Logistics:
- Exam - best is see sticky note
- Grading intent - grade like graduate course
- Watch power levels in 1965! Minor rewrite to labs 5 w/ warning + improved linearity + alternate noise temp

We're talking about linearity
- Big signals prevent cause RX to clip
- Taylor represent non-linear elements

\[ V_o(t) = V_i(t) \cdot a_0 + a_1 + V_i^2(t) \cdot a_2 + V_i^3(t) \cdot a_3 + \ldots \]

Our inputs are sinusoids

\[ V_o(t) = a_0 + a_1 V_i \cos(\omega t) + a_2 V_i^2 \cos^2(\omega t) + a_3 V_i^3 \cos^3(\omega t) \]
\[ = a_0 + a_1 V_i \cos(\omega t) + \frac{1}{2} a_2 V_i^2 (1 + \cos 2 \omega t) + \frac{1}{4} a_3 V_i^3 (3 \cos 2 \omega t + \cos 3 \omega t) \]

DC op gain DC shift 2nd harmonic gain compression 3rd harmonic

Gain compression
- In-band component of 3rd order distortion affects gain

\[ G_{out} (a_1 + 2 a_2 V_i^2) \cos \omega t \leftarrow \text{Other in-band cos \omega t terms} \]

\[ G_{out} \]

- \( a_3 \) often negative, so gain shrinks as \( V_i \uparrow \) (clipping)

\[ \text{Input } 0 \text{ to full scale, output } 0 \text{ to } \frac{G_{out}}{G} \text{ is } -1 \text{dB} \]

\[ \frac{G_{out}}{G} = 1 + \frac{3}{4} \frac{a_3 V_i^2}{a_1} \quad \text{or} \quad V_i = \sqrt{\frac{4}{3} \frac{a_3}{a_1}} \times \sqrt{10} \text{ mV} \]
Harmonic Distortion

HD2 - 2nd order harmonic distortion
\[ \frac{\text{amp.}}{\text{amp. fundamental}} \frac{\text{2nd harmonic}}{\text{fundamental}} = \frac{a_2 V_i^{1/2}}{a_1 V_i} = \frac{1}{2} a_2 V_i = \frac{1}{2} \frac{a_2}{a_1^2} V_o \]
- linear w/ \( V_i \)
- \( \text{dBc} \) is units

HD3 - 3rd order harmonic distortion
\[ \frac{\text{amp. 06 \ 3rd harmonic}}{\text{amp. 06 fundamental}} = \frac{a_3 V_i^{3/4}}{a_1 V_i} = \frac{1}{4} \frac{a_3}{a_1} V_i = \frac{1}{4} \frac{a_3}{a_1^2} V_o \]
- quadratic w/ \( V_i \)
- very similar coefficient to gain compression

THD - Total harmonic distortion
\[ \text{THD} = \sqrt{\text{HD1}^2 + \text{HD2}^2 + \cdots} \]
- 0.001% telephone
- 0.01% video
- 0.1% RF PA

Intermodulation
- we've been talking about single-tone nonlinearity
- But situation we're worried about is rogue TX near our RX

\[ V_3(t) = V_1 \cos \omega_1 t + V_2 \cos \omega_2 t \]
\[ V_3(t) = a_0 + V_1(t) \cdot a_1 + a_2 V_1^2(t) + a_3 V_1^3(t) \]
\[ = a_0 + a_1 (V_1 \cos \omega_1 t + V_2 \cos \omega_2 t) + a_2 V_1^2 \cos^2(\omega_1 t) + a_2 V_1^2 \cos^2(\omega_2 t) + 2a_2 V_1 V_2 \cos \omega_1 t \cos \omega_2 t \]

* IM2 is bad for low IF
  \[ a_2 \left( \frac{V_i^2}{2} (1+\cos(2\omega_1 t)) \right) + \frac{V_i^2}{2} (1+\cos(2\omega_2 t)) + V_i V_2 (\cos(\omega_1 t) + \omega_2 t) \text{ (2nd order)} \]

Two tone nonlinearity test

\[ \text{IM2} = \text{amp. IM w/ } V_1=V_2 \]
\[ = \frac{\text{amp. fundamental}}{\text{amp. fundamental}} \frac{a_2 V_i^2}{a_1 V_i} = \frac{a_2}{a_1} V_i = 2\text{HD2} \]
\[ \text{IM2 (dB)} = \text{IM (dB)} \text{ - IM or } V_i^2, \text{find } a_i \text{ V}_i \text{ - IM = } a_i \text{ V}_i \text{ - IM2 is V1 & IM2 = dBc = fund. } V_i = \frac{a_i}{a_1} \]
\[ \text{- can get IM2 from IM2 as dB for dB} \]
Cubic Intermodulation
- skip the $x^3 + y^3$ terms → harmonic distortion
- other terms are $3 a_3 v_1 v_2^2 (\cos \omega_1 t \cos^2 \omega_2 t) = 3 a_3 v_1 v_2^2 \cos \omega_1 t (1 + \cos 2\omega_2 t)/2 = a_3 v_1 v_2^2 \left( \frac{3}{2} \cos \omega_1 t + \frac{3}{4} \cos (2\omega_2 \pm \omega_1) \right)$

Comparing $\text{IM}_3 = \frac{\text{amp. of in-band intermod at } v_1 = v_2}{\text{amp. fund.}} = \frac{3 a_3 v_1^3}{a_1 v_1} = \frac{3 a_3}{4 a_1} \sqrt{\frac{S_i}{3}} \rightarrow 3 \text{dB}$

Like 2nd order intermod, can define voltage where 3rd order IM catches fund.

$I_{IP3} = P_{in} + IM_3 = 0 \text{dB}$
($\Rightarrow I_{IM3} = \frac{3 a_2 v_1^2}{4 a_1} \rightarrow \frac{v_i}{\sqrt{\frac{4}{3} \frac{a_i}{a_3}}}$

- can find $I_{IM3}$ for any power level from $S_{IP3}$ ... fall off @ 20dBc for 10dB input

Relation between $I_{IP3} + P_{-1dB} = \text{HD3}$

$\frac{V_i @ P_{-1dB}}{V_i @ I_{IP3}} = \left( \frac{4}{3} \frac{a_3}{a_1} \right)^{0.11}$ \quad $20 \log \frac{V_i @ P_{-1dB}}{V_i @ I_{IP3}} = -9.6 \text{ dB}$

$V_i @ I_{IP3} = \left( \frac{4}{3} \frac{a_3}{a_1} \right)^{0.11}$

- Measure $P_{-1dB}$
- Calculate $I_{IM3}$ for
- Extrapolate to $I_{IP3}$ pin backed off from $S_{IP3}$

Graphical rendering of intercept points

$\log P_{out} \quad \log P_{in}$

$I_{IP2}$ \quad $I_{IP3}$ \quad $I_{IM2}$ \quad $I_{IM3}$ component

Cascade formulas
- Involved derivation
- Simple exp.

\[ \frac{1}{\text{S}_{IP2}} + \frac{a_{11}}{\text{S}_{IP2}} \]

\[ \frac{1}{\text{S}_{IP3}} + \frac{a_{11}}{\text{S}_{IP3}} \]

Overall $S_{IP3} + S_{IP2}$

\[ \frac{1}{\text{S}_{IP2}} = \frac{1}{\text{S}_{IP2} + \text{S}_{IP3}} \]

\[ \frac{1}{\text{S}_{IP3}} = \frac{1}{\text{S}_{IP2} + \text{S}_{IP3}} \]