ringing out only works if Rn=50.
- Series-to-Parallel was promising b/c it changed apparent resistance.

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This is a copy of our matching network schematic. Up unitl now, we've put single reactive elements in the Matching Network box, and we've found that lets us cancel out reactance or susceptance that's in the load. We called that strategy ringing out the reactance because it worked at resonance, but it only worked if the resistance of the load was already z0. Today we're going to try to deal with the situation depicted here where the load is not Z0. Our secret weapon for approaching this problem is the series-to-parallel transform, because we observed that it made resistances look bigger or smaller, which reflects the underlying reality that currents or voltages in reactive parts of an RLC circuit get amplified by Q.

Series/Parallel Resistors Waste Incident Power
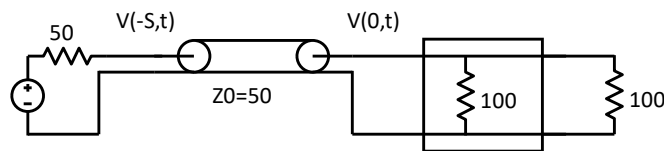
- Usually can't afford to have power in places other than the load
- Upside: resistive matches are very broadband.

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One tempting way to make this matching network is by just putting a resistor in the matching network box. You could add a series resistor if the load is less than Z0 or a parallel resistor if it's greater. While this can guarantee that the transmission line sees Z0, it forms a voltage or current divider that takes power away from the load. That's a really bad strategy in many high-speed applications for two reasons: delivering power to the load is important, and power is often scarce because of attenuation and reflections. However, one silver lining to this type of match is that it works at any frequency. As a result, you might sometimes use this matching network if you were trying to stabilize a very touchy amplifier that really needed to see 50 ohms at every frequency.

Shunt + Series LC Moves Left/Right on Smith

100

Pick C so Bn=0.5j | 100

Pick L so Xn=1j
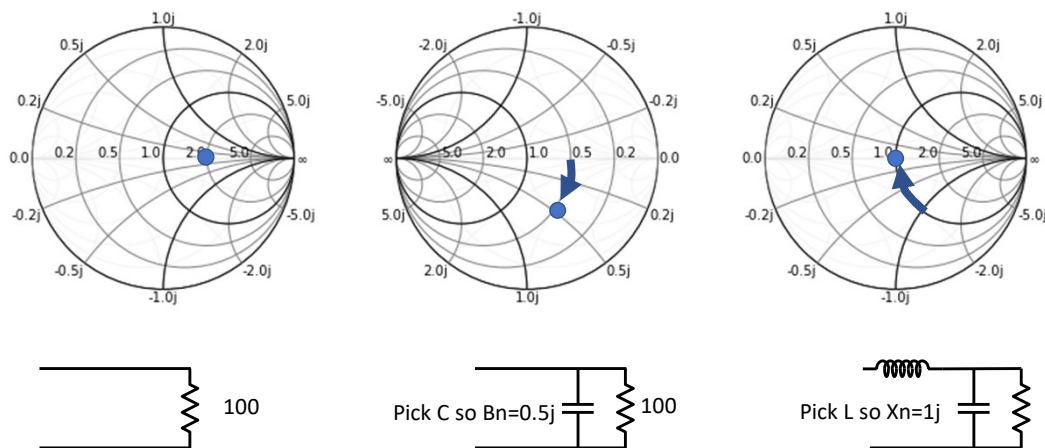
Called an L-match, works at one ω

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Instead of adding a lossy resistor to change our apparent resistance, we're going to add a lossless LC network. I'm going to depict that process on Smith Charts as we construct the network at the bottom the page. We start by depicting our load, which is at some real point other than the origin. CLICK then this shunt capacitance adds some susceptance to our load. Notice that we've switched over to admittance axes so that we can easily move along lines of constant conductance as we add this susceptance. We arrange for the suscpetance to carry us to the point where the line of constant conductance intersects with the Rn=1 line of constant resistance, which you can maybe see faintly in the background of this plot. CLICK Finally, we switch back to impedance axes and note that our load just looks like Rn=1 plus some negative reactance. We add some series inductance to cancel out the negative reactance and wind up at the origin, perfectly matched!

This type of LC network is called an L-match because it kind of looks like the letter L if you turn your head sideways. Notice that L matches only work at one frequency because the exact amount of susceptance and reactance we add with the shunt cap and series inductor changes with frequency.
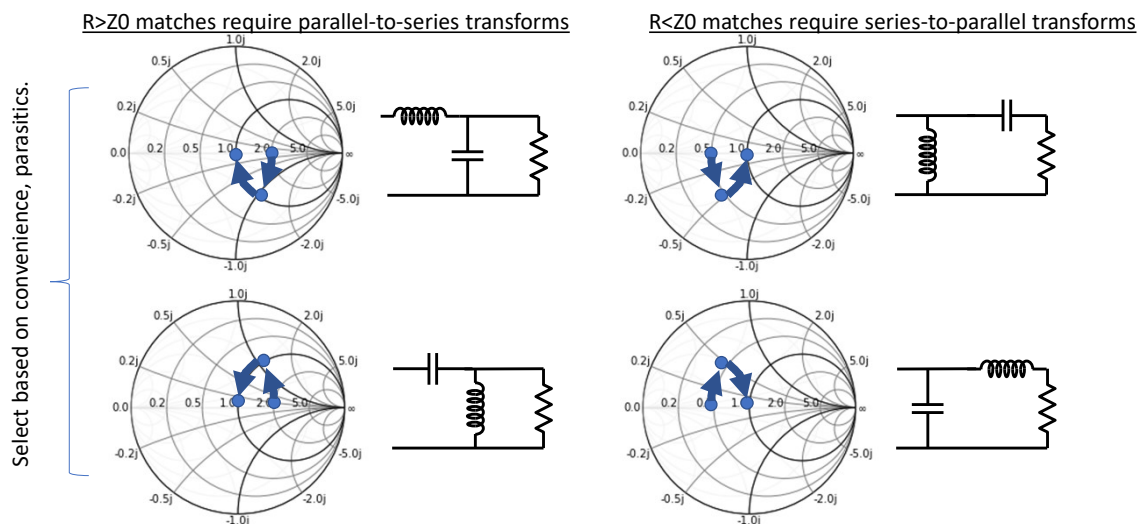
There are four different types of L-matches, and I'm going to show those matches and their effect on Smith Charts on this slide. We've gotten started with one familiar L match and one new one.

CLICK The L-match example we looked at on the last slide is on this column. There's also another circuit with the inductor and capacitor positions switched, and you can see that the circuit has the same net effect as the one at the top of the column: the same load impedance gets transformed into Z0. The only difference between the two Smith Charts is whether the L-match creates a negative reactance or a positive reactance with the first shunt component, which changes whether the arrows on the Smith chart point up or down first. Both of these networks transform a high load impedance to a lower value of Z0. One handy way to remember that these networks transform down is thinking about whether you'd analyze them with a parallel-to-series transform or a series-to-parallel transform. You'd analyze these by using a parallel-to-series transformation to create a series RLC circuit, and you can remember that the series resistor is always smaller than the parallel resistor in a transform, so this arrangement of components must reduce the apparent resistance.

CLICK You can also turn the letter L backwards to make the circuits in this column. Here we have to perform a series-to-parallel transform in order to make these networks into easy-

to-analyze parallel RLC circuits, and that will result in the resistance of the network appearing higher than the resistance of the load. Let's focus on the upper right circuit and Smith Chart pair to talk through a Smith Chart example for this column. Here the series capacitor results in a negative reactance that puts us on the Gn=1 admittance circle, then we add the negative susceptance of a shunt inductor to reach the origin.

CLICK Finally, we have to confront the fact that the circuits in the top row seem to do the same thing as the circuit in the bottom row. We pick between these rows for mundane reasons: the top row components will be differently sized than the bottom row, so they may be easier to build. Also, sometimes parasitics in our circuit will be easy to merge into either the top or bottom row. A classic example is that amplifiers always have some shunt capacitance, so the matching network with a capacitor in shunt with the load resistance is easy to implement because we get some capacitance for free.

Summary

- Series/Parallel resistors in matching networks waste incident power

- L matches
  - are made of a shunt and a series L and C
  - can match R values that aren't Z0.

- There are four types of L match networks
  - R<Z0 matches need to undergo a series-to-parallel transform
  - R>Z0 matches need to undergo a parallel-to-series transform
  - Pick shunt L vs. shunt C based on parasitics and component values.
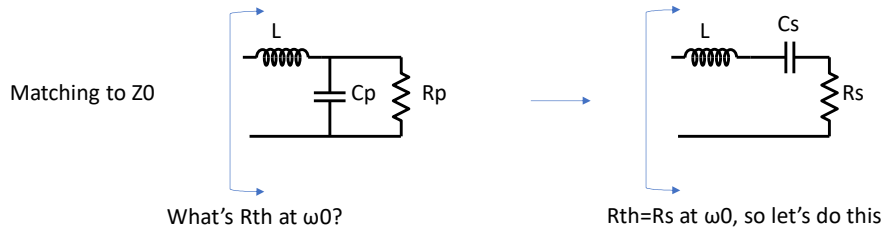
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Equations for The L-Match

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In this video we're going to derive design equations for the L-match networks from the series-to-parallel transform equations.

Apparent R Set By Series-Parallel at ω0 → Q

Matching to Z0

L · Cp · Rp

What's Rth at ω0?

L · Cs · Rs

Rth=Rs at ω0, so let's do this

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We're motivated to use a parallel-to-series transformation on this network because it isn't easy to analyze in the same way as parallel RLC or series RLC networks. However, it's easy to convert Rp, which is our load in this example, and Cp into their series equivalents Rs and Cs.

Design Inputs are R/Z0 and ω0, Outputs L & C

- Resistor Transformation ratio sets our desired Q (when Rs is set to Z0)

$$R_s = \frac{R_p}{Q^2 + 1} \text{ implies } Q_{desired} = \sqrt{\frac{R_p}{Z_0} - 1}$$

- Q and w0 uniquely specify values of L and Cs

$$\omega_0 = 1/\sqrt{LC_s} \qquad\qquad L = Z_0 \frac{Q_{desired}}{\omega_0}$$

$$Q_{desired} = \frac{\sqrt{L/C_s}}{Z_0} \qquad\qquad C_s = \frac{1}{Z_0 \omega_0 Q_{desired}}$$

- Convert Cs back to Cp (Qdesired must be same in series/parallel)

$$C_p = C_s \frac{Q_{desired}^2}{Q_{desired}^2 + 1}$$

We'll go through this design process, which just leans on the parallel-to-series transform equations, from top to bottom.

Starting at the first bullet on this page: The parallel-to-series equation specifies the series resistance as a function of the parallel resistance and Q. That means we'll be matched for some specific value of Q. We know that we want our series resistance to be Z0 in order to match our driving line, so I make that substitution and do a little rearrangement of the parallel-to-series equation to find the Q that we want as a function of our impedance transformation ratio.

Moving the second bullet: Once we know Q, we can use the Q of the RLC network and the w0 of the RLC network to find values for L and Cs. We do this by separating two equations in two unknowns.

Finally, on the third bullet: We transform Cs, which is a mathematical construct, back to the actual Cp that we'll use in our circuit.

These design equations aren't too bad, which is nice. However, this derivation is specific to the L match we used, so these design equation won't work for every L match. You can see that because things would start going wrong as early as the first line if we had to do a

series-to-parallel conversion instead of a parallel-to-series conversion. So, you'll need to redo this process for different networks, but the three bullets on the slide spell out the steps you'd use for any network: Find your desired Q from the resistor transformation, then find L and C from Q and w0, then finally un-transform the reactive component.

If Q is High, Equations Are Easier

$$Q_{desired} \approx \sqrt{R_p/Z_0}$$

$$C_s \approx C_p \text{ called } C$$

$$\sqrt{\frac{L}{C}} = \sqrt{R_p Z_0} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

- All impedances/reactances set to geometric mean of load and Z0

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Before we wrap up the video, it's worth noting that these equations get easier if Q is high. We can approximate high Q values as the square root of the transformation ratio, and our capacitor doesn't get transformed significantly when Q is high, which means that we can talk about C instead of Cs or Cp. Rearranging the design equations from the previous page we find that the characteristic impedance of the RLC network is equal to the geometric mean of the load and the impedance to which you're matching. Nice!
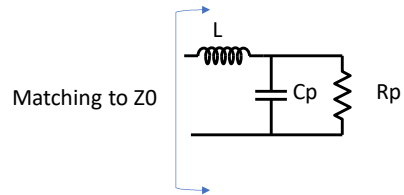
Summary

- Equations for an example parallel-to-series L match

$$Q_{desired} = \sqrt{\frac{R_p}{Z_0} - 1}$$

$$L = Z_0 \frac{Q_{desired}}{\omega_0}$$

$$C_s = \frac{1}{Z_0 \omega_0 Q_{desired}}$$

$$C_p = C_s \frac{Q_{desired}^2}{Q_{desired}^2 + 1}$$

Matching to Z0

- Need to mix and match for different topologies. Start w/ Qdesired.

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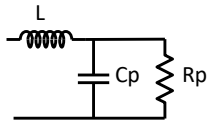# Pi and T Matches

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In this video we're going to examine slightly more complicated matching networks that give us greater control over the quality factor of our match.
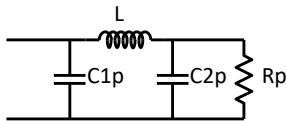
We Can't Pick Q in an L-Match

$$R_s = \frac{R_p}{Q^2 + 1} \quad \text{implies} \quad Q_{desired} = \sqrt{\frac{R_p}{Z_0} - 1}$$

L-match: Free to pick Rp/Z0 and ω0, but not Q

Makes sense, only two degrees of freedom: L, Cp

Plan: Add another component to get another degree of freedom to set Q. Called a <u>pi match</u>

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We already derived design equations for an L match, but we skimmed over a crucial limitation of L-matches, which is that Q is set by the resistance transformation ratio. That means that big transformations, say from 1kohm to 50 ohms, will have unusuably narrow bandwidths.

That limitation makes sense because we don't have many degrees of freedom when we design an L-match. There are three variables you might care about in a matching network, which are w0, Q, and the transformation ratio, but we only get to pick two component values in an L match, which are L and Cp.   Because there are only two choices we make in the circuit, we only get to specify two of the design parameters we care about.

So we're going to expand our L match by adding a new component, which will give us an additional degree of freedom. We call this type of matching network a pi match because it looks a bit like the letter pi.

Pi Structure Looks Like Two L-Matches

- Redraw L as L1 and L2, decide allocation and value of C1p w/ design.

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The key to analyzing pi-match networks is to split them into two back-to-back L-match networks. We'll decide on the allocation of L1 and L2 as well as the values of C1p and C2p as we go through our design equations in the next video.

Pi Matches Move Through "Image Resistance"
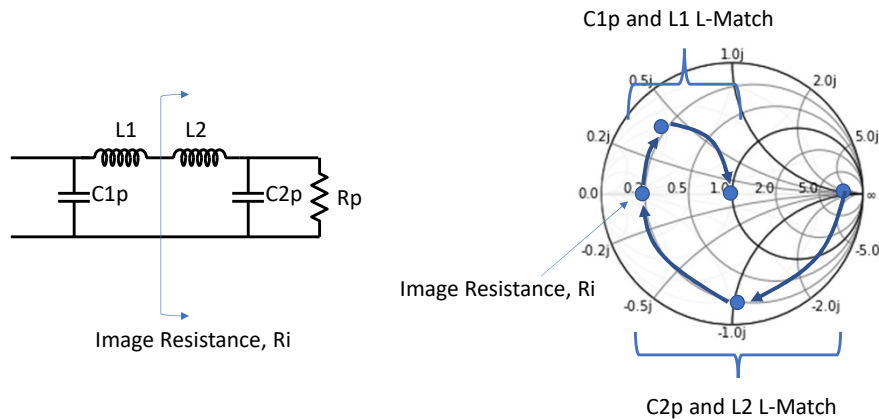
However, we can already learn something useful from splitting the pi-match network into two L-matches. I've drawn Rp, the load resistance on this Smith Chart to start with. CLICK The first L-match is a downward transforming network because the load will undergo a parallel-to-series transform when we analyze it. We see that we can imagine it taking us to another, lower real resistance. It's safe to pretend that there's a real resistance in the middle of this pi network because we can always split up L1 and L2 to arrange for our intermediate point to land at a real value just as long as the arc created by the reactance of L crosses from the bottom of the Smith Chart to the top, which is required for the second L match to work. CLICK because that intermediate resistance is made up, we call it the image resistance.

CLICK Tthe second L-match network is an upward transforming network, and we design it to take us from the image resistance to our final impedance.
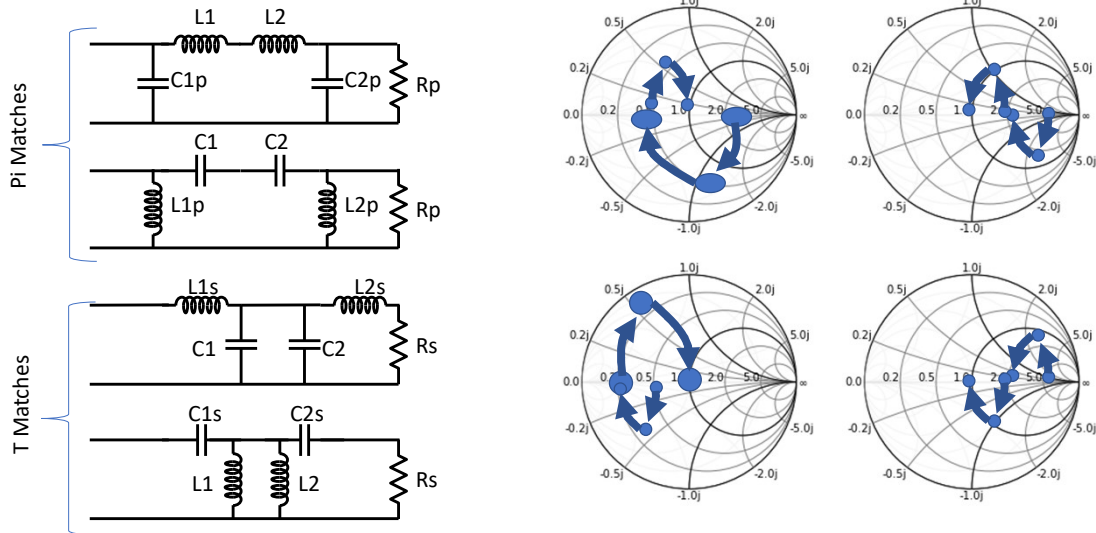
There are four types of circuits we can make that behave like pi networks. The top two on this slide are the circuit we've been looking at and a variation where we switch the capacitors and inductors. The bottom two reverse the order of L-matches, transforming the load impedance up and then down. We've labeled the load on these two Rs because it will undergo a series-to-parallel transform when we analyze the circuits. The bottom two circuits have a different shape than pi matching networks, so they're called T matching networks. Note that the two shunt components in T networks actually represent a single component that we're splitting up during analysis to make our two L-matches.

CLICK While the four types of circuits might comfortably remind us of L-matches, there are an unfamiliarly large number of possible shapes these networks can trace out on Smith Charts. As a result, I haven't drawn every possible candidate Smith Chart that a pi or T match can realize, but I've tried to capture a few of the features that might change how Smith Charts look. For instance, some of these Smith Charts represent loads with Rn being greater than 1, while others have Rn less than 1. Some charts represent networks with Ri greater than Z0, and others have Ri less than Z0. Finally, Some of these Smith Charts show networks with a positive reactance closest to the load, while others show networks that have a negative reactance first. The takeaway is that Pi and T networks give us lots of options for traversing the gamma plane.

Summary

- Pi and T matches let you pick Q

- Pi and T matches are best modeled as back-to-back L-matches

- The first L-match takes you to an intermediate "image resistance"

- There are many possible flavors of Smith Chart depending on design.
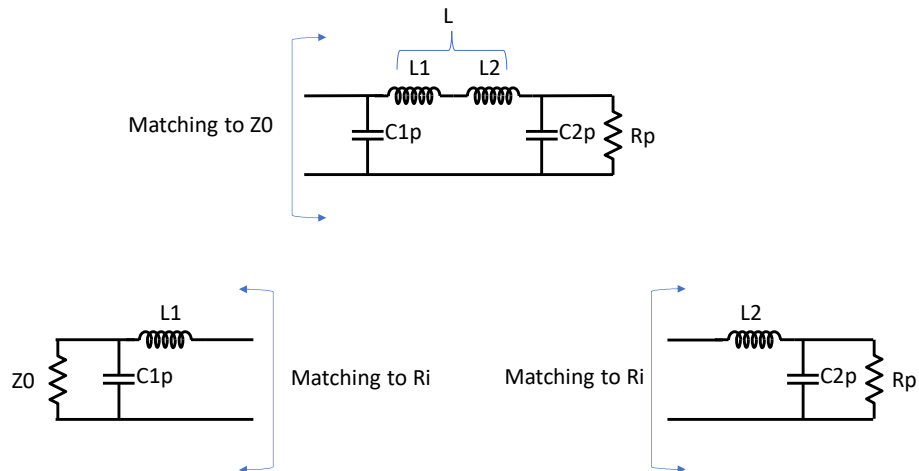
Equations for Pi and T Matches

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In this video we're going to derive some design equations for Pi and T matching networks through a far-ranging set of series-to-parallel transforms.

Split Pi Match into Two L-Matches to Ri

I've drawn our pi matching network at the top of this slide, and I've added a reminder that L1 and L2 add up to L, which is the component value that is actually in our circuit.

We start by splitting our pi network into two L-matches as in the last video, but we make a surprising decision with the left matching network. Instead of imagining we're matching from Ri to Z0, we're going to imagine we're instead matching Z0 to Ri in the middle of our network.
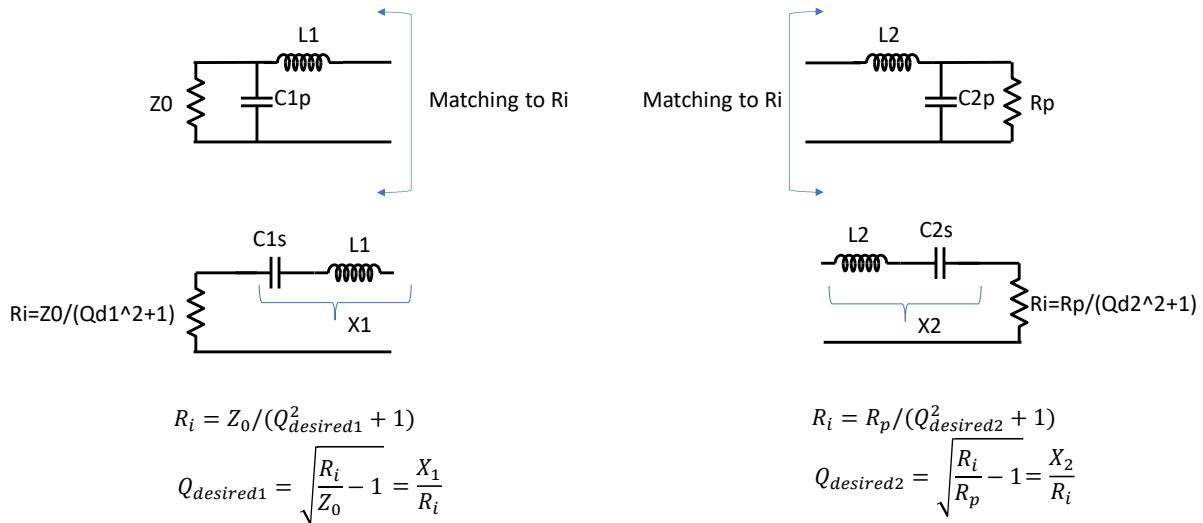
I've performed the parallel-to-series transformations for each of our L-matches on this slide. These two transformations each define a Qdesired for the L-match network in the same way that we've seen before.

CLICK However, we are going to add an additional definition of Qdesired that will be important when we try to figure out the overall Q of these two interacting L-matches. We define the series L and C in both sides as some reactance X1 and X2, which means that each L match looks like an individual component: a resistance in series with some reactance. That means we can specify a component Q for each L-match as X1/Ri or X2/Ri. So now we have two expressions for Qdesired.

I've left off the equations for transforming the capacitors from parallel to series, but that should be done as normal. As usual, if the Qdesired values are high then Cs is about the same as Cp.

Overall Q is Set By Image Resistance

C1s L1 L2 C2s

Ri=Z0/(Qd1^2+1) X1 X2 Ri=Rp/(Qd2^2+1)
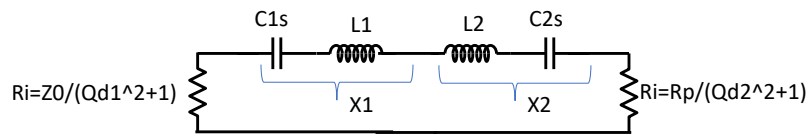
$$Q_{overall} = \frac{X_1 + X_2}{2R_i} = \frac{Q_{desired1} + Q_{desired2}}{2}$$

In series because the series LC sees 2Ri across terminals

$$\omega_0 = 1/\sqrt{L\frac{C_{1s}C_{2s}}{C_{1s} + C_{2s}}}$$

Next we bring the two parallel-to-series converted L matches together to represent the whole pi network. We can try to define an overall Q for this network, and we're going to do that by treating this as a component Q like the previous slide. First, we notice that the two reactances X1 and X2 are in series, which means we can combine them into a net reactance X1+X2. The two terminals of the compound reactance are connected by two series Ri, so the effective component resistance is 2Ri.   The component Q is given by the compound reactance divided by the compound resistance, so X1+X2 over 2Ri. This expression can be rewritten in terms of Qdesired1 and Qdesired2 from the last slide, which reveals that our overall Q is the average of Qdesired1 and Qdesired2.

We can also find the natural frequency now that this looks like a big series RLC network. Both the inductors and capacitors combine in series to give us an effective L and C which we plug into our standard equation for w0.

Now that we have these equations we can work on flipping them around to turn design parameters into L and C values.

Design Process: Pick Q, ω0, Rp, Z0 → L and Cs

- Pick Qoverall and find Ri

$$Q_{overall} = \frac{Q_{desired1} + Q_{desired2}}{2} \rightarrow 2Q_{overall} = \sqrt{\frac{R_i}{Z_0} - 1} + \sqrt{\frac{R_p}{Z_0} - 1}$$

- Use Ri and ω0 to find C1s and C2s through component Qs

$$Q_{desired1} = \frac{1}{\omega_0 R_i C_{1s}} \qquad Q_{desired2} = \frac{1}{\omega_0 R_i C_{2s}}$$

- Use ω0 to calculate L, and translate from series caps back to parallel

$$\omega_0 = 1/\sqrt{L \frac{C_{1s} C_{2s}}{C_{1s} + C_{2s}}}$$

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Unfortunately, doing so is a mess. So I'm going to gesture in the direction of design equations, but I'm not ever going to expect you to design a pi or a T network with hand analysis. These are great problems for computers and pretty joyless ones for humans.

We start by picking our desired Q, which is the great affordance granted by a pi network. By substituting in our expressions for Qdesired into the overall Q formula, we find that Ri is a function of Qoverall. It's a messy function that I'm not going to invert here, but the two quantities uniquely specify one another, which means we can go from the Q we picked to an Ri value.

Once we have Ri, we can find a component Q for each of the L matches in order to calculate values for C1s and C2s. Note that we calculate Qdesired1 and Qdesired2 in these equations using Ri, and also note that we know w0 because it's a design input that we've picked.

Finally, we use the natural frequency to find the value of L. I've omitted transforming our series capacitors back to parallel capacitors, but you have to do that too.

Like L matches, this set of design equations isn't generic for every match, and you have to make some modifications for matches that use series-to-parallel transforms instead of

parallel-to-series transforms. However, these are the basic design steps regardless of which match you use: find your compound Q, then go from Q to Ri, then go from Ri to reactance values using component Qs of each L match, and finally you use your component values and the natural frequency to pin down the last remaining component value.

Summary

- Analyze pi and T matches by
  - Breaking into two L-matches
  - Matching Z0 to Ri and Rp to Ri to wind up with big series RLC (in our example)
  - Find Qdesired for each match separately and Qoverall as fn. of Qdesired.

- Calculate L and C values for pi/T matches by
  - Pick Qdesired and use it to find Ri
  - Find Cs1 and Cs2 using component Qs and Ri
  - Find L using $\omega 0$
  - Do it with a computer

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