

The Advanced Encryption Standard (AES)

Lecture 13

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Outline

- Introduction to Finite (Galois) Fields
- AES Overview

Learning Objectives

By the end of this lecture you will...

- Have an operational understanding of the fundamental mathematics used in AES
- Understand the basic process of AES

Finite Fields

A finite field is a set F with two operators ($+$ and \cdot) satisfying the following:

- _____ : if $a, b \in F$, then $a + b \in F$ and $a \cdot b \in F$.
- _____ : 0 and 1 elements exist such that $a + 0 = a$ and $a \cdot 1 = a$ for any $a \in F$.
- _____ : For any a , there exists x and y such that $a + x = 0$ and $a \cdot y = 1$. Except there is no multiplicative inverse of 0.
- **Associative, commutative, and distributive** properties hold.
- The two most common fields are $GF(p)$ and $GF(2^n)$.

History

Galois Fields are named after French mathematician Évariste Galois. He died in a duel at age 20 after being expelled from college for political protest after the French revolution. He spent six months in prison developing his mathematical ideas.



GF(p): Arithmetic modulo p

Example: GF(13): contains 0– 12, math is all done mod 13

- _____ and _____ are the identity elements
- Additive inverse of 4 is _____ because _____.
- Multiplicative inverse of 4 is _____ because _____.
- Used in public key encryption
 - Typically 256-1024 bit numbers mod a number of this length
 - Public and private keys a and b have special property that:

GF(2^n)

- Elements are polynomials in some dummy variable x with coefficients of _____ or _____.
- Hence, each coefficient operates _____.
- Addition, subtraction, and exclusive or are all the same.
- This makes the hardware very easy because addition simplifies to _____.
- Operations are done modulo some characteristic irreducible (prime) polynomial. For AES, this polynomial is _____.

Advantages of Galois Arithmetic for Digital Hardware

Example: $GF(2^8)$ with characteristic polynomial $m(x) = x^8 + x^4 + x^3 + x + 1$.

- Each element is an _____ (easy hardware)
- Addition is 8-bit _____
- Multiplication is _____

GF(2^8) Hexadecimal Representation

- Express $a = x^5 + x^3 + x^2 + 1$ in hexadecimal.

- Express $b = x^7 + x^6 + x^2 + x + 1$ in hexadecimal

GF(2⁸) Addition

Compute $c = a + b$ where $a = x^5 + x^3 + x^2 + 1$ and $b = x^7 + x^6 + x^2 + x + 1$.

GF(2^8) Addition

Given $a = x^5 + x^3 + x^2 + 1$ and $b = x^7 + x^6 + x^2 + x + 1$.

What is the additive identity of a (i.e., what is the zero element)?

What is the additive inverse of a (i.e., $-a$)?

GF(2⁸) Multiplication

What is $a(x) \cdot x$ where $a = x^5 + x^3 + x^2 + 1$?

$GF(2^8)$ Multiplication

What is $b(x) \cdot x$ where $b = x^7 + x^6 + x^2 + x + 1$?

GF(2⁸) Multiplication

$$a = x^5 + x^3 + x^2 + 1$$

What is the multiplicative **identity** of $a(x)$ (i.e., our 1 element)?

What is the multiplicative **inverse** of $a(x)$?

Galois Field Math Summary

- Addition is _____.
- Multiplication by x is _____.
 - If the coefficient of x^8 is 0, then you are done.
 - Otherwise, you need to reduce modulo an irreducible polynomial. For AES, this polynomial is $m(x) = x^8 + x^4 + x^3 + x + 1$.

Galois Fields Worksheet

The Advanced Encryption Standard

AES Overview

From the spec:

The Advanced Encryption Standard (AES) specifies a FIPS-approved cryptographic algorithm that can be used to protect electronic data. The AES algorithm is a symmetric block cipher that can encrypt (encipher) and decrypt (decipher) information. Encryption converts data to an unintelligible form called ciphertext; decrypting the ciphertext converts the data back into its original form, called plaintext.

The AES algorithm is capable of using cryptographic keys of 128, 192, and 256 bits to encrypt and decrypt data in blocks of 128 bits.

128-bit AES

- The cipher works by taking a plain text message and scrambling it into a cyphertext message in a way that is hard to reverse.
- Scrambling is done 10 times to make it hard. The key changes from round to round.

AES Data Organization

128-bit message is organized as a matrix of 16 bytes (4x4).

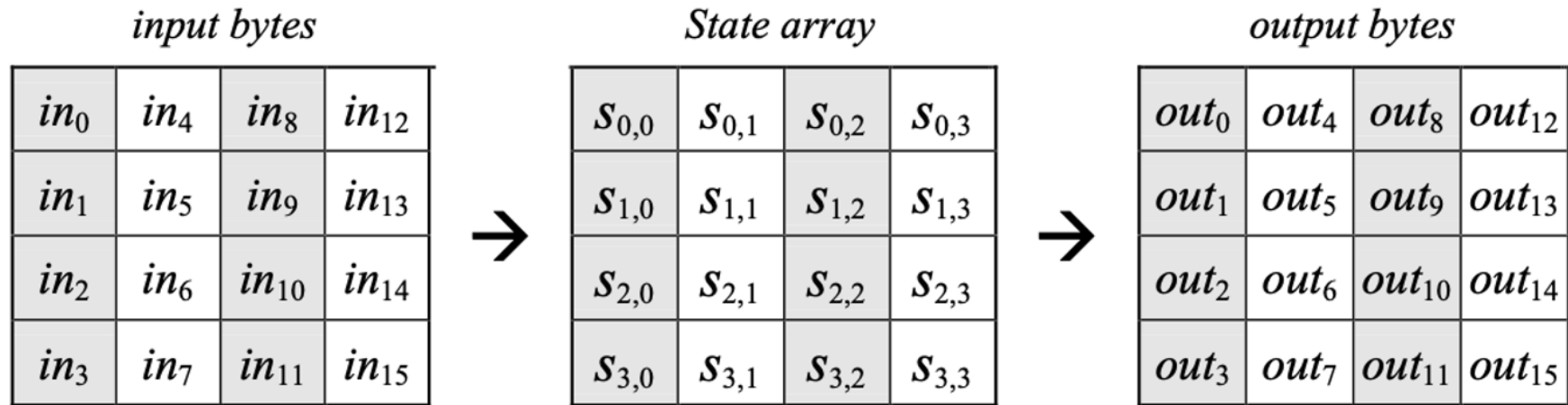


Figure 3. State array input and output.

AES Cipher Process

Each step of the cipher involves the following steps

1. **SubBytes**: take each byte and replace it with a different byte using a randomish lookup table.
2. **ShiftRows**: move bytes around in the rows
3. **MixColumns**: funky Galois multiplication on elements of the columns
4. **AddRoundKey**: XOR with the key for the current round.

AES Cipher Pseudocode

```
Cipher(byte in[4*Nb], byte out[4*Nb], word w[Nb*(Nr+1)])
begin
  byte state[4,Nb]

  state = in

  AddRoundKey(state, w[0, Nb-1])           // See Sec. 5.1.4

  for round = 1 step 1 to Nr-1
    SubBytes(state)                        // See Sec. 5.1.1
    ShiftRows(state)                       // See Sec. 5.1.2
    MixColumns(state)                     // See Sec. 5.1.3
    AddRoundKey(state, w[round*Nb, (round+1)*Nb-1])
  end for

  SubBytes(state)
  ShiftRows(state)
  AddRoundKey(state, w[Nr*Nb, (Nr+1)*Nb-1])

  out = state
end
```

Figure 5. Pseudo Code for the Cipher.¹