Impedance transforms periodically by USWR
- review
- led to Smith chart

- Justify Smith chart
- Bilinear transforms
- Examples
- A curve

- Driving sine waves onto a terminated line

- Impedance transformed by standing wave USWR pattern

Such wave has \( 2\pi \) the \( \lambda \)
high voltage looks like big impedance & drive point
\( \rightarrow \) set by line length

- Predicted impedance transform w/ equation

\[ Z(\omega) = \]

- Tested that equation on \( Z = \frac{\lambda}{4} \) series line \( \rightarrow \frac{Z}{2} \) series line

short to open \( \rightarrow \) same \( \lambda \) USWR periodic

- Wanted to capture that behavior, so using Smith Charts

- plots of the \( \Gamma \) plane

\( \Gamma \) can get to normalized impedance

\[ \Gamma = \frac{Z_{nl-1}}{Z_{nl+1}} \]

- Going to practice & observe patterns
\[ Z_{nl} = \frac{1 + \Gamma_L}{1 - \Gamma_L} = r + jx \quad \text{normalised resistance + reactance} \]

- Let \( \Gamma_L \) be \( \Re \{ \Gamma \} \) or \( \Gamma_i \) be \( \Im \{ \Gamma \} \)

\[ Z_{nl} = \frac{(1 + \Gamma_L)(1 - \Gamma_L^*)}{(1 - \Gamma_L)(1 - \Gamma_L^*)} \]

\[ = \frac{1 - |\Gamma_L|^2 - 2 \Re \{ \Gamma \} \cdot j}{1 + |\Gamma_L|^2 - 2 \Re \{ \Gamma \} \cdot j} \]

\[ r = 1 - |\Gamma_L|^2 \]

\[ x = \frac{2 \Im \{ \Gamma \}}{1 + |\Gamma_L|^2 - 2 \Re \{ \Gamma \}} \]

\[ \left( \Gamma_r - \frac{r}{r+1} \right)^2 + \Gamma_i^2 = \left( \frac{1}{1+r} \right)^2 \]

\[ \uparrow \]

- A circle shifted in Re dimension by \( \frac{r}{r+1} \)
- Radius is \( \frac{1}{1+r} \)

\[ \frac{r}{r+1} + \frac{1}{1+r} = 1 \quad \Rightarrow \text{by every circle} \]

\[ \text{Center @ } \frac{1}{x} \]

- Always hit \( 1,0 \)
- Tangent

\[ \text{Constant } x \rightarrow \text{arcs} \]

\[ \text{Constant } x \rightarrow \text{circles} \]
Examples
- draw plots on your handout
- Note non-rectangular coordinate system of $Z_{in}$ drawn on $\Gamma$ plane
  $\Gamma_{in}$ are rectangular

Admittance

$$\Gamma_l = \frac{Z_{in} - 1}{Z_{in} + 1} = \frac{1}{\frac{1}{V_{in}} + 1} = \frac{V_{in} - 1}{V_{in} + 1} \quad \text{(Just reflected by (-ve) sign)}$$

Transmission line xform

$$\Gamma(z) = \frac{\Gamma_{in} e^{2 \gamma z}}{1 + \frac{1}{\Gamma_{in} e^{2 \gamma z}}}$$

- so a transmission line in front of a point on the $\Gamma$ plane just adds phase.

$$\Gamma(z) = \frac{|\Gamma_{in}| e^{j \phi_{in}} 2 \gamma z}{1 + |\Gamma_{in}| e^{j \phi_{in}}} = |\Gamma_{in}| e^{j \phi_{in}}$$

- Rotation angle is $2 \gamma z$

- For lossless line of length $\lambda/4$, $2 \gamma z = 2 - \frac{2\pi}{\lambda}$, $\frac{\lambda}{4} = \pi$
  $l > \text{open to short}$
Plot cases on this Smith Chart. Draw loci for all possible values of k and w for examples 4-6.

- Each point described by \( x + jy \) or drawn circles
- or by \( \Re \{ \sqrt{3} + j \} \) in rectangular coordinates
- \( e^{j\phi} \) has \( r = 1 \) so \( x = 1 \) is on \( r = 1 \) circle and \( x = 1 \) arc
  but is also \( \phi = \frac{1}{2} + \frac{2\theta}{5} \)
- (2) + a \( \frac{1}{4} \) transmission line corresponds to rotation by angle of \( 2\theta_2 = 2 \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi \) about the origin