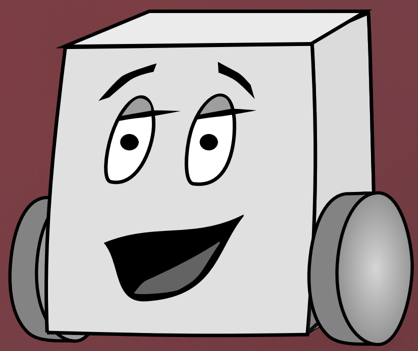

E11 Lecture 9-10: Circuits

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Outline

- Voltage, Current, Resistance
- Ohm's Law
- Kirchoff's Current Law
- Resistor Combinations
- Power
- Multimeters
- Ideal and Real Power Supplies

Voltage

- The electric force to drive electricity between two points
- Units: Volts (V)
- Technically:
  - energy / unit charge (Volt = Joule / Coulomb; V = J/C)
- Informally:
  - how hard the circuit wants to push electrons…
- Voltage is always measured between two points
  - Meaningless without a reference point
  - We typically call the reference point *ground*
  - If the ground is connected well across the system, we can treat it as 0V

Allesandro Volta

- 1745-1827
- Invented the first battery
- Granted title as Count by Napoleon in 1810 in honor of his work

en.wikipedia.org/wiki/File:Volta_A.jpg

Current

- The amount of electric charge flowing through a circuit per time.
- Unit: Amperes
  - (Amperes = Coulombs / Sec, A = C/s)
- Technically:
  - $I = dQ/dt$

Andre-Marie Ampere

- 1775-1836
- French physicist and mathematician
- A main discoverer of electromagnetism

en.wikipedia.org/wiki/File:Ampere_Andre_1825.jpg

Resistance

- A measure of the opposition to the flow of electric current
- Units: Ohm (Ω)
  - Ω = V/A
- A material with high resistance is called an *insulator*
- A material with low resistance is called a *conductor*

Georg Ohm

- 1789 – 1854
- German high school teacher
- Later joined Jesuit College in Colonge
- Determined relationship between voltage and current in a conductor
- College unsatisfied with his research and he resigned

en.wikipedia.org/wiki/File:Ohm3.gif

"Resistance is Useless"

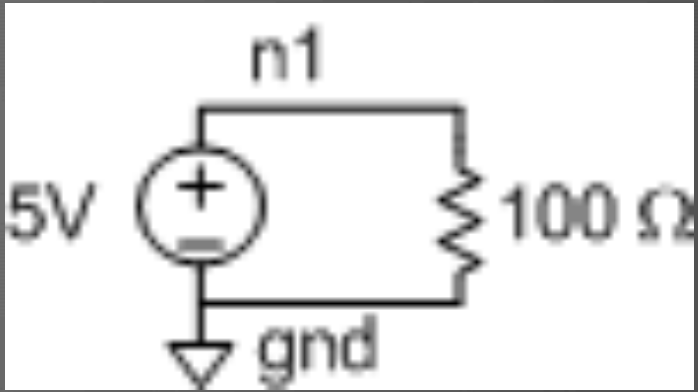
- Prostetnic Vogon Jeltz, *Hitchhiker's Guide to the Galaxy*

Circuits

- Circuits consist of nodes and elements
- Nodes:
  - Wires, at a particular voltage relative to ground
- Elements
  - Voltage sources
  - Resistors, capacitors, inductors
  - Diodes, transistors
  - Motors
  - Other sensors …

Example

- This circuit has two nodes
  - n1, gnd
- Two elements
  - A 5V voltage source
  - A 100 Ω resistor

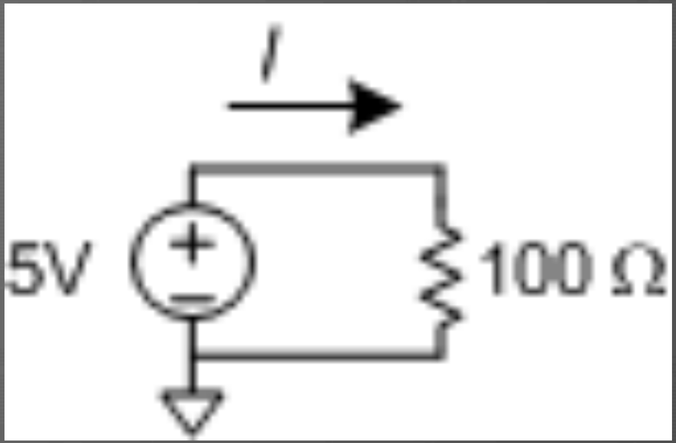


Ohm's Law

- Voltage = Current × Resistance ($V = IR$)
  - Or $I = V / R$

Example

- A 5 V power supply is connected to a 100 Ω resistor. How much current flows?
- A) 500 A
- B) 20 A
- C) 50 mA
- D) 5 mA

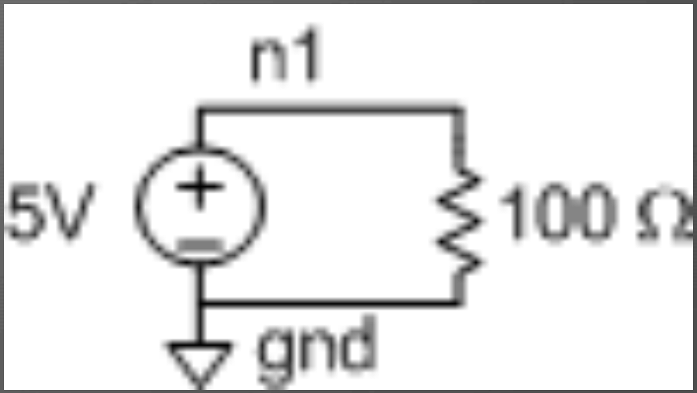


Kirchoff's Current Law (KCL)

- Charge is conserved
- It doesn't accumulate on circuit nodes
- Hence, the current flowing into a circuit node equals the current out of the node.

KCL Example

- Current flowing out of 5V supply into n1: 50 mA
- Current flowing out of n1 into resistor: 50 mA
- KCL: 50 mA = 50 mA ☺

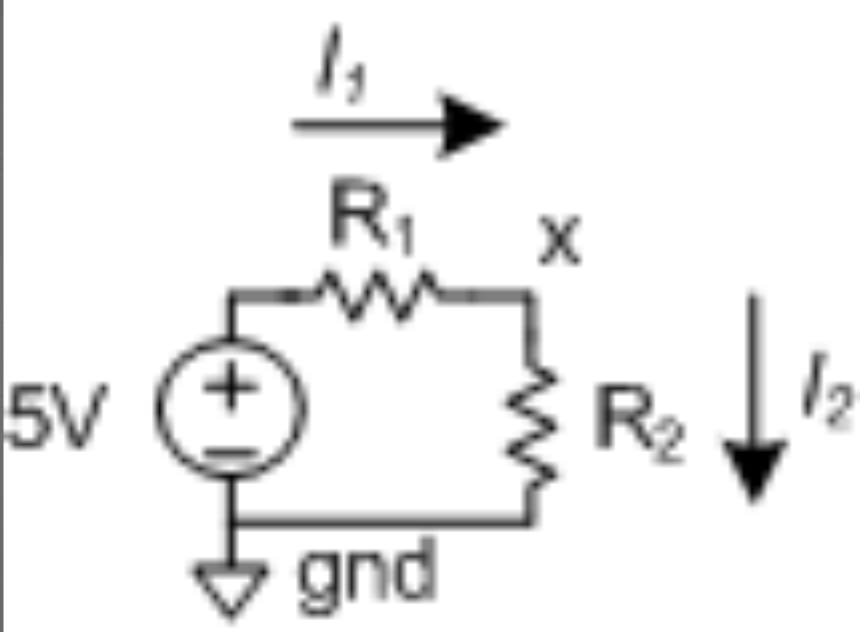


Voltage Divider

- Compute voltage at x:
- Ohm's Law: $I_1 = \frac{5 - x}{R_1}$

  $I_2 = \frac{x - 0}{R_2}$
- KCL: $I_1 = I_2$
- Solve for x:

$$\frac{5 - x}{R_1} = \frac{x}{R_2} \Rightarrow x = 5\left(\frac{R_2}{R_1 + R_2}\right)$$



Example

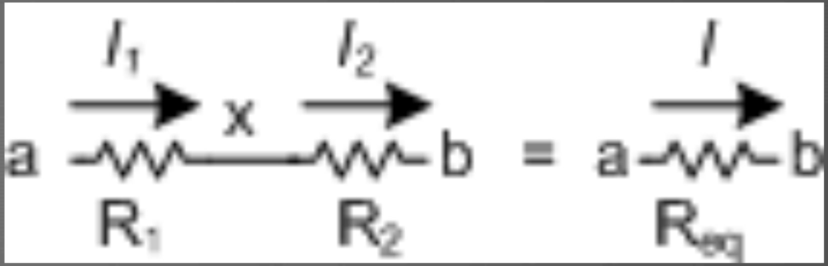
- What is the voltage at x if $R_1 = R_2 = 100\ \Omega$?
- A) 100 V
- B) 5 V
- C) 2.5 V
- D) 0.5 V

Ex: Potentiometer

- A potentiometer (pot) is a variable resistor with an adjustable tap
- Can be used as a voltage divider
  - As tap slides from top to bottom, Vout varies from 5V to 0V.

Series Resistors

- Two resistors in series are equivalent to one larger one
- Ohm's Law: $$I_1 = \frac{a - x}{R_1}$$ $$I_2 = \frac{x - b}{R_2}$$
- KCL: $I = I_1 = I_2$
- Solve for I: $$I = \frac{a - b}{R_1 + R_2}$$
- Hence, $R_{eq} = R_1 + R_2$

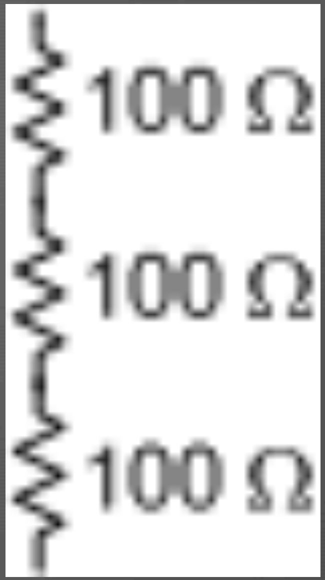


Series Resistors

- In general, any collection of resistors in series is equivalent to a single resistor with a value equal to the sum of the resistances.

Example

- What is the equivalent resistance of the circuit below?
- A) 300 Ω
- B) 200 Ω
- C) 100 Ω
- D) 33 Ω

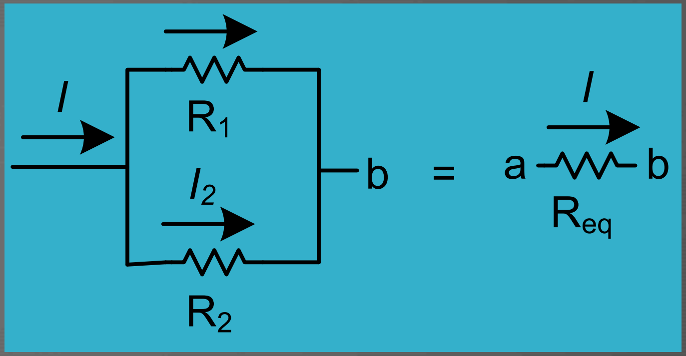


Parallel Resistors

- Two resistors in parallel are equivalent to one smaller one
- Ohm's Law:

$$I_1 = \frac{a-b}{R_1}$$

$$I_2 = \frac{a-b}{R_2}$$



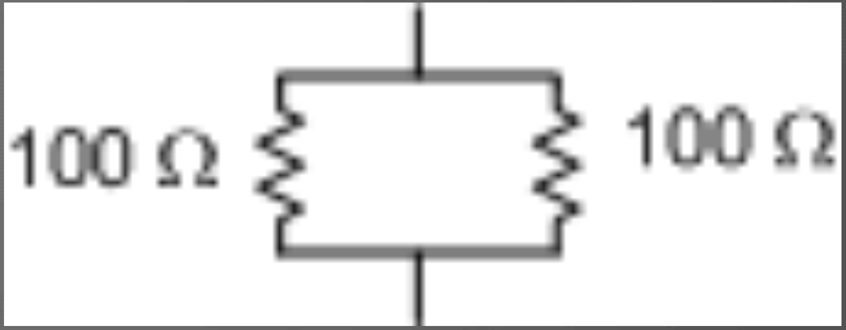
- KCL: $I = I_1 + I_2$
- Solve for I:

$$I = \frac{a-b}{R_1} + \frac{a-b}{R_2} = \frac{a-b}{\frac{R_1 R_2}{R_1 + R_2}}$$

- Hence, $R_{eq} = R_1 R_2 / (R_1 + R_2) \equiv R_1 \| R_2$

Example

- What is the equivalent resistance of the circuit below?
- A) 200 Ω
- B) 100 Ω
- C) 50 Ω
- D) 33 Ω



Example

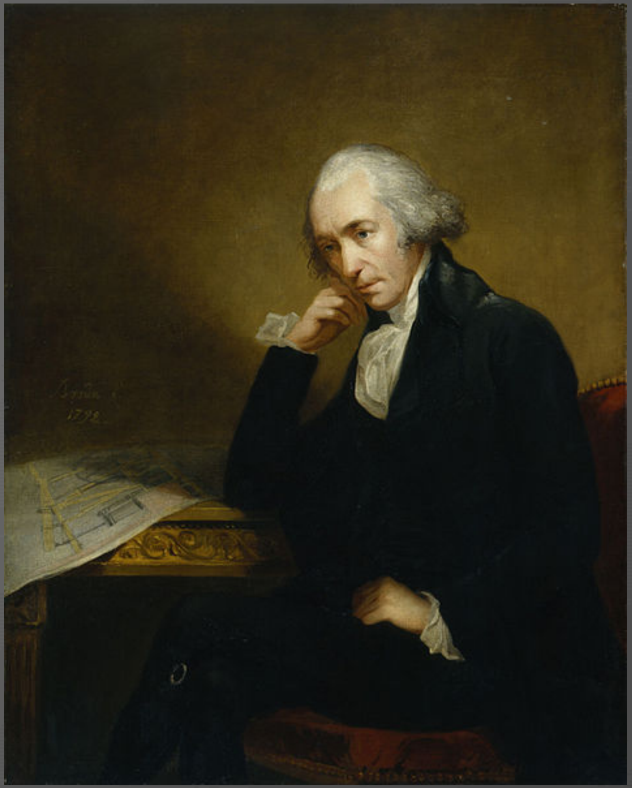
- You have a large drawer of 100 Ω resistors, but you need a 250 Ω resistor. Invent a circuit with the required resistance.

Power

- The amount of energy flowing through a circuit per time.
- Unit: Watts
  - (Watts= Joules / Sec, W = J/s)
- Technically:
  - $P = dE/dt$

James Watt

- 1736-1819
- Scottish engineer and inventor
- Home schooled
- Revolutionized steam engines
  - Condenser improved power generation



en.wikipedia.org/wiki/File:Watt_James_von_Breda.jpg

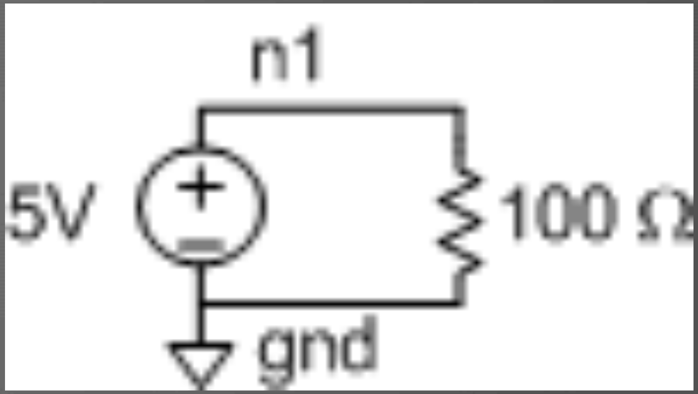
Power

- The power dissipated in a component is P = IV
  - Derivation:
    - E = QV — because Volts = Joules / Coulomb
    - dE/dt = (dQ/dt) V — differentiate both sides, assuming V const
    - P = IV — dE/dt is power, dQ/dt is current
- E = QV
  - For a resistor, V and I are related by Ohm's law, V = IR
  - Hence, $P_{resistor} = I^2R = V^2/R$

Is this a paradox that P is directly and inversely proportional to R?

Example

- How much power is delivered to the resistor?
- A) 2500 W
- B) 0.25 W
- C) 0.04 W
- D) 0.01 W



Open Circuit

- An open circuit is a circuit with no connection
  - Usually where a connection was intended
- Resistance = ∞
- No current flows

Short Circuit

- An short circuit is a circuit where two nodes are connected
  - Usually where a connection was NOT intended
- Resistance = 0
- Ex: short circuit across a power supply causes huge amounts of current to flow, might blow a fuse or start a fire!

Multimeter

- Multimeters measure:
  - Voltage (voltmeter)
  - Current (ammeter)
  - Resistance (ohmmeter)

- Some do autoscaling while cheaper ones require that you choose the right scale

fluke.com

Voltmeter

- Place meter in parallel with circuit
- Meters may be digital or analog
  - Most today are digital for cost, accuracy
- Digital:
  - A/D converter
- Analog:
  - Galvanometer
    - Resistor in series with a coil of wire in a fixed magnetic field
    - Current through coil creates a torque that rotates the coil and deflects a needle

Ammeter

- Place meter in series with circuit
- Meter includes small precision internal resistor
  - Measure voltage across the resistor
  - Deduce current

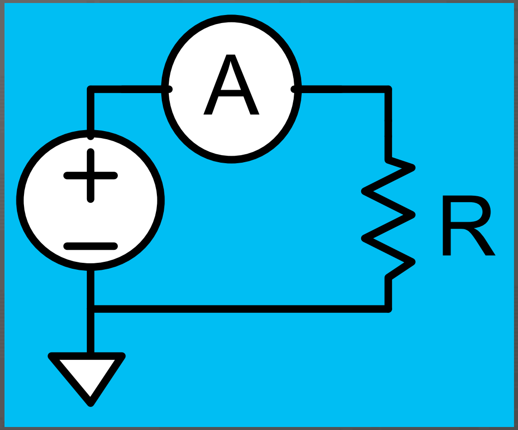
- Most multimeters have a separate terminal for measuring current
- Fuse in meter will blow if the current is too large

Ohmmeter

- Place in parallel with the resistor being measured
- Apply a small known voltage or current to the resistor
- Measure the current or voltage that flows
- Deduce resistance from Ohm's Law
- Unreliable if the resistor is in situ in a live circuit that distorts the measurements

Power Demo

- Resistors are rated for a certain amount of power
  - Typical small resistors are ¼ Watt
  - Resistors can overheat and change if power is exceeded
- Ex: 5 Ω resistor connected to a variable power supply
  - Use ammeter to measure current



- Plot I vs. V; compute P and R

A Caution about Modeling

- We have assumed that our components are ideal.
  - This is not a bad approximation for electrical components in their usual operating range.
  - If you push them too hard, they violate the assumptions.
  - Mechanical and chemical systems rarely match ideal models as well as electrical systems.

Power Supply Model

- A typical power supply is not an ideal voltage source
  - Can provide a finite amount of output current
  - Voltage droops as you pull more current
  - Examples: fuel cell, solar cell, battery

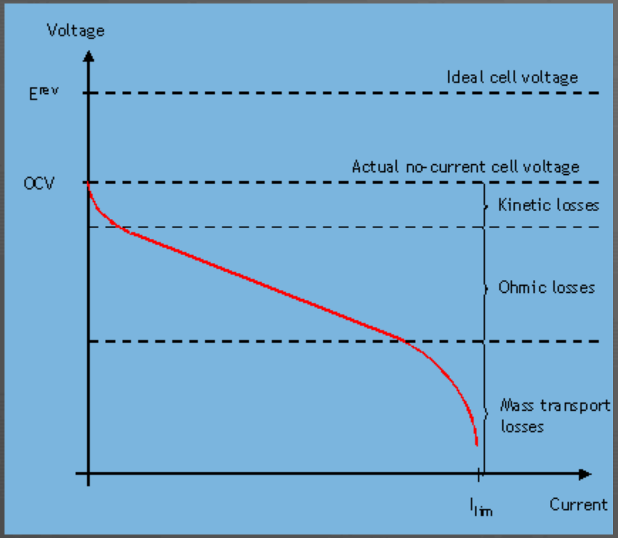
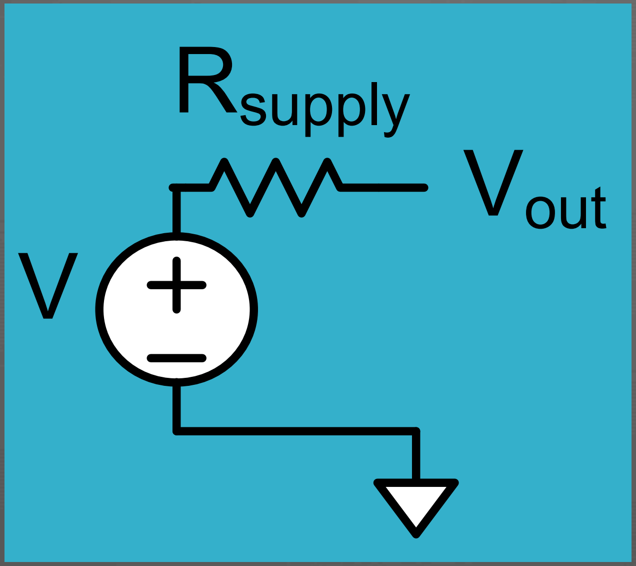


Figure from http://www.fuelcell.no/principle_fctheory_eng.htm

Power Supply Model

- **Model power supply as ideal voltage source in series with nonzero output resistance.**



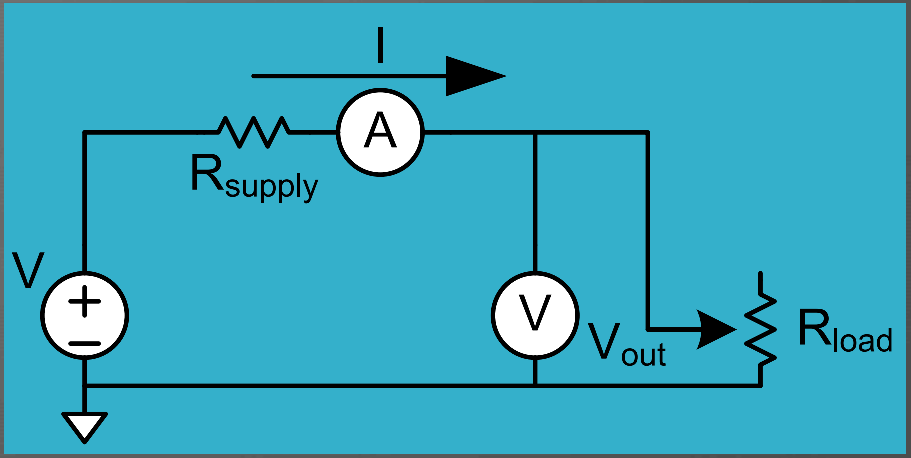
Loaded Nonideal Supply

- Suppose the load on the supply is varied. How does current and voltage change? How does power supplied to the load resistor change?

$$I = \frac{V}{R_{supply} + R_{load}}$$

$$V_{out} = V\frac{R_{load}}{R_{supply} + R_{load}}$$

$$P_{load} = \frac{V_{out}^2}{R_{load}} = V^2\frac{R_{load}}{\left(R_{supply} + R_{load}\right)^2}$$



Matched Load

- **What load resistance draws maximum power? How much power?**

$$P_{load} = \frac{V_{out}^2}{R_{load}} = V^2 \frac{R_{load}}{\left(R_{supply} + R_{load}\right)^2}$$

$$\frac{dP_{load}}{R_{load}} = \frac{\left(R_{supply} + R_{load}\right)^2 - R_{load} 2\left(R_{supply} + R_{load}\right)}{\left(R_{supply} + R_{load}\right)^4} = 0$$

$$R_{supply}{}^2 + 2R_{supply}R_{load} + R_{load}{}^2 - 2R_{supply}R_{load} - 2R_{load}{}^2 = 0$$

$$R_{load} = R_{supply}$$

$$V_{out} = V / 2$$

$$P_{load} = \frac{V^2}{4R_{load}}$$

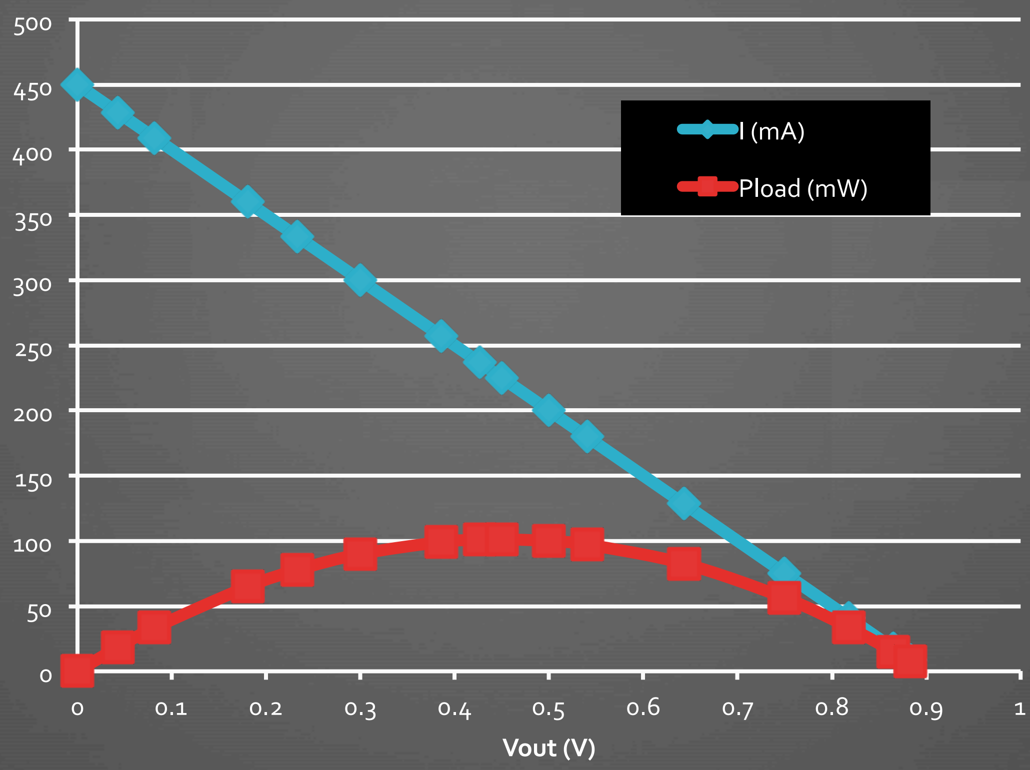
Open and Short Loads

- Open Circuit Load
  - $R_{load} = \infty$
  - $V_{out} = V$
  - $I = 0$
  - $P = 0$
- Short Circuit Load
  - $R_{load} = 0$
  - $V_{out} = 0$
  - $I = V / R_{supply}$
  - $P = 0$

Example

- A fuel cell has an open circuit voltage of 0.9 V and an effective output resistance of 2 Ω. How much power can it deliver to a matched load?
- A) 101 mW
- B) 810 mW
- C) 405 mW
- D) 1620 mW

Fuel Cell Model Predictions



Nonlinear Circuits

- In a nonlinear circuit, voltage and current are not proportional
- But nevertheless P = IV
- Ex: Fuel cell