EM waves to IV waves

Driving point or
Telegophers
Microstrips, layout /
current crowding
propagation constant

Labs will be done individually

Want to turn EM waves into measurable voltages or currents

- Assume sinusoidal: \( A \cos(wt-kz) = \psi_0(\psi_0+1) \) \( \omega / k = \frac{w}{v} \)

(Analytical rep)

- Travelling in \( \hat{x} \) \( \Rightarrow \hat{E} \) in \( \hat{x} \) \( \Rightarrow \hat{B} \) in \( \hat{y} \) (more vector math - pointing)

- Varies with time \( \partial \hat{E} / \partial t = 0 \), \( \partial \hat{B} / \partial t = 0 \)

\[
\nabla \times \hat{E} = \begin{vmatrix}
\hat{\nabla} & \hat{y} & \hat{z} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
\hat{E}_x & \hat{E}_y & \hat{E}_z
\end{vmatrix} = \hat{y} \frac{\partial \hat{E}_x}{\partial z} = -j k \hat{E}_x (x,y) \hat{y} ~ \nabla \times \hat{B} = \hat{x} \frac{\partial \hat{B}_y}{\partial z} = -j k \hat{B}_y (x,y) \hat{x}
\]

\[
\nabla \times \hat{E} = -\frac{2B}{j\epsilon} = -\mu \frac{\partial H}{\partial t}
\]

- \( j k \hat{E}_x (x,y) \hat{y} = -\hat{k} \hat{\omega} \hat{B}_y (x,y) = -\hat{k} \omega \mu H_y (x,y) \)

\[
\frac{E_x}{H_y} \approx k \hat{z} \approx \frac{\mu \omega}{\epsilon \omega} = \frac{k}{\epsilon} [\Omega] \quad \text{impedance of free space}
\]

- \( \mu \omega \omega = \frac{\mu \omega}{\epsilon \omega} = \sqrt{\mu} [\Omega] \quad \text{measures ratio of } E \text{ to } H \text{ at any point}
\]

\[
\text{Can't be measured with a multimeter}
\]
Let's put the wave in a wire

\[ V = \int E \cdot dl = \frac{h \cdot E}{c} \]

From before:

\[ \int kE_x = \int \omega M H_y \]
\[ \alpha \int kE_x = \int \omega M H_y \cdot \frac{a}{b} \cdot b \]
\[ \int kV(z,t) = \int \omega L' I(z,t) \]

- Propagating EM wave is same as propagating voltage/current wave

\[ \frac{V(z,t)}{I(z,t)} = Z_0 = \frac{\omega L}{k} = \frac{\omega L}{\sqrt{\mu \varepsilon}} = \sqrt{\frac{1}{\sigma}} \]

\[ k \text{ is free space impedance } \frac{\omega L}{k} \text{ line affected by environment} \]

\[ \int kV = \int \omega L' \left( \frac{\omega}{k} CV \right) \rightarrow k^2 V = \omega^2 L' C \]
Summary \[ Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \quad k = \frac{c}{\sqrt{LC}} \quad \Rightarrow \quad v = \frac{1}{\sqrt{LC}} \]

This environment called a transmission line

A structure that supports transverse electromagnetic (TEM) plane waves

Idealized, practical example is microstrip line

- Issue 1: coupling risk of unwanted distributed fringe (smaller is better)
- Issue 2: corners

Aside from current crowding and skin effect change R vs. \( \frac{L}{C} \)

Limits on impedance control of geometry, can't just make wider to lower R, change \( \frac{L}{C} \)

Other lines exist - coplanar waveguide, differential microstrip, BIL, etc.

Homework

Waveguides similar, but don't support TEM
- What Zin do we see driving this line? (Zo would be nice)
- Solve for general case to add loss

- In addition Zin = \frac{1}{Y}
- Y = Cs

- IF ZY constant create a constant k line
- IF ZY ≪ 1 then Zin ≈ \sqrt{2Y}

- Lossless line
  \[ Zin = \frac{Z + Zin}{\sqrt{1 + \frac{4}{2Y}}} \]

- Lossy line
  \[ Zin = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \]

- Characteristic impedance talks about \( V_H \) at a point, might want \( V(x)/V(x_0) \)
- Different points from \( \gamma \) - propagation constant

- Voltage divider: \( V_{HI} = V_H \frac{Z_0}{Zo} \frac{1}{YdZ} \) → \( V_{HI} \)
  \[ \frac{V_{HI}}{V_H} = \frac{Z_0}{Zo} \frac{1 - ZdZ}{Z + Zo \gamma \omega} \]

- In limit of sinusoidal expansion
  \[ \frac{\delta V}{\delta Z} = V_{HI} = V_H (1 - \frac{Zo}{Z_0} \delta Z) \]

- 1st order DE complex propagation constant
Propagation constant

$$V = V_0 e^{-\alpha z} e^{-j\beta z}$$

**exp. loss as we travel along line**

**phase as we travel along line**

$$\rightarrow \text{delay is } \frac{\partial \phi}{\partial t}$$

$$\rightarrow \text{freq. dependent } \rightarrow \text{dispersive}$$

- Finding $\alpha + j\beta$
  - Generally $\alpha \ll \beta$ 
  - Generally $\alpha \ll \beta$ 

**If $RG \ll \infty$**

$$\alpha = \frac{\sqrt{\frac{R}{Z_0}} + 6Z_0}{2}$$

$$\alpha = \sqrt{\frac{R}{Z_0}}$$

$$\beta = \omega \sqrt{LC}$$

**If $RG \ll \infty$**

$$\beta = \omega \sqrt{LC}$$

**if $RC = GL$**

$$\text{ inherent}$$

- frequency independent delays

- $\alpha = \sqrt{LC}$

- $\beta = \omega \sqrt{LC}$

- so non-dispersive

$$\varphi = \frac{VLC}{N_0} \text{ per}$$