Near Field E&M

- Simple probe case
- Plane waves
- Use Maxwell from last time

Wave Derivation
- Per usual
- They do it?

Lumped vs. Distributed
- KVL & KCL from Maxwell
- Relationship to c
- Boundary

Near Field Probe Intro
- What are they
- Why used...
- EMC
- How used
- Complex case
- Gotcha
- Cabling

1st 1/2 of class is about how Maxwell meshes with our assumptions about circuits

- Easiest way to observe this, which is still relevant for your RF practice, is to observe near-field radiation from circuits

- Do so w/ new field probes (pass around)

- B sensitivity of B probe works by inducing current
- E probe works by induced voltage - other side of amp displacement current

B probes
- Are they orientation sensitive? — Yes, face down towards current
- What do you trade off? E vs. sensitivity? — Even resolution
- B doesn’t care about orientation, already precise

EMC
- Near field radiation can indicate a source of far field radiation

- Smaller/delicate probes
- Radiated coupling/noise ID
- E -> impedance

 shielded vs. E

- Not a guarantee! -> Frequency response
- Propagation
2 Examples of new field measurements

- Issue: small signal, need to isolate from noise \( \rightarrow \) spec an. or FFT

  - Can aid your signal by turning it off or changing S

- Issue: cabling creates big stray fields, setup counts

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Do you have a radiation problem? Maybe... eg use fiber to cap could form harmonic circuit?

- Where are frequency components from? \( \rightarrow \) switching activity

  \( \rightarrow \) nonlinearity

- \( E/B \) difference from \( \Delta t \) vs. \( dB/dt \)

Simple example:

- Find magnetic field \( \propto \) current @ distance \( d \)

- Will a B probe measure this? If so @ what orient?

  - Maxwell!

\[
\oint B \cdot dl = \mu_0 \int J \, dt \quad \text{for each } d\theta
\]

\[
2\pi B = \mu_0 I \quad \Rightarrow \quad B = \frac{\mu_0 I}{2\pi}
\]

- Easiest example

  - is simple, look up

  - is tricky, FEn or give up

- Biot-Savart

- B probe does not measure \( B \) DC. If it were AC, then feedback.
- Everything we've seen is quasi-static & instantaneous — no reason for B to leave L\n
- I promised high freq. + wave behavior — no current reflectors

Maxwell in free space:
\[
\nabla \cdot \mathbf{E} = 0 \quad \text{no q}
\]
\[
\nabla \cdot \mathbf{B} = 0
\]
\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}
\]
\[
\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\]

Take
\[
\nabla \times \left( \nabla \times \mathbf{E} \right) = \nabla \times \left( \frac{\partial \mathbf{B}}{\partial t} \right) = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}
\]

- Recall \( \nabla \cdot \nabla \mathbf{E} = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\nabla^2 \mathbf{E} \)

- Gives
\[
-\nabla^2 \mathbf{E} + \varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0
\]

- Symmetry
\[
-\nabla^2 \mathbf{B} + \varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0
\]

- Iconic examples of a class of PDE called wave equations

\[
\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}
\]

Assume \( \phi \) \( \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2} \)

\( \psi \) is any function

\( \Lambda \) imaginary constant \( \pm i \)

\( v = \pm 1 \)

\( \Lambda \) propagates

\( \Lambda \) constant \( z = \lambda \)

- Speed for us in our E&M wave equations is \( \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \) speed of light

- Changes when not in free space by \( \sim 10-50\% \)

- Can have propagating \( \mathbf{E} + \mathbf{B} \) fields & \( \mathbf{E} - \mathbf{B} \) fields make voltages & currents via Maxwell
- When do we use static models vs. when wave models
- Get there by deriving \( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \)
  - \( \nabla \cdot \nabla \times \mathbf{B} = \nabla \cdot \mathbf{J} = \nabla \cdot \mathbf{J} = 0 \)
  -\( \nabla \cdot \mathbf{J} \) - No current divergence: current \( \mu = \) current arch (kcl)

Revised: \( \nabla \times \mathbf{B} = \mu \mathbf{H} \) & let \( M_0 \to 0 \)

\( \nabla \times \mathbf{E} = -\frac{\partial \mathbf{H}}{\partial t} = 0 \) so \( \nabla \times \mathbf{E} = \mathbf{0} \) & no curl \( \implies \) potential gradient

\( \nabla \phi \equiv \mathbf{E} \implies \int \nabla \phi \cdot d\mathbf{e} = 0 \implies \mathbf{KVL} \)

- If \( M_0 \) or \( E_0 \to 0 \) then \( C \to \infty \) - these approximations hold when waves are fast

So figure about waves if \( \frac{1}{C^2} \ll \frac{1}{\lambda^2} \)

\[ \frac{c}{2} \ll \frac{\lambda}{2} \]

Some of highest \( \mu \) wavelengths

- fiber optic: \( \varepsilon_r = 10 \), \( \mu_r = 1 \)

- \( \lambda \) square light pulses \( @ \) 10 Gbps
- \( \lambda \) standard near-infar length

Example: Thin hole resister is 1 cm, \( M_0 = 1 \), \( \varepsilon_r = 4 \)

\( @ 1 \text{ MHz} \quad \frac{c}{2} = \lambda \cdot (1 \text{ MHz}) \rightarrow \lambda = 1.5 \text{ cm} \quad \text{not a wave} \)

\( @ 10 \text{ MHz} \quad \frac{c}{2} = \lambda \cdot (10 \text{ MHz}) \rightarrow \lambda = 1.5 \text{ cm} \quad \text{wave-like} \)

- Not a lumped R
- Many elements