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Common Emitter Dynamics and the Miller Effect

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E151 – Analog Circuit Design

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In this video series we're going to take a straightforward approach to figuring out the dynamics of our simplest amplifier, which is directly calculating its transfer function. We're going to find out that's a huge pain. So we're going to try to make some physical sense of the transfer function using an effect called the Miller Effect, and we'll use that to make a simplifying approximation. Finally, we'll wrap up talking about amplifier step responses. This is all a bit of a grab bag, but it's all important context for what a dynamic response looks like. We'll follow up with a more unified analysis of dynamics in the next video series.

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Common Emitter Amplifier Transfer Function

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In this video we're going to try to analyze the dynamic behavior of our simplest amplifier, the common emitter. That means we'll try to guess what it's Bode plot looks like based on circuit analysis. It might be tempting to do that by just finding the transfer functions for all of our circuits. This video is motivation to _not_ do that. We're going to find out that the common emitter transfer function is very complicated, so we'll start looking for ways to simplify it next.

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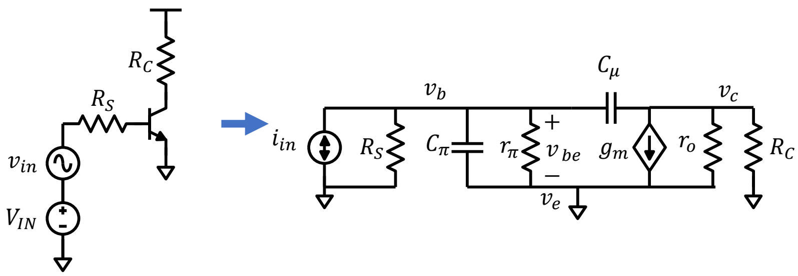
The CE Transfer Function is Complicated



$$i_{in} = v_b C_\pi s + \frac{v_b}{R_S||r_\pi} + C_\mu s(v_b - v_c)$$

$$C_\mu s(v_b - v_c) = g_m v_b + \frac{v_c}{R_C||r_o}$$

$$\frac{v_{out}}{v_{in}} = \frac{\left(1 - \frac{sC_\mu}{g_m}\right)}{1 + s\left(C_\mu(R_C||r_o + R_S||r_\pi + g_m(R_C||r_o)(R_S||r_\pi)) + C_\pi(R_S||r_\pi)\right) + s^2 C_\mu C_\pi (R_C||r_o)(R_S||r_\pi)}$$

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I've drawn a common emitter on the left here and put its dynamic small signal model in the center of the slide. I took an unusual step when I made the small signal model, which was converting the input voltage source into an input current source. This just saves us a bit of algebra.

CLICK We set this problem up by writing two nodal equations. This top one balances current at the base, and the bottom one balances current at the collector. You can see that these are two variables in two unknowns. Surely simplifying these will be straightforward and quick, and it will yield a super tractable expression.

CLICK So just a little bit of algebra happens, and we get this monstrosity. This expression is completely accurate, so its good that we have it around as a point of comparison for our approximations. However, this is complicated enough that its almost unususable, so don't worry about memorizing it. Do notice however, that we have a second order system with a right-half-plane zero.

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CE Transfer Has Real, Widely-Spaced Poles

$$\frac{v_{out}}{v_{in}} = \frac{\left(1 - \frac{sC_\mu}{g_m}\right)}{1 + s\big(C_\mu(R_C||r_o + R_S||r_\pi + g_m(R_C||r_o)(R_S||r_\pi)) + C_\pi(R_S||r_\pi)\big) + s^2 C_\mu C_\pi (R_C||r_o)(R_S||r_\pi)}$$

No overshoot, so we assume $D(s) = (\tau_1 s + 1)(\tau_2 s + 1) = \tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)s + 1$

Assume widely separated poles, ie: $\tau_1 \ll \tau_2$, so $D(s) \approx \tau_1 \tau_2 s^2 + \tau_1 s + 1$

$$\tau_1 = \frac{1}{p_1} = C_\mu\big((R_C||r_o) + R_S||r_\pi + g_m(R_C||r_o)\,(R_S||r_\pi)\big) + C_\pi(R_S||r_\pi)$$

$$\tau_2 = \frac{1}{p_2} = \frac{C_\mu C_\pi (R_C||r_o)(R_S||r_\pi)}{C_\mu(R_C||r_o + R_S||r_\pi + g_m(R_C||r_o)\,(R_S||r_\pi)) + C_\pi(R_S||r_\pi)}$$

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We can simplify this expression with a few highly accurate assumptions. We start by breaking the transfer function down. We can see that we have a right half-plane zero on the top of the expression. That zero turns out to be quite bad for stability, but we'll deal with it later. We also see that we have two poles.

CLICK Those two poles are real. One way you can justify this is that you've never seen an amplifier have a ringing step response in class. So that means we can write this denominator polynomial, D(s), as the product of two first order terms. Multiplying that out gives us this second order expression.

CLICK The next thing to note is that those two poles are widely separated. So one is very dominant, it is at a much lower frequency than the other. We are just asserting this for now, but we can check it at the end of the slide. With this assumption we can rewrite our expression for the denominator, and we can particularly note that the first time constant is just the coefficient for the s sterm in D(s).

CLICK That means that we can just pick our first time constant out of our transfer function. Like we said before, it's the coefficient for s. We could almost just stop here. We know our 3dB frequency is going to be set by this pole because it's much slower than the other pole in the system, so we generally care most about having a good estimate of this for bandwidth estimation purposes.

CLICK However, higher order poles can matter a lot for stability, so we'll soldier on for a bit. We can find our second pole location by taking the ratio of the s^2 coefficient to tau1. We can see that our assumption of wide separation is correct by focusing on the gm*(Rc||ro) term in the denominator. That term is amplified by the voltage gain, which we usually design to be quite large, so it's going to divide down the top easily.

This is all great, and we do have some simpler expressions now, but this is still hard to use in the lab. One thing that's particularly frustrating is that both tau1 and tau2 depend on both Cpi and Cmu, so we can't use our trick of Thevenizing from one cap to calculate time constants quickly. That's an important detail: it's tempting to associate each cap with a pole, but that approach falls apart in complicated circuits.

… Another is that it's impossible to make an underdamped system in a circuit without an inductance, and this is just a big mess of Rs and Cs.
… That means the Cmu term is amplified by the amplifiers voltage gain, which we presume is big.

Summary

- The common emitter has a complicated transfer function

$$\frac{v_{out}}{v_{in}} = \frac{\left(1 - \frac{sC_\mu}{g_m}\right)}{1 + s\left(C_\mu(R_C||r_o + R_S||r_\pi + g_m(R_C||r_o)(R_S||r_\pi)) + C_\pi(R_S||r_\pi)\right) + s^2 C_\mu C_\pi (R_C||r_o)(R_S||r_\pi)}$$

- The CE has two, real, widely-spaced poles & one right-half-plane zero.

$$\tau_1 = \frac{1}{p_1} = C_\mu(R_C||r_o + R_S||r_\pi + g_m R_L(R_S||r_\pi)) + C_\pi(R_S||r_\pi)$$

$$\tau_2 = \frac{1}{p_1} = \frac{C_\mu C_\pi (R_C||r_o)(R_S||r_\pi)}{C_\mu(R_C||r_o + R_S||r_\pi + g_m R_L(R_S||r_\pi)) + C_\pi(R_S||r_\pi)}$$

- Poles aren't associated with specific caps.
- This is unsustainable, we need faster, lab-suitable BW estimation.

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The Miller Effect and the Miller Approximation

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In this video we're going to discuss a common circuit behavior that will help us simplify the common emitter transfer function.

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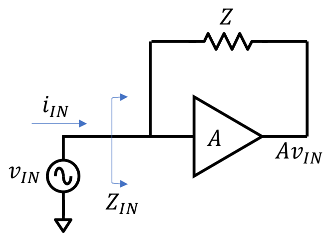
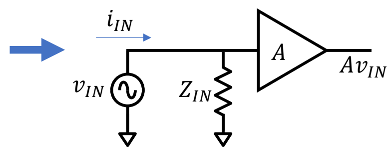
Z in Feedback Around Gain Looks Small



$$i_{in} = \frac{v_{in} - Av_{in}}{Z}$$

$$Z_{IN} = \frac{Z}{1 - A}$$

The Miller Effect



The Miller Approximation

(Note: ignores feed-forward current in Z)

- Note: if A is negative, then $z_{in} = Z/(1 + |A|)$
- Note: if $Z = 1/Cs$, then $C_{in} = (1 - A)C$.
- i.e.: Miller Effect makes caps look bigger!

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We're going to talk about a circuit behavior called the Miller Effect, and it's easiest to see the Miller Effect in action using this super simplified, single-ended, ideal amplifier. We're curious how having this impedance Z in feedback around the amplifier affects the amplifier's input impedance. Note that the input impedance would be infinite if we were just looking into a perfect amplifier, so this Z is almost guaranteed to lower the input impedance.

CLICK We start that calculation by noting that our test current has to flow through Z, so we can find it by taking the voltage across Z divided by the impedance. The voltage across Z is pretty easy to calculate, because our ideal amplifier sets the voltage on the far side to A*vIN, so we just take the voltage difference divided by Z.

CLICK That means we can do a simple rearrangement to find that ZIN is Z/(1-A). This is super interesting. It means we can modify the size of the impedance perceived by vIN by changing A. That includes making the impedance infinite if our gain is exactly one. That makes sense because there will never be a voltage across Z if A is equal to one. This relationship, that ZIN=Z/(1-A) is called the Miller Effect.

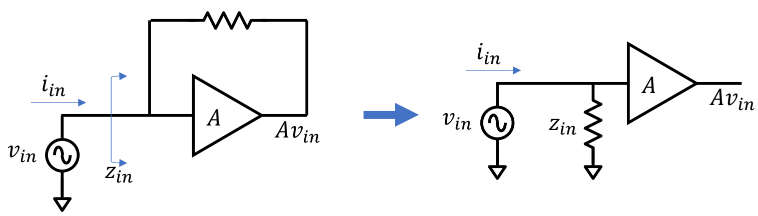
CLICK We can take this one step further, and say that we can represent our circuit with a simplified version that pulls ZIN out of feedback and instead puts it in shunt with the amplifier A. This is called the Miller Approximation. It loses some crucial information, like the fact we'd expect vIN to shoot some feed-forward current through Z to the output node, but getting rid of feedback makes the circuit MUCH easier to analyze.

CLICK I want to call out a few cool details about the Miller Effect. First, note that if your gain is negative, which is the case for most of our amplifiers, then the denominator of the Miller Effect expression adds together instead of subtracting. That's nice for analysis because it avoids the weird infinite impedance case I mentioned earlier.

Second, if your impedance Z is a capacitor, then your impedance is 1/Cs and you can flip the Miller expression upside down. That means your input cap is your feedback cap multiplied by (1-gain). In other words, you perceive a capacitance that is bigger than your feedback cap by a factor of your voltage gain. This happens because the voltage gain increases the swing across the cap, so the total voltage change it perceives is increased, and you need to push more charge through it to accomplish that voltage change. This effect is also why Cmu is so bad for amplifiers, it gets Millerized, which makes it seem much larger than Cpi if the voltage gain is high.

Summary

- Miller Effect changes the input impedance of amplifiers w/ feedback.
- Comes from gain of amplifier increasing swing across the element.
- Miller Effect with negative gain makes caps look bigger.



$$z_{in} = \frac{Z}{1 - A}$$

The Miller Approximation for a Common Emitter

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In this video we're going to use the Miller effect to make an equivalent circuits for the common emitter amplifier and see if we can simplify our bandwidth calculation.

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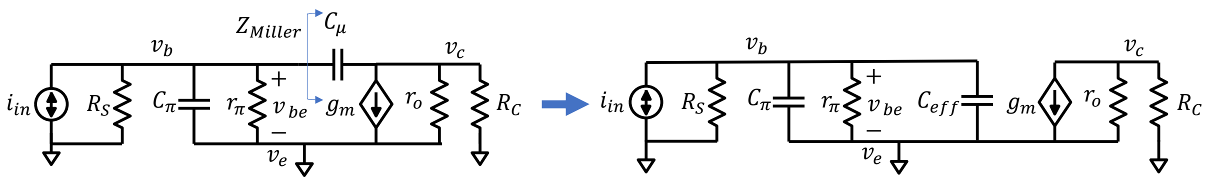
Miller Effect Happens Between Base/Collector



$a_v = -g_m(R_C||r_o)$

$v_c = a_v v_b$, so C_μ is in feedback around a_v

$C_{eff} = C_\mu(1 + g_m(R_C||r_o))$

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I've copied our model for the dynamic common emitter here, and we're going to see if we can make it simpler using the Miller Approximation.

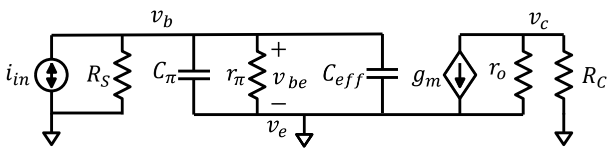
CLICK First, we recall that our gain is –gm*(RC||ro).

CLICK But we can also recall our definition of av, which is that it's the change in voltage at the output node, vc, divided by the input node, vb. So our gain relates the behavior at vb and vc, which means Cmu is in feedback around this amplifier because it connects our input and our output nodes. All the rest of the CE is just a fancy way of drawing the gain triangle from the Miller Effect video, and Cmu has that gain fall across it.

CLICK So we're going to try to find a miller representation for Cmu and attach that to ground.

CLICK Doing so gives us this schematic, where Ceff is a Millerized Cmu.

Miller Approx. Ignores Other C_μ Effects



With $C_{eff} = C_\mu(1 + g_m(R_C||r_o))$

Miller Approx: $v_o = -g_m(R_C||r_o)\frac{1}{(R_\pi||R_S)\left(C_\pi + C_\mu(1 + g_m(R_C||r_o))\right)s + 1}\left(\frac{v_{in}}{R_S}r_\pi||R_S\right)$

$$\tau_1 = (R_\pi||R_S)C_\pi + (R_\pi||R_S)C_\mu + g_m(R_\pi||R_S)(R_C||r_o)C_\mu$$

Full fn: $\frac{v_{out}}{v_{in}} = \frac{\left(1 - \frac{sC_\mu}{g_m}\right)}{1 + s\left(C_\mu(R_C||r_o + R_S||r_\pi + g_m(R_C||r_o)(R_S||r_\pi)) + C_\pi(R_S||r_\pi)\right) + s^2 C_\mu C_\pi (R_C||r_o)(R_S||r_\pi)}$

Lost this!

Lost this!

$$\tau_1 = (R_\pi||R_S)C_\pi + (R_\pi||R_S)C_\mu + (R_C||r_o)C_\mu + g_m(R_\pi||R_S)(R_C||r_o)C_\mu$$

Lost this!

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I've copied our simplified schematic from the previous page to see if we can find the transfer function more easily.

CLICK it turns out that we can! That's because our output voltage is given by the usual gm generator current times some resistance, and all the cap is not in parallel at the input. So the dynamic term in the middle of the page says that our base voltage is going to roll off in a first order way at higher frequencies, which reduces our overall gain. The third term with vin is a bit complicated, but it's just some nonsense that is involved with turning our Norton input back into a Thevenin input, and it turns into a boring divider between rin and Rs.

We can estimate our bandwidth pretty easily with this approximation because we're just pretending the common emitter is a first order system. So we grab tau from our s term and call it a day. Notice that the Millerized Cmu is featured prominently in our bandwidth estimation.

CLICK We can compare this against the full function we derived earlier. I've pulled out the tau1 value and written it in the same way as the Miller version in the middle of the page to facilitate that comparison.

CLICK When we do that comparison, we see some big differences. First, we've clearly lost some nuance, because we don't have our feedforward zero or our second pole. Maybe that's fine. The feedforward zero is at a really fast speed, and we were assuming the first pole was dominant anyway. However, we also lost a term in our estimate of the first pole, so our Miller Approximation is going to have a slightly smaller time constant than our real expression.

None of these are great behaviors for an approximation. Losing the second pole and the zero can affect our stability, and if we're estimating bandwidth then we'd prefer to have a conservative underestimate to an overestimate. So the Miller Approximation is not conservative, and it loses a lot of nuance, but it's super quick and it gives us an easy way to estimate the effect of feedback capacitances.

Summary

- C_μ is Millerized in a common emitter
- Miller approximation: pretend the input cap (C_π and Millerized C_μ) determine the whole transfer function.
- Gives us a first order transfer function, loses zero and second pole
- Not a conservative approximation, but good because it's very simple.

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Step Responses in Amplifiers

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In this video we’re going to talk about step responses in amplifiers.

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Step Responses Are Sums of Pole Responses

$$\frac{Y(s)}{X(s)} = H(s) = K\frac{N(s)}{D(s)} = K\frac{\left(\frac{s}{z_1}+1\right)\left(\frac{s}{z_2}+1\right)\left(\frac{s}{z_3}+1\right)\ldots}{\left(\frac{s}{p_1}+1\right)\left(\frac{s}{p_2}+1\right)\left(\frac{s}{p_3}+1\right)\ldots} = K\left(\frac{f_1(s)}{\frac{s}{p_1}+1}+\frac{f_2(s)}{\frac{s}{p_2}+1}+\frac{f_3(s)}{\frac{s}{p_3}+1}\ldots\right)$$

Value of f_i is set by z_i

p_i either real or conjugate pairs

Apply inverse Laplace to each term to get: $y_h(t) = \sum_{1}^{n} C_i \exp p_i t$

Fast poles settle before slow poles start. Can approximate most systems as 1st or 2nd order.

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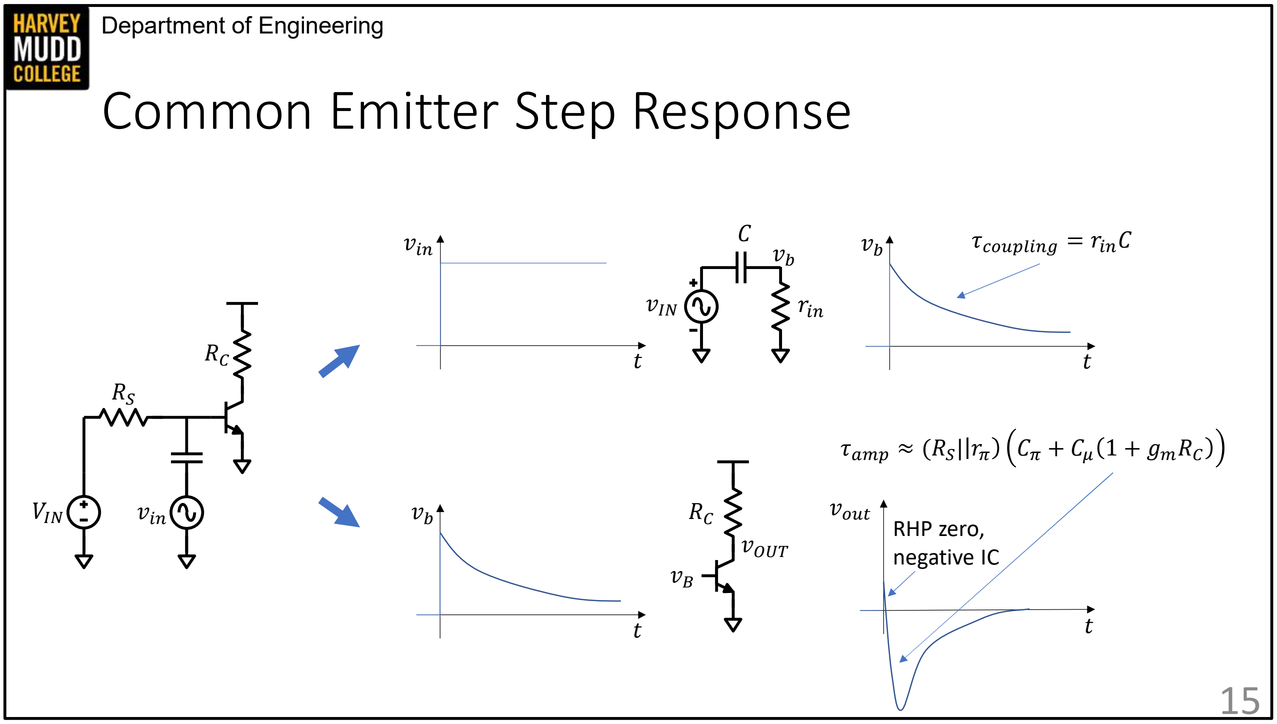
We often focus on the frequency domain when we're doing amplifier design because using impedances make frequency domain analysis so easy. However, the step response performance of amplifiers can be more important than their bandwidth in some applications.

We can start to analyze step responses to amplifiers by noticing that the dynamic gains we've been calculating are transfer functions, and by going back to the basics of what our transfer functions mean. This slide shows a bunch of representations of a transfer function H. One of the most important things to remember is that transfer functions are defined as the ratio of some output Y to some input X. The second representation on here acknowledges that any transfer function is going to be a ratio of a numerator polynomial to a denominator polynomial, multiplied by some gain. You can expand those numerator and denominator polynomials in terms of the individual zeros and poles that make them up. Note that this is essentially the tau*s+1 factor form of each singularity, but I've replaced the time constants by pole and zero locations, which are the inverse of the time constants. We can imagine doing a giant partial fraction expansion to split this up into a bunch of pole terms. It's worth noting that the numerators of these terms will be set by the zero polynomial, and also that the p values are either real or complex conjugate pairs for real-valued signals.

CLICK The super important relationship we get from this exercise is that the homogeneous solution to y, which is part of its response to a step or an impulse, is given by a weighted sum of pole responses. Note that the Ci values are going to be determined by the zero behavior.

CLICK This has a bunch of super interesting implications. It means that we can essentially ignore the behavior of widely separated poles. Fast poles with small time constants might completely settle out before slow ones start, so they'll look essentially like steps in our output. That type of response, with a fast partial and a slow tail settling, is called a doublet response.



That's a bit abstract, but we can take a quick look at how it plays out in a common emitter. One important and common situation is where we're trying to measure the step response of an AC coupled amplifier like the one I've drawn on the left.

CLICK We can analyze it by breaking it into a few parts. We know our amplifier model presents rin from the base to ground, so a small signal model reveals that vb is just related to vin by a high pass filter. When we put a step on vin, we expect vb to have a decaying exponential response. We can figure out the time constant of that response pretty quickly: it's just rin times C. One aside: If our input impedance had some cap in parallel with vb, then we'd expect a capacitive divider here too, but coupling caps are usually big enough that the capacitive division effect is small.

CLICK We can then treat that vB input as the signal driving out. The initial step in vb triggers the step response of the amplifier, which is much faster than the coupling network. We see that behavior in the sketch I've put on the right side here. The amplifier has a quick step response with a time constant set by the first pole of the common emitter. Then the amplifier is able to follow the slow decay of vb exactly.

There are a few details I want to call out here. One is that the gain of the amplifier is negative, so an increase of the input leads to a large decrease in the output. The second is that there's a little error right at the start of the output voltage step, where it increases before becoming negative. That behavior is the consequence of having a feedforward zero. You can justify its existence by thinking about the initial value theorem with the full CE transfer function, the zero in the numerator has a negative sign that flips your gain around in the initial condition.

You can also imagine current shooting through the feedback cap to the output impedance, which increases the output voltage, before the amplifier has a chance to respond and decrease the voltage. You might notice that this second argument holds for anything that is in parallel with the amplifier, and that's a good observation. Generally, any two parallel paths with different frequency responses will have a zero associated with them.

Regardless, this behavior is called feedforward dynamics, and the right half plane zero is called a feed forward zero.

Summary

- Step responses are made of a sum of decaying pole responses.
- Zeros set initial conditions in step responses.
- Zeros come from parallel paths in circuits.
- AC coupled step responses: fast amp settling + slow coupling settling.
- RHP zero in CE is a “feedforward zero”, sets IC in wrong way.