- Filter description
- Filter Synthesis
- Transform
- Relation to s parameters
- Laplace Picture
- 2-port U/VNA

We're talking about s-parameters - generalization of Π to 2-port systems. Going to talk more.

- Need to go over filters for design project... ok s-parameter example.

- Filters are linear systems that are frequency selective.

\[ w_1, w_2, w_0, w_{s1}, w_{s2} \]

\[ 0 \text{dB} \]

- pass band
- stop band
- filtering region
- transition region

\[ \begin{align*}
\text{insertion loss} & \quad \text{stop-band rejection} \\
(1/\text{A}^2) & \quad (1/\text{A}^2)
\end{align*} \]

- \[ W_0 \] called the center frequency.

- \[ w_1, w_2 \] define edge of pass band.

\[ \text{in general: } w_1, w_2 \text{ often } -3\text{dB} - \frac{1}{2} \text{ power BW} \]

- Bandwidth \[ \text{BW} = w_2 - w_1 \]

- Fractional BW means \[ w_2/w_1 \] determined from \[ W_0 \].

\[ \text{eg: } \text{FBW} = \frac{\text{BW}}{w_0} \]

- \[ W_0 = \sqrt{w_1 w_2} \]

\[ \frac{1}{1+e^2} \]

- Max pass band deviation of \[ \frac{1}{1+e^2} \] 3 min stop band \[ 1/\text{A}^2 \]
Lecture 12 - Filter Design

- Steepness of rolloff in transition region $\propto c_0^n$ where $n$ is called order

$L$ is related to # of poles used to make filter

$$\text{Filter prototype } |H(cs)|^2 = \frac{1}{1 + c^2 F(cs)}$$

$F$ function $F(cs)$ is polynomial or ratio of polynomials w/order $n$

$F$ specific function determines shape of filter

$F$ order $\rightarrow$ poles $\rightarrow$ Energy storage $\leftrightarrow$ # stored states in memory

Butterworth

Chebyshev I

Chebyshev II

Elliptic

$H(cs)^2

\frac{F(cs)}{\left(\frac{c_0}{w_p}\right)^n}
\rightarrow$

passband edge

Chebyshev polynomial

$C_n(w_p)$

Invers Chebyshev polynomial

$1 - C_n(w_p)$

Chebyshev rational polynomial $N(cs)/D(cs)$

$-$ same

$-$ phase for these usually nots

$-$ Bessel-Thomson filters approximate linear phase (pretty well)

@ expense of ripple $F(cs)$ are Bessel functions

$-$ Butterworth often called maximally flat $\sim$ $-$ type II Chebyshev technically flatter
- How do we make filters?

- By tradition, start w/ low pass & learn about high pass transformation

- 1st pick $A$, $\varepsilon \rightarrow n$ (a allowable stopband monotonicity)

$$\varepsilon = \sqrt{\frac{100}{10^6} - 1}$$

- Lowpass vs. highpass plots or equations:
  - Butterworth: $n = \frac{\ln A_s / \varepsilon}{\ln \cos \omega_p}$
  - Chebyshev: $n = \frac{\cosh^{-1} A_s / \varepsilon}{\ln \cos \omega_p}$
  - Elliptic $\rightarrow$ elliptic integrals

- Now need to turn into circuit

- Need $n$ energy storage elements

$$n = 3: \quad \frac{L_1}{L_2} \frac{L_3}{L_4} \quad \frac{L_5}{L_6} \quad \frac{C_1}{C_2} \frac{C_3}{C_4} \frac{C_5}{C_6} \quad \text{or} \quad \frac{L_1}{L_5} \frac{L_2}{L_4} \frac{L_3}{L_6} \frac{C_1}{C_2} \frac{C_3}{C_4} \frac{C_5}{C_6}$$

- Get component values from a filter table

- Tables normalized to $Z_{in} = 1\Omega$ & $\omega_p = 1$ rad/s

- Un-normalize $Z$ by multiplying $Z_0$ by $Z_0$ (50 ohm)

$$L = L_0 Z_0 \quad C = C / Z_0$$

- Un-normalize $\omega$ by dividing all elements by $\omega_p$

( recall $Z_{in} = \sqrt{\frac{1}{C}} + \omega_0 \approx \frac{1}{\sqrt{2C}}$ )

Example butterworth:
Go from low pass to high pass, band pass, band stop

\[ \frac{1}{\omega_0^2} \rightarrow \frac{1}{\omega_c^2} \rightarrow \frac{1}{\omega_c^2} \frac{1}{1 + \frac{BU}{\omega_c^2}} \rightarrow \frac{1}{1 + \frac{L \cdot BU}{\omega_c^2}} \]

\[ \frac{1}{\omega_0 C} \rightarrow \frac{1}{\omega_c C} \rightarrow \frac{1}{\omega_c C} \frac{1}{1 + \frac{BU}{\omega_c C}} \rightarrow \frac{1}{1 + \frac{C \cdot BU}{\omega_c C}} \]

High pass  Band pass  Band stop

Lo band elements ~ parallel tanks open & series ring short

\( Y = 0 \)  \( Z = 0 \)

\[ |H(s)|^2 = |S_{21}|^2 \]

\[ S_{11} \text{ goes low when } S_{21} \text{ is high} \rightarrow \text{no reflection in pass band} \]

\[ \text{reflect otherwise } \frac{1}{s} \text{ lossless filters} \]

\( L \) can make lossy RC or RL ladders