

# Common Base Amplifiers and Cascode Amplifiers

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E151 – Analog Circuit Design

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We have been taking a tour of single transistor amplifiers in order to find more amplifiers variants that we can substitute into our multistage amplifiers. In this video series we're going to conclude that tour by introducing an amplifier called the common base. We'll look at some practical quirks that arise when you bias and use the common base, and then we'll wrap up by looking look at a type of circuit called a cascode, which is a real word that describes a cascade of cathodes, where a common emitter and common base are combined together to achieve some interesting amplifier performance specifications.

# Common Base Amplifiers

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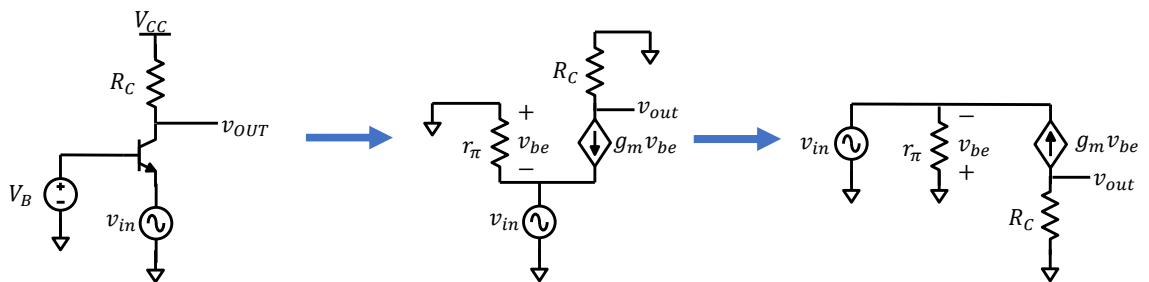
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In this video we're going to introduce a new single stage amplifier called a common base amplifier.

## Common Base Amplifiers Have Unusual Specs

- Low  $r_{in}$ , modest  $a_v$  and potentially interesting  $r_{out}$  (more later).
- Applications in RF circuits, compound amplifiers, current buffers.
- Assuming  $r_o$  is infinite here, though not hard to derive if finite.



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Common base amplifiers aren't going to see a ton of use in this class, but they are popular in a variety of applications. They have low input resistance, which is useful for interfacing with high impedance current sources or with very low output impedance elements like 50 ohm antennas.

Common bases have interesting output impedances because they are potentially quite high. We won't see this in our analysis because we're going to assume  $r_o$  is infinite for simplicity, but the impedance looking down into the collector is very high. However, the collector resistor falls in parallel with this very high resistance, so this resistor loaded amplifier never exhibits high output resistance.

We're going to analyze the amplifier parameters of this amplifier and, as usual, we need to start by making a small signal model. Pause the video and draw a small signal model of the common base.

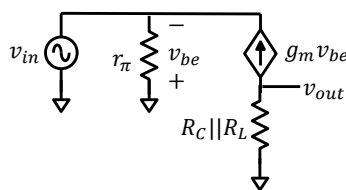
CLICK Here's a small signal model where we directly substitute the hybrid-pi model into the circuit

CLICK and here's a simplified version where I've rearranged the elements to make the analysis simpler.

... we won't see the high rout because ro is infininte

## Common Base $r_{in}$ and $r_{out}$

- For  $r_{in}$  set  $R_S = 0, R_L$  proper value
- For  $r_{out}$   $R_L = \infty, R_S$  is proper value
- $R_L$  parallel with  $R_C$
- Use  $v_{in}$  as test source

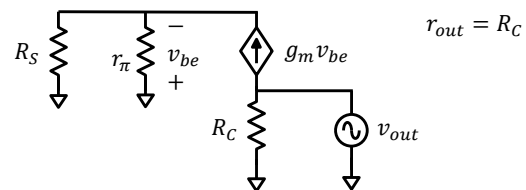


$$i_{in} = i_{r\pi} + i_{gm}$$

$$i_{in} = \frac{v_{in}}{r_{\pi}} + g_m v_{in}$$

$$r_{in} = \frac{1}{g_m} || r_{\pi}$$

$g_m$  zeroed by shorted  $v_{in}$ .  
If  $r_o$  finite, this is a left-right pattern



$$r_{out} = R_C$$

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This is a pretty simple small signal model, so I'm going to turn you loose on the  $r_{in}$  and  $r_{out}$  calculations soon. As a reminder, for  $r_{in}$  we set  $R_S$  to be zero and  $R_L$  to a proper value for the amplifier, which falls in parallel with  $R_C$ . We use a voltage test source. Pause the video and calculate the input resistance.

CLICK The input current in this amplifier is the sum of the current in the  $g_m$  generator and the current in  $r_{\pi}$ . Note that when I wrote this equation I implicitly flipped around the direction of  $v_{be}$  and the  $g_m$  generator because I presumed the  $g_m$  current was flowing out of the node at the top of the amplifier. We can sub in expressions for the  $r_{\pi}$  and  $g_m$  current because  $v_{in}$  directly controls  $v_{be}$ , which is the same pattern we saw at the output of the emitter follower and everywhere else we've attached a voltage source to an emitter. Rearranging the equation we find the input resistance is  $1/g_m$  in parallel with  $r_{\pi}$ .

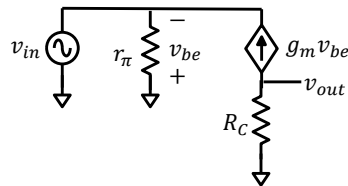
CLICK To find  $r_{out}$  we let  $R_L$  be infinite and set  $R_S$  to a normal value, which connects from the base to ground. Pause the video and try to find  $r_{out}$ .

CLICK The  $g_m$  generator gets shut off by  $R_S$  grounding  $r_{\pi}$ , which means the output voltage source only sees  $R_C$ . That's a quick calculation, but it would be more interesting if  $r_o$  were finite. That would mean we'd see a left-right pattern looking down into the collector, and that pattern would lead to a Thevenin impedance as high as  $\beta \cdot r_o$ . That's interestingly

high!

## Common Base $a_v$

- Set  $R_S = 0, R_L = \infty$
- Must use test voltage



$$v_{out} = -i_{gm}R_C$$

$$v_{out} = -(-g_m v_{in})R_C$$

$$a_v = g_m R_C$$

Finally, we'll find the voltage gain. We let the source impedance be zero and the load impedance be infinite as usual, then apply a test voltage source. Pause the video and try to find the voltage gain.

CLICK The output voltage is going to be  $-i_{gm}R_C$ , and we can find  $i_{gm}$  easily because  $v_{in}$  controls negative  $v_{be}$  directly. Note that I haven't done the current reversal that I did on the last slide, instead opting to write out all the negative signs in this equation. This means the gain is positive  $g_m R_C$ . The positive gain is an interesting change of pace that can have some nice properties in feedback or noise cancelling systems.

## Summary

- Common base amplifiers have low  $r_{in}$ , which is useful in specialized applications.
- Common base  $r_{out}$  is more interesting if we allow finite  $r_o$ .

$$r_{in} = \frac{1}{g_m} || r_{\pi}$$

$$r_{out} = R_C$$

$$a_v = -g_m R_C$$



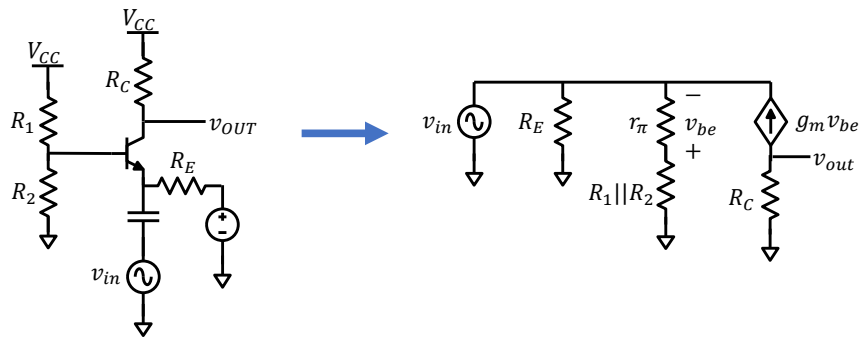
# Common Base Miscellany: Biasing, T-Models and $a_i$

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In this video we're going to look at a collection of three interesting facts about common base amplifiers.

## Biasing Common Bases is Hard

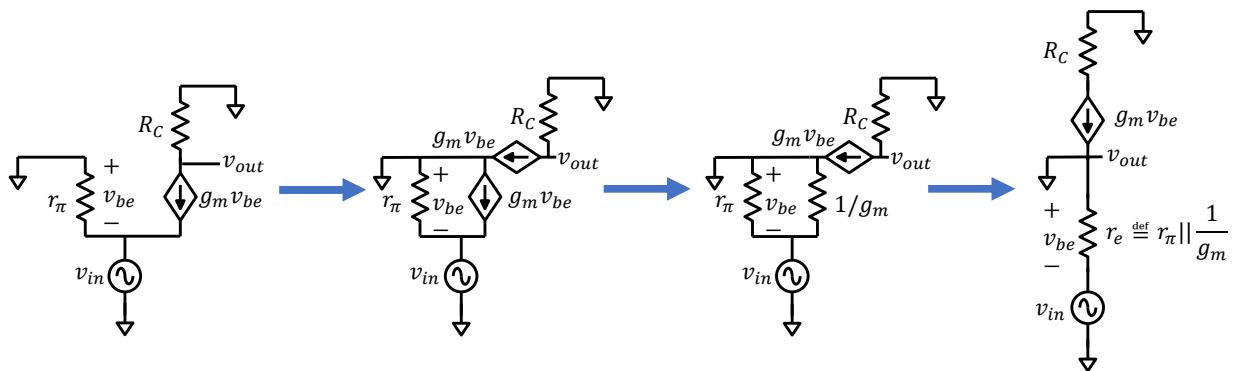
- Bias resistors create an effective  $g_m$  divider. Fix w/ cap, but take care.
- Need  $R_E$  to capacitively couple input, but  $R_E$  makes a current divider.



The first of these is that biasing common base amplifiers is more challenging than you'd expect. When you put a voltage divider on the base, it appears as a resistor to ground in the small signal model. This means  $v_{in}$  no longer controls  $v_{be}$  directly, and instead it's related to  $v_{be}$  through a voltage divider. This is a great opportunity to calculate changes in the gain using our effective  $g_m$  transform, but I won't chase that through here. A capacitor at the base can short out the  $R_1 || R_2$  resistor in the mid-band, but that capacitor is prone to introducing instability into the amplifier for reasons we'll discuss later.

In addition, if we need to bias the emitter, then the emitter biasing resistor appears in parallel with the input impedance, creating a current divider. You might be tempted to set  $R_E$  to zero, but that would result in the voltage source shorting the emitter input to small signal ground. You might also be tempted to set it to infinity, effectively removing this biasing circuit from the amplifier, but the capacitively coupled input doesn't provide a DC path to ground to bias the amplifier, so we're stuck with the bias network stealing a bit of current. Fortunately, the small input resistance affords us leeway in picking  $R_E$ .

# T-Model Can Be Used to Analyze CB



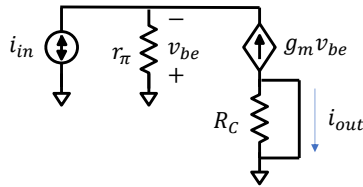
Second, though this is a bit of a detour, I wanted to show you this alternate way of analyzing a common base. We're not going to use it, but it's handy in case you see someone else use it in the future.

You can transform the small signal model of the common base by pretending the current source flows into the base and then out to the emitter. Recall that our models only capture the terminal behavior of transistors, so we can do these transforms as long as  $i_b$ ,  $i_c$  and  $i_e$  are the same.

The  $g_m$  generator between the base and the emitter is in parallel with the control voltage across  $v_{be}$ , so we can replace it with  $1/g_m$ . We see that in the third subcircuit here. Collapsing  $r_{pi}$  and  $1/g_m$  in parallel reveals a fairly simple model where  $v_{in}$  sees a single resistor to the grounded base. That resistor value is  $r_{pi}$  in parallel with  $g_m$ , which is sometimes called  $r_e$ .

## Current Gain, $a_i$ , of Common Base is $\approx 1$

- $a_i \stackrel{\text{def}}{=} \partial i_{out} / \partial i_{in}$  is measured w/ test current, Norton  $R_S = \infty$ ,  $R_L = 0$
- Common base  $a_i \approx 1$ , so they make good current buffers.



$$i_{out} = -i_{gm} = g_m v_{be}$$

$$i_{in} = \frac{v_{be}}{r_{\pi}} - g_m(-v_{be})$$

$$v_{be} = \frac{i_{in}}{\frac{1}{r_{\pi}} + g_m} = \frac{r_{\pi} i_{in}}{\beta + 1}$$

$$i_{out} = g_m \frac{r_{\pi} i_{in}}{\beta + 1} = \frac{\beta}{\beta + 1} i_{in}$$

Third, the current gain of a common base is about 1, which means you can think of common base amplifiers as current buffers. Said another way, any current that goes into them comes out from a high output impedance, as if driven from a current source.

We measure current gain in a way similar to voltage gain. We imagine a Norton test current with infinite source impedance drives into the amplifier input, and then measure how much current goes through a shorted load attached to the output. We start our analysis by noticing that  $i_{out}$  is equal to  $-i_{gm}$ , given by  $g_m \cdot v_{be}$ . We can find  $v_{be}$  by writing KCL at the top node, and we follow that math through and sub it into our  $i_{out}$  expression to find  $i_{out}$  is  $\beta / \beta + 1$   $i_{in}$ , so the output current is an almost exact copy of the input current.

## Summary

- Base bias in common base causes an effective  $g_m$  divider
- Emitter bias of common base creates current divider.
- Can analyze a common base by transforming circuit into a T-model
- Common base current gain is  $\frac{\beta}{\beta+1}$

# Cascode Amplifiers

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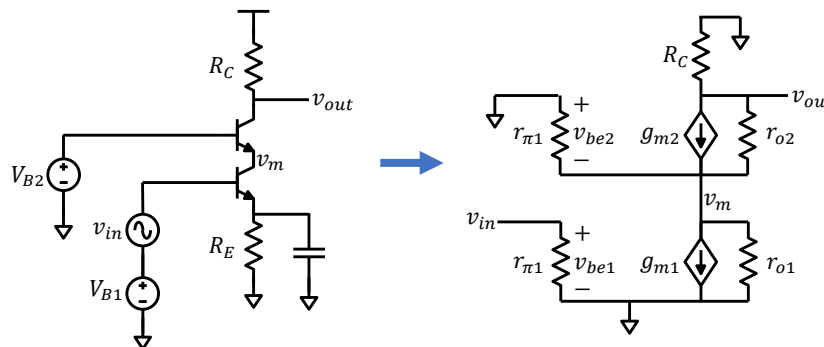
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In this video we're going to see one of the most common applications of a common base, which is as part of a compound amplifier called a Cascode. Cascode is short for cascade of cathodes, a portmanteau courtesy of old-timey engineers. Compound amplifiers are single-stage amplifiers that feature two transistors that interact in some interesting way, and they can often be analyzed by breaking them into multiple stages. We'll find  $r_{in}$ ,  $r_{out}$  and  $a_v$  for cascodes like we have for other stages.

## Cascodes are CE with CB on Top

- Will find they have modest  $r_{in}$ , high-ish  $r_{out}$ ,  $a_v \approx -g_m R_C$
- However,  $r_{out}$  has the potential to be very large, can use for high gain.
- Have very important properties when we study dynamics.



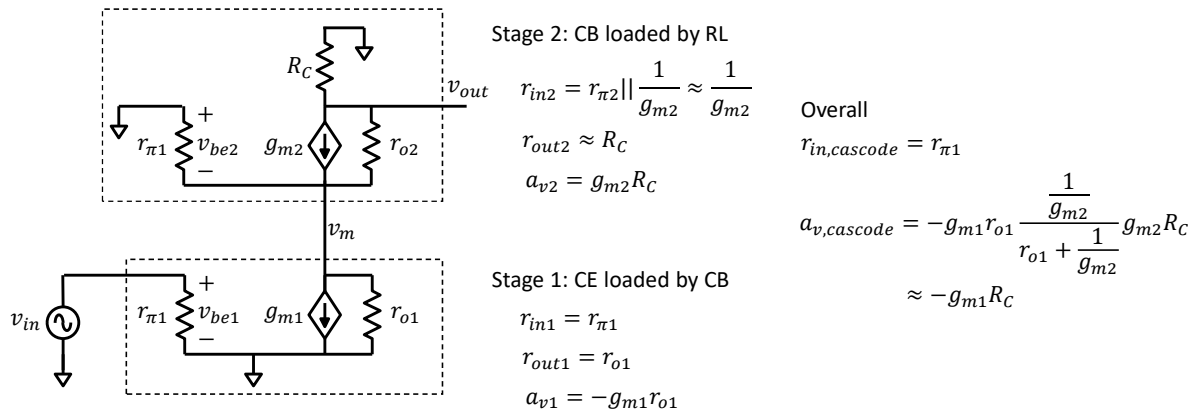
The cascode is an extremely common amplifying structure, so much so that I've joked that there are only two tricks in analog design: cascoding and differential signals. So that makes it surprising that we won't find any spectacular amplifier parameters when we analyze it: the  $r_{in}$  will be middling,  $r_{out}$  will be sort of high and  $a_v$  will be the same as a common emitter. However, like the common base with finite  $r_o$ , we'll see that the output impedance is given by something huge in parallel with  $R_C$ . Later we'll try to leverage that super high output resistance to make an amplifier with very high voltage gain. Cascodes also operate at much higher frequencies than common emitters for reasons we'll understand when we study dynamics.

We're going to assume  $r_o$  is finite during this analysis because it doesn't make much sense if we don't.

The first step in finding amplifier parameters is to draw a small signal model for the common emitter with degeneration. Pause the video and try drawing one now.

CLICK This is my version!

# Analyze Cascode $a_v$ & $r_{in}$ as 2 Stage Amplifier



Now we're going to find the voltage gain and input resistance of the cascode. We'll do that by analyzing the cascode amplifier as a two stage amplifier. We draw an amplifier model box around the bottom resistor and call it a common emitter that is loaded by our second stage, a common base. We draw an amplifier model box around the upper transistor to discover that our second stage is a common base.

Knowing this, pause the video to identify the parameters of each of these amplifier stages. You may approximate  $r_{o2}$  as infinite when finding  $r_{out2}$ , but otherwise assume it's finite. Or, if you're feeling adventurous, let  $r_{o2}$  be finite for your whole analysis.

CLICK The parameters are here. Common emitters remain straightforward, so we know the  $r_{in1}$  is  $r_{\pi 1}$  by inspection. Finding  $r_{out1}$  and  $a_{v1}$  is also simple, but requires us to show great faith in our amplifier model. Though the emitter of the common base is right above the boundary of our model, we make sure to only consider things inside our amplifier model box when finding  $r_{in}$ ,  $r_{out}$  and  $a_v$ . The interaction between this and the common base will be taken into account when we do multistage analysis. That means our  $r_{out}$  is given by  $r_{o1}$  because  $v_{be1}$  is shorted to zero by  $R_S$  when we analyze the output resistance, and  $a_{v1}$  is given by the  $g_{m1}$  generator driving into  $r_{o1}$ .

The common base looks pretty normal, so we're not surprised to find that  $r_{in2}$  is



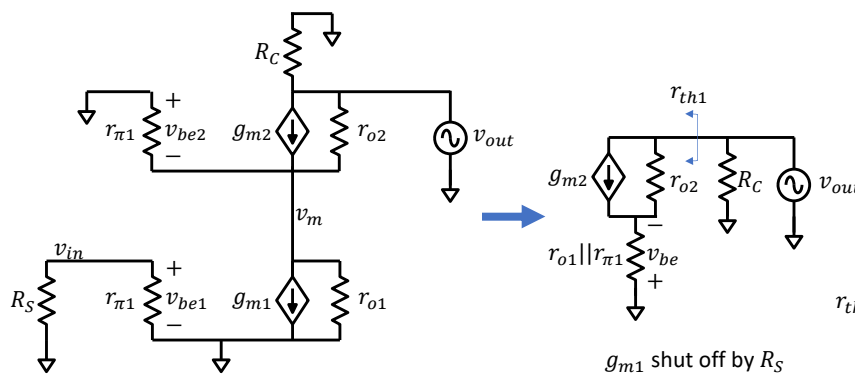
$r_{pi2} \parallel 1/g_{m2}$ ,  $a_{v2}$  is  $g_{m2} * R_c$  and  $r_{out2}$  is about  $R_c$ . The fact we have a finite  $r_{o2}$  means that approximated  $r_{out2}$  is no longer exact, but if you followed the analysis all the way through then you should note that  $R_c$  is in parallel with something really big, so the approximation is pretty good. You should also note that  $R_S$  slips into your answer.

Great, now that we've found the behavior of each stage we need to find the overall  $r_{in}$  and  $a_v$  of a cascode. Pause the video and do that.

CLICK The overall  $r_{in}$  is just the input impedance of the first stage, so it's  $r_{pi1}$ . The overall voltage gain is given by the gain of the first stage times the gain of the second stage times the interstage loading. Note that I've approximated  $r_{in2}$  as  $1/g_{m2}$ , ignoring  $r_{pi2}$ , which is fine because  $r_{pi2}$  is beta times bigger than  $1/g_{m2}$ . We can simplify this expression significantly if we note that  $r_{o1}$  is much much bigger than  $1/g_{m2}$ , which means the interstage loading cancels out a factor of  $g_{m2} * r_{o1}$  from our final expression. That means the gain of a resistively loaded cascode is about the same as a common emitter.

The gain and input resistance of this cascode amplifier are the same as a common emitter, but our output swing is guaranteed to be worse because we need to keep two transistors in forward active instead of just one. Right now, cascodes don't look like a good deal. They'll look more interesting later. However, one thing I wanted to emphasize is that you pay a little bit of output swing in order to gain the upcoming benefits of a cascode.

# Cascode $r_{out}$ Uses Left-Right Pattern



Find  $r_{th1}$  first, then put  $R_C$  in parallel:

$$v_{be2} = -i_{th1}(r_{o1} || r_{\pi1})$$

$$i_{th1} = i_{g_m2} + i_{r_{o2}}$$

$$i_{th1} = g_{m2}v_{be2} + \frac{v_{th1} - v_{be2}}{r_{o2}}$$

...

$$r_{th1} = r_{o2} + r_{o1} || r_{\pi1} + g_{m1}(r_{o1} || r_{\pi1})r_{o2} \approx r_{o2} + r_{\pi1} + \beta_1 r_{o2}$$

$$r_{out} = r_{th1} || R_C \approx R_C$$

Finally, we're going to find  $r_{out}$  of the cascode. This requires us to give up on our two stage model because we didn't find the  $r_{out}$  of a common base with a finite  $r_o$ , so we're kind of cleaning up some homework from earlier videos by letting  $r_o$  be finite here. If  $r_o$  was infinite, then  $r_{out}$  would be  $R_C$  and our two-stage model would be perfectly accurate. Letting  $r_o$  be finite is going to reveal an interestingly high impedance that I've referred to in the past.

Pause the video and try to find  $r_{out}$ .

CLICK I started finding  $r_{out}$  by redrawing the circuit. Doing so reveals that  $g_{m1}$  is shut off by  $R_S$ , so we have a significantly simpler model.

CLICK That model is comprised of a left-right pattern in parallel with  $R_C$ , so we're going to find  $r_{th1}$  first and then  $R_C$  back in parallel at the end of the analysis. Because the left-right pattern is relatively new, I'll step through the analysis of it line-by-line.

CLICK First we note that if we used an input test current to find  $r_{th1}$ , then all that current would wind up in the  $r_{o1} || r_{\pi1}$  tail resistor, which means our control voltage is  $-i_{th1}$  times  $r_{o1} || r_{\pi1}$ .

CLICK  $i_{th1}$  has to split between the gm generator and  $r_{o2}$ .

CLICK so we substitute expressions for  $i_{gm2}$  and  $i_{ro2}$  in terms of  $v_{be}$  and  $v_{th1}$ .

CLICK We could sub in our expression for  $v_{be2}$  and then rearrange these expressions for a while, and I've skipped all those algebra steps. The result is our usual left-right pattern result,  $r_{left} + r_{right} + g_m * r_{left} * r_{right}$ . In our case,  $r_{left}$  is  $r_{o2}$  and  $r_{right}$  is  $r_{o1} || r_{pi1}$ , which gives us this expression. There are a lot of  $r_o$  terms in it, so we assume it's going to be big, but

CLICK if we simplify it by noting that  $r_{pi1}$  is much smaller than  $r_{o1}$ , we find that this impedance is  $r_{pi1}$  plus  $(\beta + 1)r_{o2}$ . Beta times  $r_o$  is a really big number! Often on the order of 10 mega-ohms. That is interesting, and it means that a Norton model of the collector of a cascode would be an almost ideal current source.

CLICK However, that giant output resistance doesn't matter in this configuration because it's in parallel with RC. It might be cool to see if we could replace RC with a current source to allow this really big  $r_{th1}$  to interact with the current from our gm generator, and we'll be looking at ways to do that in the next two video series.

There's a little bit of chicken and egg reasoning going on in my assertion that the gm generator is off. I'm asserting that  $v_{be}$  is zero since the gm generator is off, and the gm generator is off because  $v_{be}$  is zero. I confront that reasoning by asserting that if everything on the left side of the circuit is attached to ground, there's no reason for non-ground values in the middle.

If you add  $r_o$  to this problem, then  $v_{be}$  is no longer zero and you have to do more analysis to find an expression for  $r_{out}$ . However, that analysis is a classic example where our small signal models pay off. I won't go over that solution in these videos, but encourage you to try finding  $r_{out}$  with  $r_o$  included and small signal patterns in mind.

## Summary

- Cascodes are compound CE/CB amplifiers w/ modest  $r_{in}$ ,  $r_{out}$  and  $a_v$
- But  $r_{out}$  has interesting properties: something big in parallel with  $R_C$
- Cascode amplifier parameters

$$r_{in} = r_{\pi 1}$$

$$r_{out} = R_C || (r_{\pi 1} + (\beta_1 + 1)r_{o2})$$

$$a_v = -g_{m1}(R_C || (r_{\pi 1} + (\beta_1 + 1)r_{o2})) \approx -g_{m1}R_C$$