More Differential Circuits and Offsets

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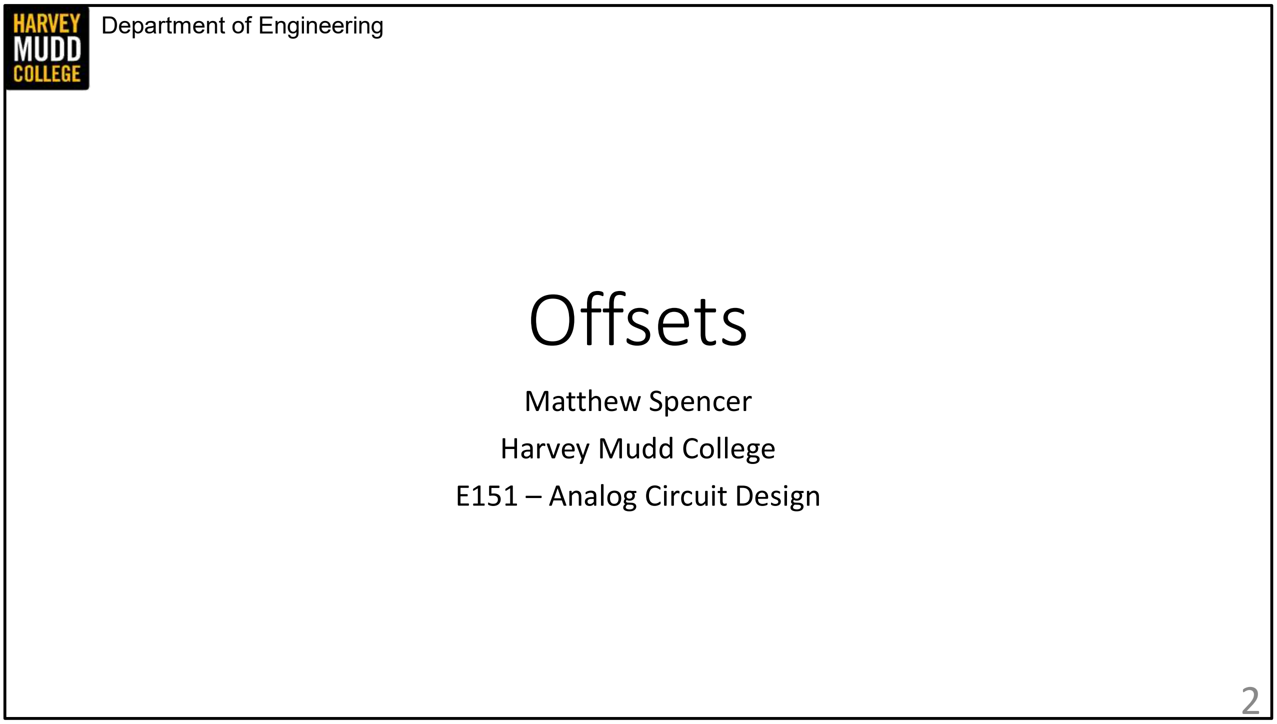
E151 – Analog Circuit Design

In this video series we're going to wrap up our discussion of differential circuits by adding a few implementation details. We'll talk about what happens when the sides of the differential circuit aren't exactly matched, and we'll talk about what to do when we want a single ended output from an emitter coupled pair. In the next video series we're going to talk about output stages, which are amplifiers that are designed to drive high power or low impedances, then we'll introduce our first op-amp!



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Offsets

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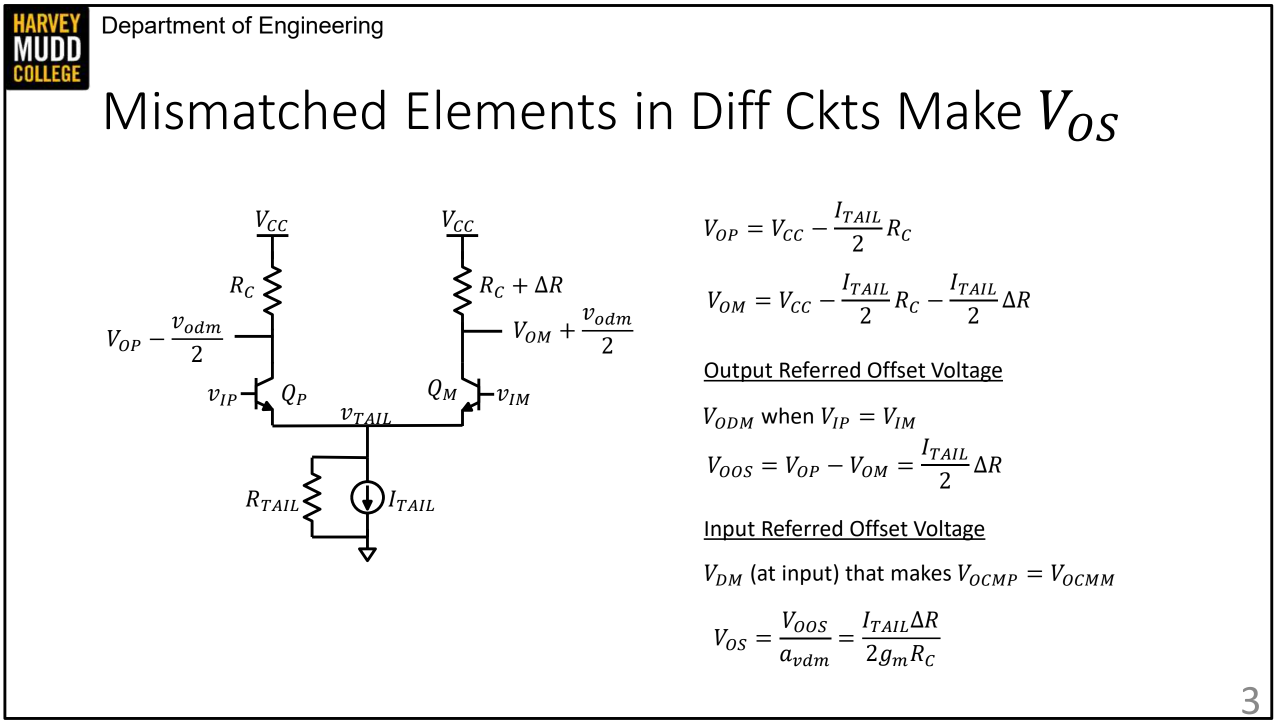
In this video we’re going to talk about offsets, which are unbalanced currents or voltages in our amplifiers that are caused by asymmetries between the two differential halves of the circuit.



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Mismatched Elements in Diff Ckts Make V_{OS}

$$V_{OP} = V_{CC} - \frac{I_{TAIL}}{2} R_C$$

$$V_{OM} = V_{CC} - \frac{I_{TAIL}}{2} R_C - \frac{I_{TAIL}}{2} \Delta R$$

Output Referred Offset Voltage

V_{ODM} when $V_{IP} = V_{IM}$

$$V_{OOS} = V_{OP} - V_{OM} = \frac{I_{TAIL}}{2} \Delta R$$

Input Referred Offset Voltage

V_{DM} (at input) that makes $V_{OCMP} = V_{OCMM}$

$$V_{OS} = \frac{V_{OOS}}{a_{vdm}} = \frac{I_{TAIL} \Delta R}{2 g_m R_C}$$

I've drawn an emitter coupled pair here to kick off our discussion of offsets. It has slightly mismatched collector resistors, the right resistor is a little bigger than the left resistor by an amount DeltaR. We're going to analyze the large signal effects of this mismatched component. We're skipping the small signal analysis of it because it's surprisingly complex, and involves another two gains called the differential to common mode conversion gain and the common mode to differential conversion gain. We'll indicate that we're doing large signal analysis with our usual large signal notation.

Fortunately, the analysis is simplified by the fact that QP and QM are in forward active and DeltaR is a small value. That means the tail current still splits evenly between QP and QM because the collector voltage of a BJT in forward active doesn't affect the collector current.

CLICK Knowing that, it's easy to calculate the two output voltages. Each is below VCC by the resistance times ITAIL/2. These two voltages are different from one another, which is new; during half circuit analysis we assumed the halves of the circuit were identical so that the output voltages were both equal to the output common mode voltage.

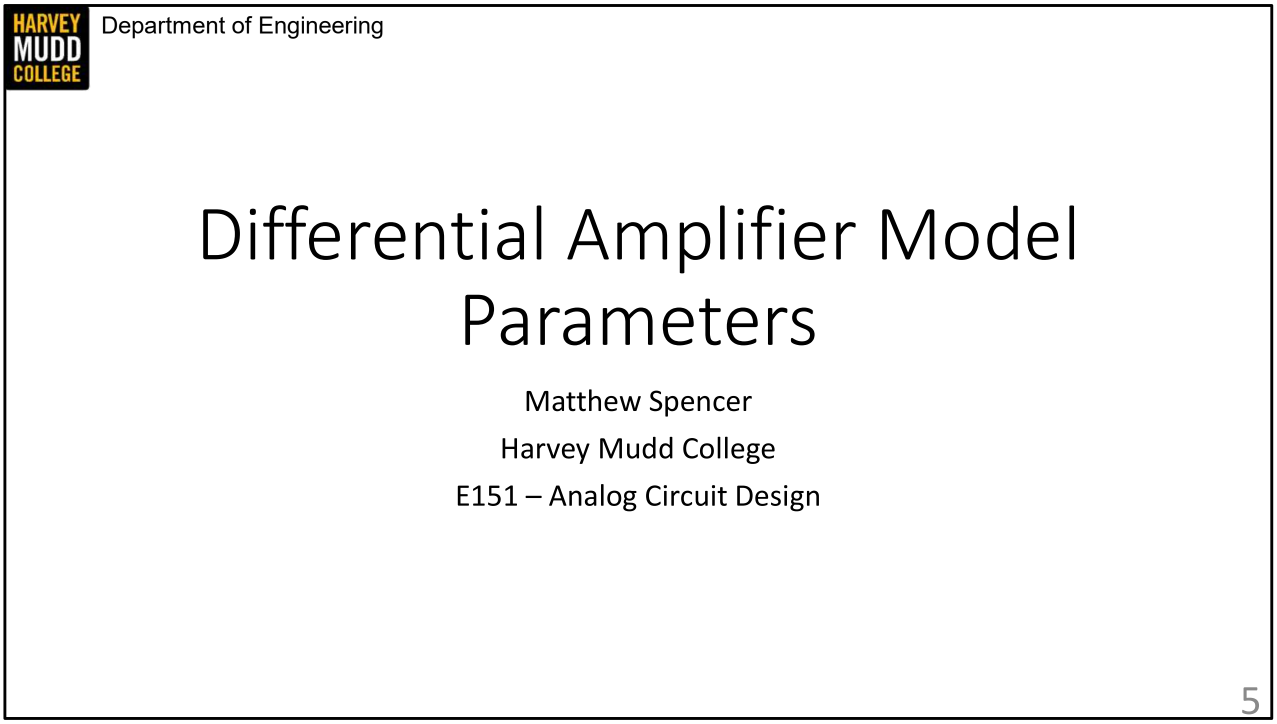
CLICK This difference in output voltages has a special name, it's the output referred offset voltage. You find the output referred offset voltage by finding the difference between VOP and VOM when the inputs are equal to one another.

CLICK Another common measure of offset in amplifiers is given by the input-referred offset voltage. This is the voltage difference you would need to apply at the input in order to make the output voltages equal. Put another way, it's the input differential mode required to cancel the output-referred offset. Because DeltaR is small, the output offset is a small signal. That means you can calculate the input difference that would cancel it using the small signal differential voltage gain. Dividing VOOS by avdm gives us an expression for the input referred offset VOS.

A few notes. We're sort of ignoring that the offset created a small common mode shift at the output too. The output common mode is not halfway between VOP and VOM. That's OK because the common mode shift happens on the collector, and collector voltage doesn't affect the circuit's behavior significantly. It's also worth noting that there are all kinds of other offsets we could have. Different gm values could create different gains on either side of the structure, different beta values could create an input offset current, and different IS values would result is a small voltage. Managing this mismatch is a major task of circuit designers and there are lots of tuning and cancelling techniques that you can learn about in the future!

Summary

- Mismatched elements create offset voltages (or currents).
- We're analyzing large signal, DC offsets.
- Output referred offsets are differences in voltage at the output.
- Input referred offsets are the differences in input voltage needed to cancel output referred offsets. Find them as $V_{OS} = V_{OOS}/a_{vdm}$.



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Differential Amplifier Model Parameters

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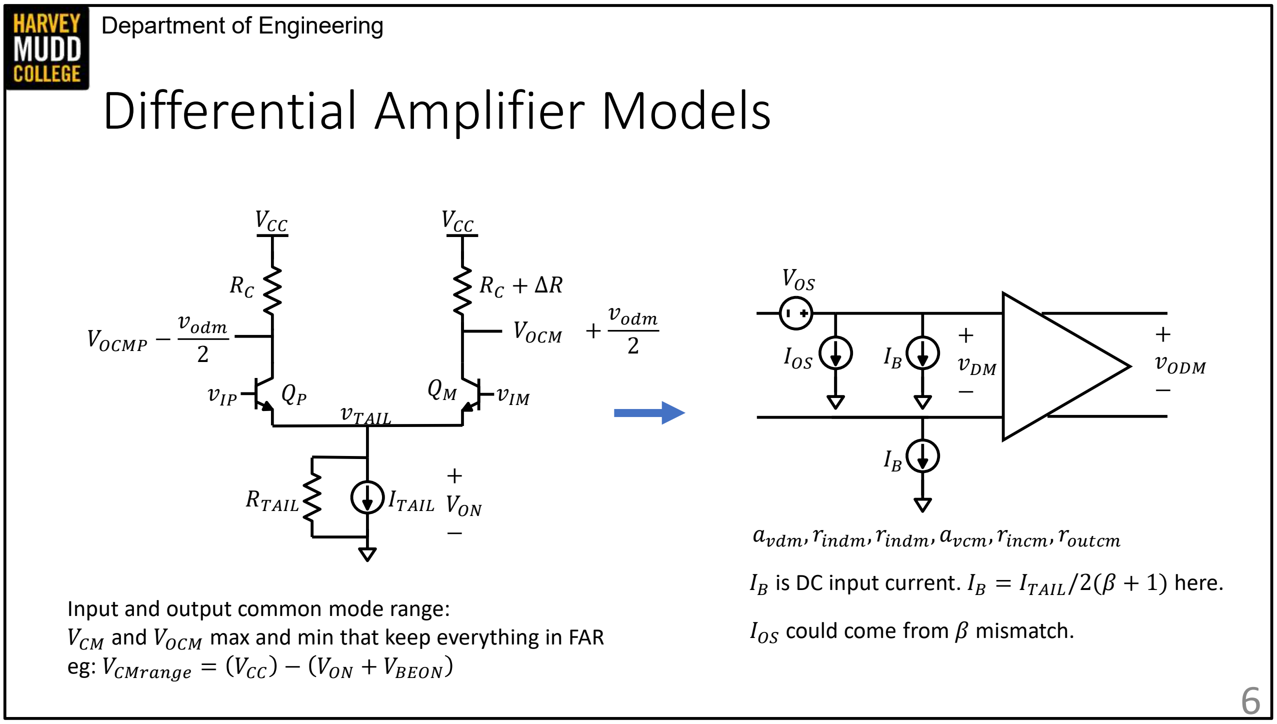
In this video we're going to learn about design parameters that we use to describe differential amplifiers, which are an abstraction of the differential circuits we've been looking at.



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Differential Amplifier Models

$a_{vdm}, r_{indm}, r_{indm}, a_{vcm}, r_{incm}, r_{outcm}$

I_B is DC input current. $I_B = I_{TAIL}/2(\beta + 1)$ here.

I_{OS} could come from β mismatch.

Input and output common mode range:
V_{CM} and V_{OCM} max and min that keep everything in FAR
eg: $V_{CMrange} = (V_{CC}) - (V_{ON} + V_{BEON})$

Thus far we've been looking at differential amplifiers like the one pictured here, perhaps with offsets as shown here by the extra DeltaR on the right resistor.

CLICK We can abstract this into a differential amplifier model just like we abstracted our single-stage amplifiers into amplifier models.

CLICK We've already seen a bunch of small signal parameters parameters that describe this differential amplifier. These are the differential and common mode parameters we've been finding with our half-circuits.

CLICK And we can add in our voltage offsets from last video to the amplifier model too. I've chosen to represent the voltage offset of this amplifier as an input referred offset, and that's a pretty common choice. Though sometimes when you're analyzing chains of amplifiers it's helpful to refer all the offsets to either the input or the output.

CLICK The input bias current is another parameter used to describe amplifiers. This is the DC bias current that flows from the inputs into the amplifier. In our amplifier, this would be given by ITAIL/2(beta+1) because each transistor has an emitter current of ITAIL/2, and that implies a base current that is beta+1 times smaller. Note that if this amplifier were AC coupled, the input bias current would be zero because it's a DC, large-signal parameter.

CLICK If beta is different for QP and QM, then the input current will be different on the two sides of the amplifier. Adding an input offset current lets us represent that type of mismatch.

CLICK Finally, though this won't appear on our differential amplifier diagram, the input and output common mode can't move such that transistors fall out of forward active. We can look at the input common mode for an example of this. To that end, I've labeled the tail current source with it's on voltage, VON, which needs to be maintained across it to keep the current source working. That means the input common mode needs to stay above VON + VBEON in order to guarantee that vTAIL is always above VON. The input common mode needs to stay below VCC-VCESAT+VBEON to keep QP and QM from saturating, though you might notice that quantity is above the voltage rails. Because the common mode is very unlikely to move above VCC, we call the maximum input common mode voltage VCC. Subtracting the minimum from the maximum gives us VCC-VON-VBEON as our input common mode range.

avcm, rincm, routcm, avdm, rindm, routdm, input common mode range, output common mode range, input-referred offset, input bias current, input offset current, Department of Engineering

Summary

- Differential amplifiers are described by
  - Differential and common mode small signal parameters: $a_{vdm}, r_{indm}, r_{indm}, a_{vcm}, r_{incm}, r_{outcm}$
  - An offset voltage applied to one of the inputs
  - (offset voltage can be attached to one output line if you want output referred)
  - An input bias current applied to both input lines
  - An input offset current applied to one of the input lines.
  - An input common mode voltage range.
  - An output common mode voltage range.

Differential-to-Single-Ended Conversion Circuits

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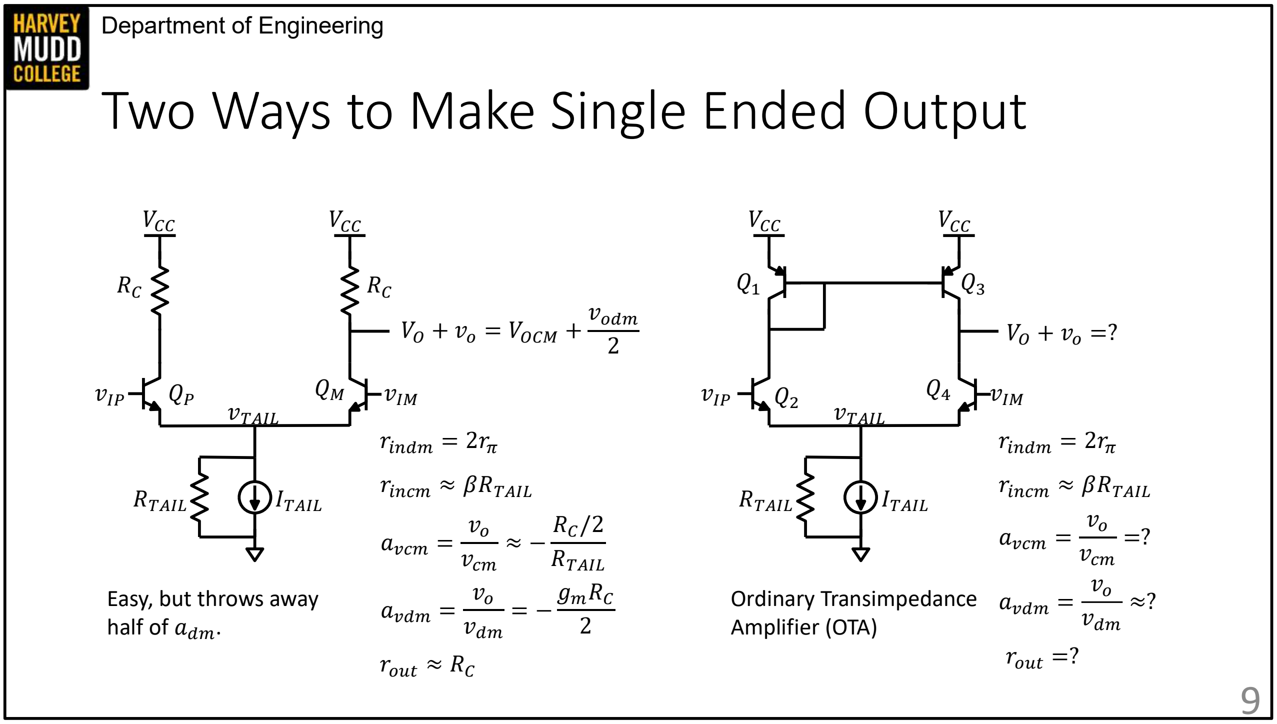
In this video we're going to look at ways to convert from differential signals to single-ended signals, which will lead us into a pretty tricky amplifier analysis.



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Two Ways to Make Single Ended Output

$V_O + v_o = V_{OCM} + \frac{v_{odm}}{2}$

$r_{indm} = 2r_\pi$

$r_{incm} \approx \beta R_{TAIL}$

$a_{vcm} = \frac{v_o}{v_{cm}} \approx -\frac{R_C/2}{R_{TAIL}}$

$a_{vdm} = \frac{v_o}{v_{dm}} = -\frac{g_m R_C}{2}$

$r_{out} \approx R_C$

Easy, but throws away half of a_{dm}.

$V_O + v_o = ?$

$r_{indm} = 2r_\pi$

$r_{incm} \approx \beta R_{TAIL}$

$a_{vcm} = \frac{v_o}{v_{cm}} = ?$

$a_{vdm} = \frac{v_o}{v_{dm}} \approx ?$

$r_{out} = ?$

Ordinary Transimpedance Amplifier (OTA)

9

Though there are lots of interesting ways to use differential signals, we're ultimately interested in building an op-amp, and op-amps have single ended output. So we need to convert our differential signal into a single ended signal.

One super easy way is pictures on the left here. We can just take the signal from one side of our differential amplifier. As long as our load is high enough resistance, this won't change the operation of our amplifier at all.

CLICK The top level analysis of this approach is that it's super easy, but we throw away half of our gain. We can see that our single ended small signal output vo is going to be equal to vodm/2, so we won't see our full differential mode gain to the output. This is an OK tradeoff if you really need simplicity, but we're going to see an amplifier that lets us capture our whole differential mode voltage gain very soon.

CLICK Before we do that, let's think about the small signal parameters of this amplifier. The differential inputs don't know where we're picking voltages out of this circuit, so the differential input impedance looks about the same as an emitter coupled pair with a differential output.

CLICK We can make the same argument about the common mode input impedance.

CLICK The common mode voltage gain starts to look a little weirded because we have to write a new definition for it. This equation defines the gain as the ratio of our single ended output to a common mode input. OK, that's a weird definition, but we expect both the left and right output nodes to have the same voltage for a common mode input, so our half circuit wouldn't change, and we wouldn't expect any change in our common mode gain compared to differential outputs.

CLICK The differential gain is a bit differential from the emitter coupled pair with two ouptuts. Because we're only taking output on one side, we are comparing vodm/2 against a full vdm at the input. That results in our differential gain getting cut in half.

CLICK Finally, we don't have separate differential and common mode rout values because our output is single ended. However, the single ended output is easy enough to approximate: we see RC in parallel with the collector of a transistor, and looking down that collector probably has a high impedance. That's good enough for now, though it's worth noting that the collector impedance is quite tricky because QM has the whole left side of the transistor dangling off of the emitter.

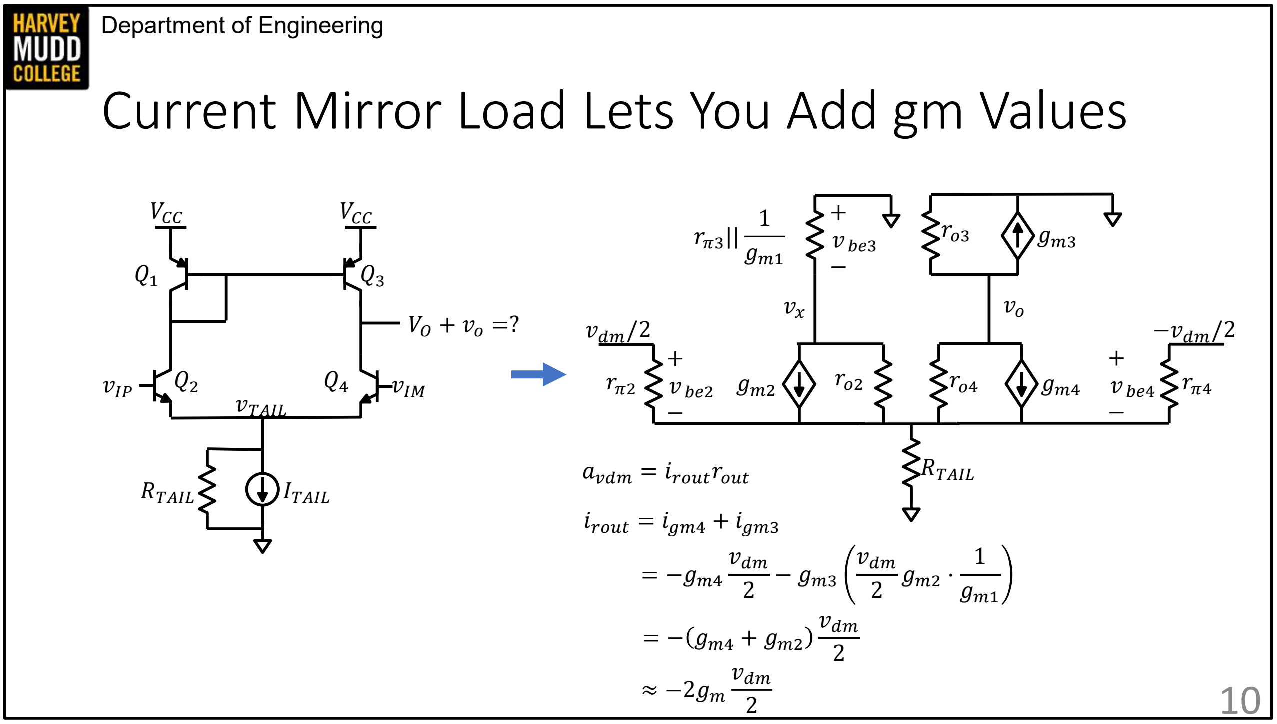
CLICK I promised you an amplifier that made use of our full differential gain, and this is it! Here we load our emitter coupled pair with a current mirror. That mirror is going to capture current steered to the P side of the amplifier and reflect it back to the M side, which is pretty cool. We'll analyze that behavior in a minute.

CLICK But first, it's worth noting that this mirror-loaded emitter coupled pair is extremely common and useful. So much so that it's called an ordinary transimpedance amplifier. You know something is common when it has ordinary in its name!

CLICK Starting in on the small signal behavior of this amplifier, we can make the same arguments as we did before about the input impedances. The bases of Q2 and Q4 don't know anything about the collectors, so our differential and common mode rin don't have any surprises.

CLICK However, this amplifier has a lot of other surprises. It's not immediately apparent how to find output impedance, common mode or differential gain, or even our large signal output voltage. We'll spend the next two slides figuring out rout and avdm, which are pretty crucial to using this amplifier. VO actually can't be solved analytically in any meaningful way, and it's going to be very sensitive to temperature and process variations, so it's common to use feedback to set the value of VO. We'll ignore avcm for now, but it's small.

… we actually won't ever find avcm in this video series. Also, vo needs to be set by feedback to work properly.



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Current Mirror Load Lets You Add gm Values

$$a_{vdm} = i_{rout} r_{out}$$

$$i_{rout} = i_{gm4} + i_{gm3}$$

$$= -g_{m4} \frac{v_{dm}}{2} - g_{m3} \left(\frac{v_{dm}}{2} g_{m2} \cdot \frac{1}{g_{m1}} \right)$$

$$= -(g_{m4} + g_{m2}) \frac{v_{dm}}{2}$$

$$\approx -2 g_m \frac{v_{dm}}{2}$$

10

OK, we can make a sprawling small signal model for this OTA, and we notice right away that our small signal model isn't symmetric. That means we can't fall back on half-circuit analysis, which is a shame.

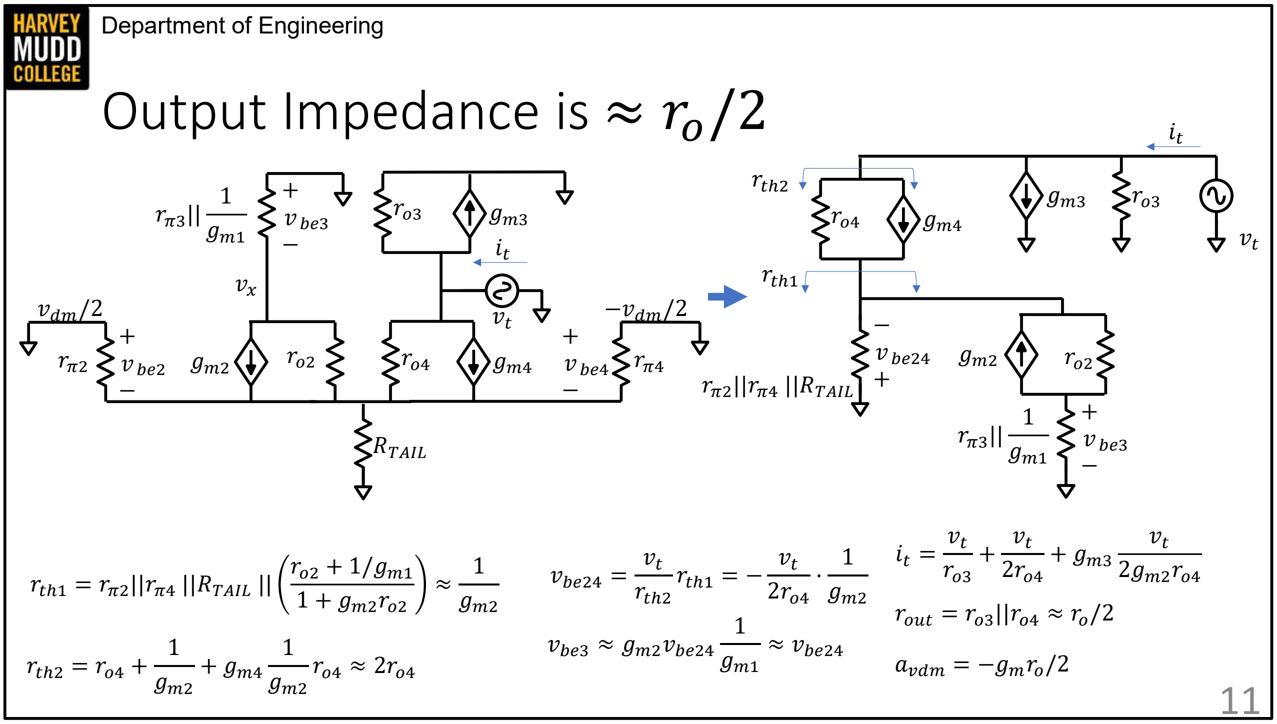
CLICK We're going to start our analysis strategy by noting that any change in small signal current at the vo node is going to result in a really big change in vo. That's because extra current that gets injected there has to make its way to ground through the output impedance of the amplifier, and just glancing at this circuit our output impedance is probably on the order of ro. So, fine, if we can find how much extra current gets injected into the output when vdm changes, and then we find the output impedance (which we want anyway), we can calculate our gain.

CLICK Writing KCL at the vo node, we see that the excess current that needs to worm its way to ground through resistors is given by the current generated in gm3 and gm4.

CLICK the gm4 current is easy enough to calculate. Our inputs are still differential, so the tail node is still a differential small signal ground, which means gm4 is driven by vdm/2. gm3 is a bit trickier, but it's worth noting that the 1/gm1 from the diode connected device at the collector of Q2 is part of a current mirror. So any current that goes into that resistor will come out of gm3. We can see that clearly by noting that we're taking the current through gm2, vdm/2 times gm2, multiplied by the impedance of 1/gm1 to find the control votlage for gm3.

CLICK If gm1 and gm3 are matched, then they cancel out of our expression and we see that gm3+gm4 gets multiplied by vdm/2.

CLICK Simplifying further, if gm4 and gm2 match well, then we find that our output current is about equal to gm times vdm. Mirroring our current has let us capture our full differential input voltage, hooray!



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Output Impedance is $\approx r_o/2$

$$r_{th1} = r_{\pi 2}||r_{\pi 4}\ ||R_{TAIL}\ || \left(\frac{r_{o2} + 1/g_{m1}}{1 + g_{m2} r_{o2}}\right) \approx \frac{1}{g_{m2}}$$

$$r_{th2} = r_{o4} + \frac{1}{g_{m2}} + g_{m4}\frac{1}{g_{m2}} r_{o4} \approx 2r_{o4}$$

$$v_{be24} = \frac{v_t}{r_{th2}} r_{th1} = -\frac{v_t}{2r_{o4}} \cdot \frac{1}{g_{m2}}$$

$$v_{be3} \approx g_{m2} v_{be24} \frac{1}{g_{m1}} \approx v_{be24}$$

$$i_t = \frac{v_t}{r_{o3}} + \frac{v_t}{2r_{o4}} + g_{m3}\frac{v_t}{2 g_{m2} r_{o4}}$$

$$r_{out} = r_{o3}||r_{o4} \approx r_o/2$$

$$a_{vdm} = -g_m r_o/2$$

OK, so we know the current going into rout, now we need to find a value for rout. You might be able to guess that that's a bit involved by the horrible small signal model I've redrawn on the right. That small signal model is just a redrawing of the fully differential circuit on the left, and you can see that by looking at … show some similar parts: ro3, gm3, etc …

With that model drawn, we can launch into the small signal analysis, but before we do I want to warn you that I'm never going to expect you to do this kind of big derivation under duress. However, you are responsible for the results of this derivation, which you can already see up in the title of the slide. The output impedance is ro/2.

CLICK OK, understanding this circuit is much easier if we break it up into a few Thevenin impedances. rth1 and rth2 are good places to break the circuit up.

CLICK rth1 looks down into a 1/gm pattern because the control voltage for gm2 falls in parallel with it as vbe24. That means we can use our usual 1/gm pattern, which is ro2 +1/gm2 over 1+gm2ro2. Simplifying that expression gives us about 1/gm2.

CLICK Knowing rth1 makes it easier to find rth2, which is just given by a left-right pattern. ro4 is the left resistor and the rth1 is the right resistor in the pattern. If gm4 is about gm2, then this looks like roughly 2ro4.

CLICK knowing both rth2 and rth1 makes it easy to find vbe24. We know the current running in the rth1 branch is given by vt/rth2, and multiplying that by rth1 gives us vbe24.

CLICK That's significant because vbe3 is about the same as vbe24. Almost all the current that flows in rth1 runs through the gm generator, which is responsible for the 1/gm impedance. That's important, because knowing vbe3 is the last piece of the puzzle we need to calculate i_t.

CLICK We can write KCL at the top node, finding the current in the ro3 branch, the current in the rth2 branch using our rth2 value, and finally the current in gm3 using our value for vbe3.

CLICK Dividing through by vt gives us something that looks like a parallel combination of resistors. But if we assume gm3 and gm2 are about the same, then the two ro4 terms will add together so that we get ro3 in parallel with ro4. If those are the same as one another, then we find the output impedance is ro/2.

CLICK Circling back to our iout expression form the previous page, we can see that the differential voltage gain is given by –gm*ro/2. Remember this avdm value and this rout value, it will save you doing this math on the fly when you see these in the wild.

Summary

- You can get single ended outputs by
  - grabbing one side of a resistively loaded emitter coupled pair
  - by building a mirror loaded emitter coupled pair, which is called an ordinary transimpedance amplifier (OTA).

- Mirror loads in OTAs combine the gm of the input devices by reflecting current from the left side of the load to the right side.

- The output impedance of an OTA is $r_{o3}||r_{o4}$, so $a_{vdm} \approx g_m r_o/2$.