Motivate
- 2 ports
- Deliver to 1 port
- Transmission

Define
- wave vs port
- Linear!

5 parameter power
- point @ power deba
- squaring
- conservation

VNA
- directional coupler
- vs. freq.
- phase errors
- SOLT why

Revisiting driving point impedance

\[ Z_{in} = Z_0 \]

\[ V = 0.11 \, \text{V/s} \]

Zin looks like open

reflectance "sets" apparent load impedance

get information from reflection about load

Talking about 5 parameters
- describe power Xfer in RF systems

- A way of abstracting away a lumped model
- describe input-output behavior assuming linearity
- close relatives to Z & Y parameters which you may have seen

Introducing 2 ports

- Thevenin used to describe systems w/ 1 port
  - Abstract details to an IV relationship
  - Often systems we care about have 2 ports

Eg: \[ V_2 - \text{Thevenin} @ 1 \text{ fails } V_c \] need to know load @ 2
  - So use more complex Thevenin model

\[ \begin{align*}
Z_{in} &= Z_{11} + Z_{12} + Z_{21} + Z_{22} \\
\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} &= \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \\
\end{align*} \]  
- "matrixize" ohm's law
- Thevenin
- dependent sources
* Calculate Z-params by using w/ leads
  ~ if \( x_1 \) or \( x_2 = 0 \) then easy to find

\[ Z_{21} = Z_{21} \text{ w/ } Z_{12} \text{ open} \]

\[ Z_{22} = Z_{22} \text{ w/ } Z_{12} \text{ open} \]

\[ Z_{22} = R_2 \]

\[ Z_{12} = R_2 \]

~ Just a resistor in this case.

\[ y = \frac{R_1}{R_2} \]

\[ Z_{12} = R_2 \]

\[ Z_{21} = \frac{R_2}{R_1} \]

~ These are fine if you can make open ckt. w/ src.

~ Hard to do @ RF w/ pth fringing yields non-linear effects

~ Unstable into shorts or opens.

~ At RF, usually go with a different set of parameters that
  are easier to measure

~ S-parameters of scattering parameters

~ Describe incident & reflected waves instead of port \( S \)-param

~ Generalization of \( \Pi \) to two ports

\[ \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \]

~ Define \( a_1 \) in terms of full wave
  ~ amplitude (implicitly sinusoidal)

\[ a_1 = \frac{V_{i1}}{\sqrt{Z_0}} \]

~ Convenient for power flow

\[ a_2 = \frac{V_{i2}}{\sqrt{Z_0}} \]

\[ |a_1|^2 = \frac{V_{i1}^2}{Z_0} \]

~ Power in t.line

\[ b_1 = \frac{V_{i1}}{\sqrt{Z_0}} \]

\[ b_2 = \frac{V_{i2}}{\sqrt{Z_0}} \]

\[ \begin{bmatrix} \frac{V_{i1}}{\sqrt{Z_0}} \\ \frac{V_{i2}}{\sqrt{Z_0}} \end{bmatrix} \]

\[ b_1 = S_{11} a_1 + S_{12} a_2 \]

\[ b_2 = S_{21} a_1 + S_{22} a_2 \]

\[ Z_{12} \text{ term port } Z_{12} \text{ equals } S_{21} a_2 \]

\[ b_1 = S_{11} V_{i1} \text{, } b_2 = V_{i2} \text{, } Z_{11} = \frac{V_{i1}^2}{S_{11}} \]
- Really similar argument (term part 1) shows $S_{22}$ is $\frac{1}{2}$
- $S_{11}$ and $S_{21}$ are same w/ appropriate terminations

$$S_{21} = \frac{V_r}{V_{in}}$$  
measure this to get a type of gain called

"Transducer power gain" or more later
w/ port 2 terminated

- Measure by terminating ports in $Z_0$

Let's do an example:
- Find S-parameters for

$$S_{11} = \frac{R_2 Z_0 + R_1 - Z_0}{R_2 Z_0 + R_1 + Z_0}$$
$$S_{22} = \frac{R_2 Z_0 (R_1 + Z_0) - Z_0}{R_2 Z_0 (R_1 + Z_0) + Z_0}$$

$$S_{21} = \frac{R_2 Z_0}{R_1 + R_2 Z_0} \left(1 - S_{11}\right)$$
$$S_{12} = \frac{Z_0}{R_1 + Z_0} \left(1 - S_{22}\right)$$

- Power conserved in lossless 2-port

$$\frac{1}{2} |V_1|^2 = \frac{1}{2} |V_2|^2 + \frac{1}{2} \text{Re}\{V_1^* I_2\}^2$$ 2-port

- Can also express port voltages in terms of waves

$$V_1 = \sqrt{Z_0} (a_1 + b_1)$$
$$V_2 = \sqrt{Z_0} (a_2 - b_2)$$

$$I_1 = \sqrt{Z_0} (a_1 - b_1)$$
$$I_2 = \sqrt{Z_0} (a_2 - b_2)$$

- Invert these expressions to get port IV expressions for waves

- I find these confusing

$$a_1 = \frac{V_1 + Z_0 I_1}{\sqrt{Z_0}}$$
$$a_2 = \frac{V_2 - Z_0 I_2}{\sqrt{Z_0}}$$

$$b_1 = \frac{V_1 - Z_0 I_1}{\sqrt{Z_0}}$$
$$b_2 = \frac{V_2 + Z_0 I_2}{\sqrt{Z_0}}$$

- VNA needs cal.