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Small Signal Patterns and CE with Degeneration Resistor

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E151 – Analog Circuit Design

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In this video series we're going to take a brief sidetrack to work through some common small signal circuits that show up often. Knowing the solutions to these circuits should help you solve many BJT small signal problems quickly. We're then going to derive the amplifier parameters of another new type of amplifier called the common emitter with resistor degeneration so that we can use it in our multistage designs.

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Small Signal Patterns

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In this video we're going to look at some common small signal circuits and find their Thevenin resistances. This practice will be handy for finding amplifier parameters. I'm going to offer you the chance to work through these examples as I go through this video and doing so is great practice.

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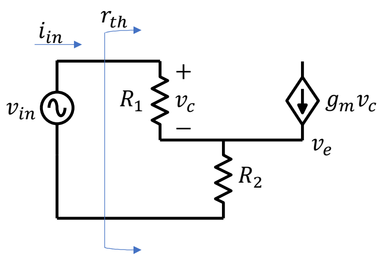
The R_E Boosting Pattern



$$v_e = i_{in}(g_m R_1 + 1)R_E$$

$$v_c = i_{in} R_1$$

$$v_{in} = v_e + v_c$$

$$r_{th} = R_1 + (g_m R_1 + 1)R_2$$

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The first small signal circuit we're going to look at is called the RE boosting small signal pattern. It is pictured above. You'll note that we leave the far side of the current source floating, and that's because current sources have infinite differential impedance, so they perfectly hide impedances or voltages on the other side of them. Also, note that we're using a generic vc to describe the control voltage of the gm generator, though it will be some variation on vbe in most circuits you analyze.

Pause the video and try to find rth.

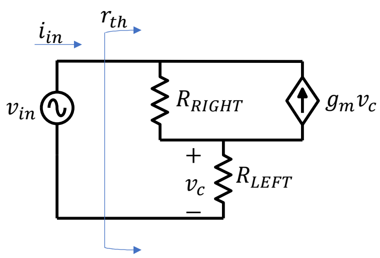
CLICK We solve this circuit by noticing that the emitter voltage is just R2 times the current running through R2, and that the current running through R2 is the R1 current plus the gm current. We're taking a shortcut here by noting that the gm branch is indirectly controlled by iin because vc is equal to gm*R1.

We then note that vc is equal to iin times rpi and that our total input voltage is ve+vc. Subbing into that equation and dividing off iin reveals that rth for this structure is R1+(gm*R1+1)R2. We saw exactly this result for the input resistance of the emitter follower, where R1 was rpi and R2 was a parallel combination of a bunch of stuff including RE. The fact R1 was equal to rpi caused the second term to be beta+1.

This structure has the effect of boosting the apparent impedance of R2, which sits in series with R1. That R2 boosting effect is because the gm generator supplies much of the current to R2 that is needed to set its voltage to the right level. That means the vin source doesn't need to send as much current as if the gm generator weren't there. That reduction in current sent from the vin source turns into a boosted resistance as seen from the port. This resistance boosting comes from a type of feedback: raising vin raises vc, which raises igm, which raises ve, which reduces vc. So the gm generator is kind of fighting to keep vc small.

That's all interesting, but the main takeaway is that if you see a pattern that looks like a generator driving into a node shared with a resistor with a control voltage, then you can use this Thevenin resistance.

The Left-Right Pattern



$$v_c = i_{in} R_{LEFT}$$

$$\frac{v_{in} - v_c}{R_{RIGHT}} = i_{in} + g_m v_c$$

$$v_{in} = i_{in} R_{RIGHT} + g_m v_c + v_c$$

$$v_{in} = i_{in}(R_{RIGHT} + R_{LEFT} + g_m R_{RIGHT} R_{LEFT})$$

$$r_{th} = R_{RIGHT} + R_{LEFT} + g_m R_{RIGHT} R_{LEFT}$$

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The second small signal circuit we're going to look at is called the left-right circuit, and that name is going to seem weird for a bit because it's named after a circuit we'll see much later in the course.

Pause the video and try to find rth.

CLICK We solve it by first noting that all the current that flows into the top two branches must also flow out of them, so iin has to be flowing in RLEFT. That means vc is equal to iin*RLEFT. Then we write KVL at the top node of the circuit, observing that the current in RRIGHT has to be equal to the sum of iin and gm*vc. Rearranging then subbing in vc reveals that rth for this structure is RRIGHT+RLEFT+gm*RRIGHT*RLEFT. We haven't seen this one in any circuits yet, but it's coming right up. You can imagine that RRIGHT might be a ro value from a BJT, because that falls in parallel with gm generators in our BJT model. With that hint, be sure to look for this pattern in the output impedance of cascode amplifiers.

The third term, which is the product of RRIGHT and RLEFT can be quite large. As is the case with the RE booster, that boosted impedance comes from feedback. Increasing vin should increase iin, but the total current seen by RRIGHT is reduced by the current supplied by the gm generator, which means iin is smaller for a given change in vin than it would be without the gm generator.

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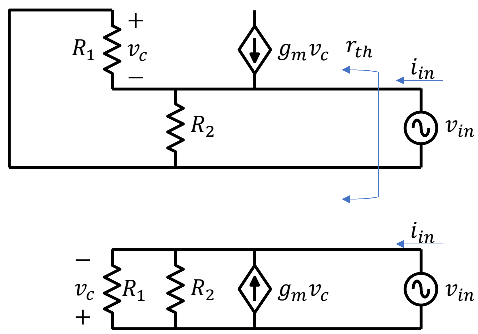
The $1/g_m$ Pattern



$$i_{in} = -g_m(-v_{in}) + \frac{v_{in}}{R_1} + \frac{v_{in}}{R_2}$$

$$\frac{v_{in}}{i_{in}} = \left(g_m + \frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

$$r_{th} = R_1 || R_2 || \frac{1}{g_m} \approx \frac{1}{g_m}$$

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The final small signal pattern we're going to look at is called the 1/gm pattern. I've drawn it in a "BJT way" on top and a "cleaned up way" on the bottom. These circuits are the same if you assume the gm generator attaches to ground on the other side, and we're free to pick that because it makes no different to how much current the generator produces.

Pause the video and try to find rth.

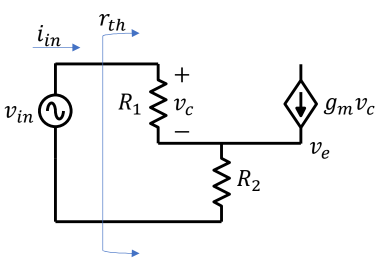
CLICK In this circuit we're apply a voltage source directly to the control resistor R1, so the gm generator is producing a current that's proportional to vin. We note this when we write a KCL equation to express iin as the current through each branch of this circuit. The gm generator is producing a current in the opposite direction of iin, so we're subtracting it from the KCL equation, but vin is also negative, which will cancel out that behavior. The other currents are just vin over resistances.

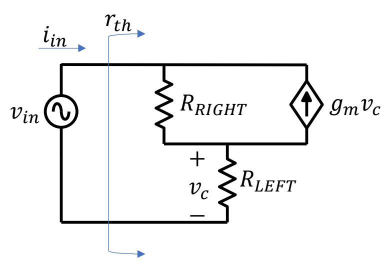
You can rearrange this equation to find that vin/iin, which is rth, is one over the sum of some conductances. That is the same form as a parallel combination, so we can rewrite rth as R1 in parallel with R2 in parallel with 1/gm. 1/gm is usually quite small, so this equation is often approximated as 1/gm.

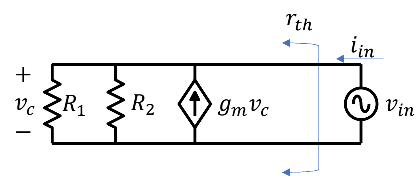
We saw this in the emitter follower circuit almost exactly, though there was a minor difference in the emitter follower result that we'll explain in the next video. In general, you should expect this any time you change a gm generators control voltage by wiggling the node the gm generator feeds. Making that more concrete for our class, you should expect to see an impedance that looks like 1/gm any time you wiggle an emitter.

Summary

- We saw three small signal patterns that appear in many BJT circuits:







$r_{th} = R_1 + (g_m R_1 + 1) R_2$

$r_{th} = R_{RIGHT} + R_{LEFT} + g_m R_{RIGHT} R_{LEFT}$

$r_{th} = R_1 || R_2 || \frac{1}{g_m} \approx \frac{1}{g_m}$

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Small Signal Pattern Transforms

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In this video we're going to add a handful of transforms to our toolbox that will enable us to recognize our small signal patterns in even more circuits.

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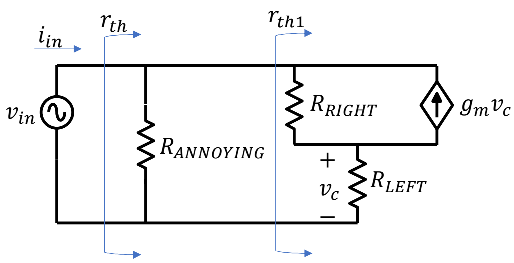
The "There's some Stuff in Parallel" Transform



$r_{th} = r_{th1} || R_{ANNOYING}$

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OK, the first transform we're going to talk about is useful when an annoying element is in parallel with an otherwise appealing small signal model. I've drawn a circuit above that looks like an annoying resistor in parallel with a great left-right pattern. If the annoying resistor weren't there, we could just use our earlier left-right result, but we derived that result assuming all of iin passed through RLEFT, and that's not true because the annoying resistor will steal some of iin.

CLICK So we're going to analyze this by changing the circuit we're looking at. First we're going to find rth1 looking only into the left-right branch, then we'll just note that the annoying resistor is in parallel with rth1. The takeaway is that you can peel off parallel elements to look at just a branch of interest just as long as you add them back at the end.

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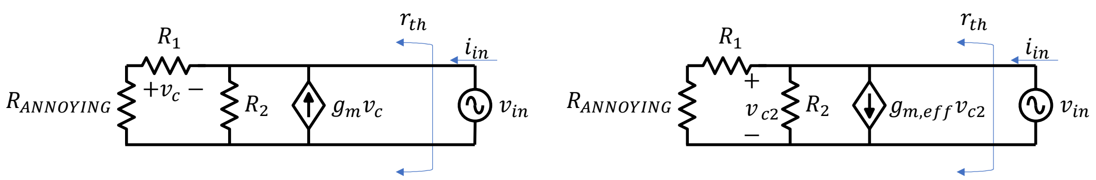
The Effective gm Transform

- Including gm reversal



$$g_{m,eff} = \frac{R_1}{R_{ANNOYING} + R_1} g_m$$

$$r_{th} = R_2 || (R_1 + R_{ANNOYING}) || \frac{1}{g_{m,eff}}$$

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The second transform we're going to see has to do with gm generators with inconvenient control variables. The circuit on the left looks almost like our 1/gm pattern, but the annoying resistor means our control voltage isn't between vin and ground. We can fix that by redefining our control voltage, which we see on the right. I've made two changes in that circuit: first, the control voltage now runs from vin to ground, but I've made sure we wind up with the same current in the gm generator by defining an effective gm that is a resistor divided version of gm. Second, I've reversed the direction of the current source by reversing the polarity of the control voltage. By redefining our control voltage we've made our math a little bit easier and given ourselves leeway to eliminate the node between R1 and the annoying resistor by combining those two in series.

In addition to making our life easier, redefining the control voltage like this gives us a bit of physical insight. The divider is reducing the voltage that's applied across the control resistor, so thinking of it as reducing the effect of gm is an apt summary of its effect.

This is a super common transform. We've already seen it at work in our emitter follower output resistance, where R1 was rpi and RANNOYING was the source resistance. This effect is why we add RS/beta to 1/gm in the emitter follower rout.

Summary

- Take inconvenient parallel elements off of your analysis and add them back in parallel at the end.
- Redefine the value of gm in order to use more convenient control variables.

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Common Emitter with Degeneration Resistor

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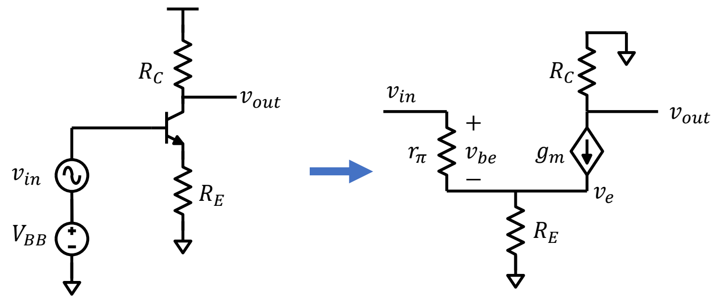
In this video we're going to find the small signal parameters of an amplifier called the common emitter with degeneration. Like the emitter follower, this is going to be a lot of ground even though the slides will go quick. Also like the emitter follower, you need to remember both the analysis and the results on these slides. Take it slow and work the math when you are asked. Fortunately, we'll see some of our small signal patterns in the common emitter with degeneration, which will hopefully lighten our mental load.

CE w/ Degen is CE w/ Non-Grounded Emitter

- Will find they have high r_{in}, high-ish r_{out}, $a_v \approx -R_C/R_E$.
- Good for precise voltage gains, linearity. (a_v doesn't depend on g_m!)
- Assuming $r_o = \infty$ to simplify math.



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The common emitter with degeneration is a common emitter with the bypass cap on the emitter node removed. That means the emitter is no longer grounded. We'll find that common emitters with degeneration have high rin, fairly high rout and an av that is the ratio of the collector resistor to the emitter resistor. That's exciting because it's easy to find resistors with fairly precise relationships in their ratios, and also because our gain doesn't depend on gm! We expect that to dramatically enhance the linearity of this amplifier. However, RC/RE is often smaller than gm*RC, which is why RE is often referred to as a degeneration resistor. The gain is degenerated relative to gm*RC because of RE.

We're going to assume ro is infinite throughout our analyses to simplify our math. Including ro doesn't change our results much, but it makes the math much harder. You'll be examining the effect of ro on common emitters with degeneration on your homework. That's going to be a slog, but being comfortable wrestling with a tough small signal model is a skill that I want you to practice.

The first step in finding amplifier parameters is to draw a small signal model for the common emitter with degeneration. Pause the video and try drawing one now.

CLICK This is my version!

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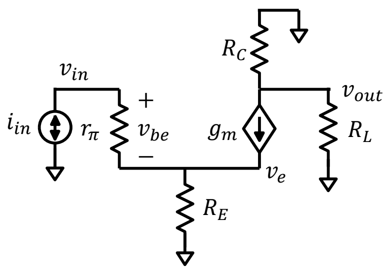
CE w/ Degen r_{in}

- Set $R_S = 0$, R_L proper value
- R from v_{out} to ground is $R_E || r_o || R_L$
- Use i_{in} as test source: easier than v_{in}



$$v_{be} = i_{in} r_\pi$$

$$v_e = (i_{in} + i_{gm}) R_E$$
$$= i_{in}(1 + g_m r_\pi) R_E$$

$$v_{in} = v_e + v_{be}$$
$$= i_{in}(1 + g_m r_\pi) R_E + i_{in} r_\pi$$

$$r_{in} = r_\pi + (\beta + 1) R_E \approx \beta R_E$$

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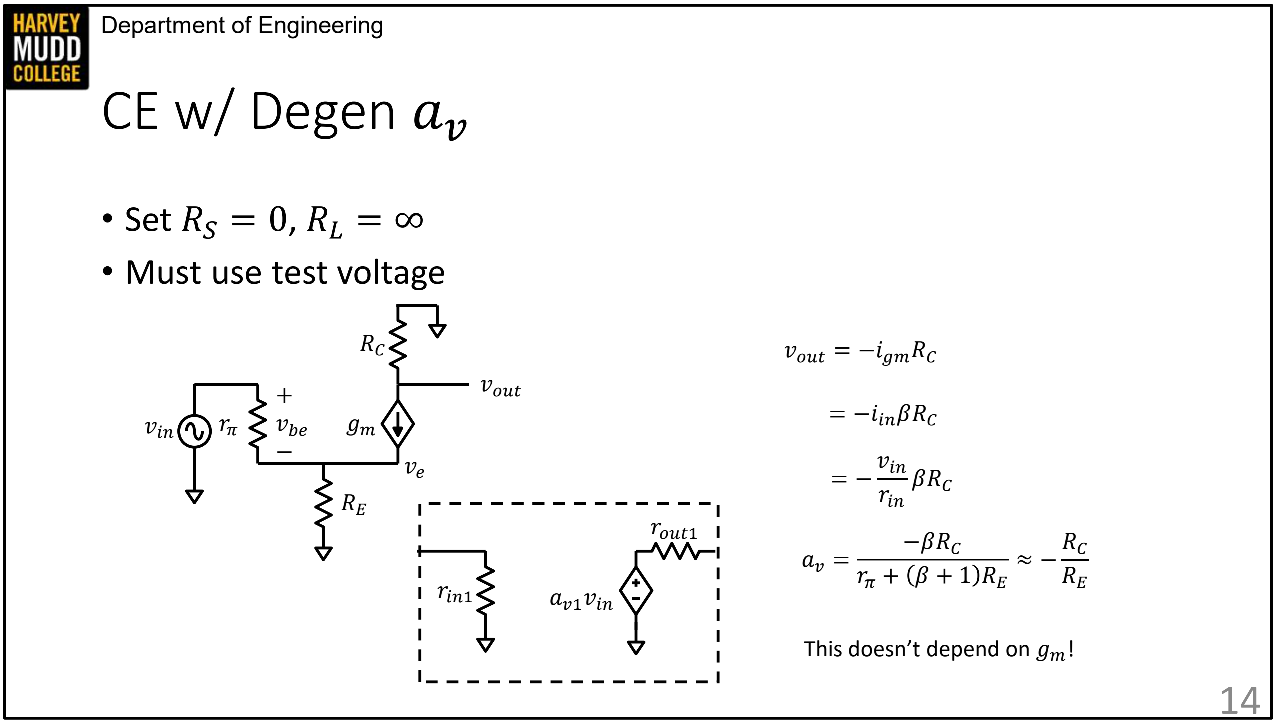
We set up our analysis to find rin by remembering the test conditions we use for rin. We set RS to zero so it doesn't add to the rin that we calculate, and we set RL to an appropriate value so that any feedback effects in our amplifier get captured by rin. I'm applying a test current source to find the input resistance.

Given this setup, I'd like you to pause the video and try to find rin. Feel free to use our small signal patterns if any are relevant.

CLICK I've included my solution on the right. You'll recognize this math from two places: the emitter follower input and the RE boost small signal pattern. We find the emitter voltage by noting the input current and the gm current flow in RE, then we relate the gm current to the input current, and finally we add the vbe voltage to the ve voltage to find vin. Because this math is identical to the emitter follower and the RE boost pattern, we wind up with the same rin expression. That's rpi + (beta+1)RE, or approximately beta*RE.



Now we're going to find the voltage gain of the emitter follower. We recall the we calculate voltage gain as small signal vout over small signal vin with zero source resistance and infinite load resistance. We set the resistances that way so that we're not measuring either the source or load divider by accident. I've set that up in the small signal model below, note that the resistor from vout to ground has gone back to ro||RE because we removed the load. Pause the video and try to find the gain of this amplifier.

CLICK This is a somewhat new derivation, so I'm going to show it step-by-step. I started by finding that vout is given by the gm generator current passing through RC. vout is negative because the gm generator current runs up through RC from ground.

CLICK We note that igm is beta times iin, but we're stuck because we don't know iin.

CLICK So we play the same trick we played with the emitter follower, where we replace this detailed transistor level schematic with the amplifier model for just a second, because the rin we found on the previous page relates vin and the iin that flows into the base. That's exactly what we want, so we know the current flowing into here is vin/rin.

CLICK Substituting that expression in gives us the gain, which is beta*RC / (rpi + (beta+1)*RE). Because beta is big and RC and RE are often of a similar order to rpi, we can make a super quick lab version of this equation: av is equal to –RC/RE. I find this super quick lab version to be easy to remember, which is great, and it also doesn't depend on gm, which is double great because it eliminates our gm nonlinearity. That nonlinearity can sneak back into our expression through rpi, which you'll recall is beta/gm, but addining degeneration seems to have greatly improved our linearity.

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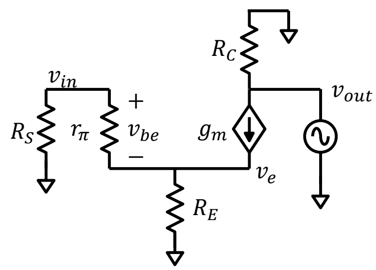
CE w/ Degen r_{out}

- Set $R_L = \infty$, R_S to proper value
- Test v_{out} happens to be easier than i_{out}



$r_{out} = R_C$

g_m source is shut off because v_{be} has ground on both sides of it.

Adding r_o makes a great problem for small signal patterns.

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Finally, we're going to find rout of the emitter follower. Our rout test conditions are an infinite load resistance, so it doesn't steal any current from our output, and an RS that is normal for the amplifier. We're choosing to use a test voltage source for this analysis because it makes our life easier. Pause the video and try to find rout.

CLICK rout is just RC. That's true because the gm generator is shut off when we ground vin through the source resistance.

There's a little bit of chicken and egg reasononing going on in my assertion that the gm generator is off. I'm asserting that vbe is zero since the gm generator is off, and the gm generator is off because vbe is zero. I confront that reasoning by asserting that if everying on the left side of the circuit is attached to ground, there's no reason for non-ground values in the middle.

If you add ro to this problem, then vbe is no longer zero and you have to do more analysis to find an expression for rout. However, that analysis is a classic example where our small signal models pay off. I won't go over that solution in these videos, but encourage you to try finding rout with ro included and small signal patterns in mind.

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Summary

- CEs w/ Degeneration are amplifiers with high r_{in} and linear gain.

- CE w/ Degeneration amplifier parameters

$$r_{in} = r_\pi + (\beta + 1)R_E \approx \beta R_E$$

$$r_{out} = R_C$$

$$a_v = \frac{-\beta R_C}{r_\pi + (\beta + 1)R_E} \approx -\frac{R_C}{R_E}$$