

Grade overview & power levels

Linearity

Harmonic distortion & P-1dB

Intermodulation
↳ IIP2 & IIP3

Logistics:

- Exam ~ ~~test~~ is sec sticky note
- Grading intent ~ grade like graduate course
- Watch power levels in lab 5! When rewrite to lab 5 w/ warnings & improved immunity & alternate noise temp

- We're talking about linearity
- Big signals ~~prevent~~ cause RX to clip
- Taylor represent non-linear elements

$$V_o(t) = V_i(t) \cdot a_1 + a_0 + V_i^2(t) a_2 + V_i^3(t) a_3 + \dots$$

our inputs are sinusoids

$$V_o(t) = a_0 + a_1 V_i \cos(\omega t) + a_2 V_i^2 \cos^2(\omega t) + a_3 V_i^3 \cos^3(\omega t)$$

$$= a_0 + a_1 V_i \cos(\omega t) + \frac{1}{2} a_2 V_i^2 (1 + \cos 2\omega t) + \frac{1}{4} a_3 V_i^3 (3 \cos \omega t + \cos 3\omega t)$$

\uparrow DC op \uparrow gain \uparrow DC shift \uparrow 2nd harmonic \uparrow in-band gain compression \uparrow 3rd harmonic

Gain compression

- in-band component of 3rd order distortion affects gain

~~Gain~~ $G = \underbrace{\left(a_1 + \frac{3}{4} a_3 V_i^2 \right)}_{G_{eff}} \cos \omega t$ ← gather in-band $\cos \omega t$ terms

- a_3 often negative, so gain shrinks as $V_i \uparrow$ (clipping)
- Measure w/ P-1dB, value of V_i st $\frac{G_{eff}}{G}$ is -1dB

In lab, test by ramping input tone & watching gain

$$\frac{G_{eff}}{G} = 1 + \frac{3}{4} \frac{a_3}{a_1} V_i^2, \quad 20 \log \left(1 + \frac{3}{4} \frac{a_3}{a_1} V_i^2 \right) = -1 \text{ dB} \implies \frac{3}{4} \frac{a_3}{a_1} V_i^2 = -0.11$$

or $V_i = \sqrt{\frac{4}{3} \frac{|a_3|}{a_1}} \times \sqrt{0.11}$

Harmonic Distortion

HD2 - 2nd order harmonic distortion

$$HD2 = \frac{\text{ampl } 2^{\text{nd}} \text{ harmonic}}{\text{ampl fundamental}} = \frac{a_2 V_i^2 / 2}{a_1 V_i} = \frac{1}{2} \frac{a_2}{a_1} V_i = \frac{1}{2} \frac{a_2}{a_1^2} V_o$$

if output referred

- linear w/ V_i
- dBC is units

HD3 - 3rd order harmonic distortion

$$HD3 = \frac{\text{ampl. of } 3^{\text{rd}} \text{ harmonic}}{\text{ampl. of fundamental}} = \frac{a_3 V_i^3 / 4}{a_1 V_i} = \frac{1}{4} \frac{a_3}{a_1} V_i^2 = \frac{1}{4} \frac{a_3}{a_1^2} V_o$$

- quadratic w/ V_i
- very similar coefficient to gain compression

THD - Total harmonic distortion

First, $\frac{P_{\text{distortion}}}{P_{\text{fundamental}}} = \frac{P_{HD1}}{P_{\text{fundamental}}} + \frac{P_{HD2}}{P_{\text{fundamental}}} + \dots = HD1^2 + HD2^2 + \dots$

$$THD = \sqrt{HD1^2 + HD2^2 + \dots}$$

10% telephony
5% video
0.001% audiotape
0.1% RF PA

Intermodulation

- we've been talking about single-tone nonlinearity
- But situation we're worried about is rogue TX near our RX

ie $V_i(t) = V_1 \cos \omega_1 t + V_2 \cos \omega_2 t$

$$V_o(t) = a_0 + V_i(t) \cdot a_1 + a_2 V_i^2(t) + a_3 V_i^3(t)$$

$\rightarrow x^3 + x^2y + y^2x + y^3$
 \rightarrow next page

$$a_1 (V_1 \cos \omega_1 t + V_2 \cos \omega_2 t) + a_2 V_1^2 \cos^2(\omega_1 t) + a_2 V_2^2 \cos^2(\omega_2 t) + 2a_2 V_1 V_2 \cos \omega_1 t \cos \omega_2 t$$

IM2 is bad for low IF

$$a_2 \left(\frac{V_1^2}{2} (1 + \cos(2\omega_1 t)) \right) + \frac{V_2^2}{2} (1 + \cos(2\omega_2 t)) + V_1 V_2 (\cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t)$$

Two tone nonlinearity test

same as harmonic distortion

intermodulation (2nd order)

$$IM2 = \frac{\text{ampl. IM w/ } V_1=V_2}{\text{ampl. fundamental}} = \frac{a_2 V_i^2}{a_1 V_i} = \frac{a_2}{a_1} V_i = 2HD2$$

(HD2 + 6dB)

- IM amp $\propto V_i^2$, fund $\propto V_i \rightarrow$ IM catches up
- IIP2 is V_i st $IM2 = 0$ dBC = fund. $V_i = \frac{a_1}{a_2}$
- can get IM2 from IIP2 as dB for dB

Cubic intermodulation

- skip the $x^3 + y^3$ terms \rightarrow harmonic distortion

$$\begin{aligned}
 & \text{- other terms are } 3 a_3 V_{i1} V_{i2}^2 (\cos \omega_1 t \cos^2 \omega_2 t) \\
 & = 3 a_3 V_{i1} V_{i2}^2 \cos \omega_1 t (1 + \cos 2\omega_2 t) / 2 \\
 & = a_3 V_{i1} V_{i2}^2 \left(\frac{3}{2} \cos \omega_1 t + \frac{3}{4} \cos (2\omega_2 \pm \omega_1) \right)
 \end{aligned}$$

(9.1dB)

comparing $\frac{IM3}{P_{fund}}$ = $\frac{\text{amp. of in-band intermod. } \omega_1/V_1=V_2}{\text{amp fund.}}$ = $\frac{3 a_3 V_i^3}{4 a_1 V_i} = \frac{3}{4} \frac{a_3}{a_1} S_i^2 = 3 HD3$ HD3 + 10dB

Like 2nd order intermod, can define voltage² where 3rd order IM catches fund.

$IIP3 = P_{in}$ st $IM3 = 0\text{dBc}$ \Rightarrow $IM3 = 1 = \frac{3}{4} \frac{a_3}{a_1} V_i^2 \Rightarrow V_i = \sqrt{\frac{4}{3} \frac{a_1}{a_3}}$
 (ie: 3rd intermod amp = fund)

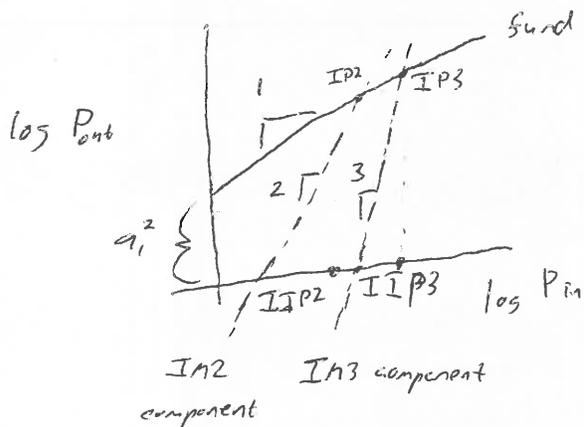
- can find $IM3$ for any power level from $IIP3$... fall off @ 20dBc for 10dB input

Relation between $IIP3$ & P_{-1dB} & HD3

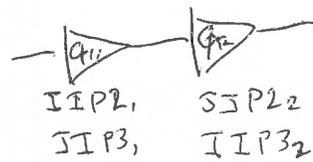
$$\left. \begin{aligned}
 V_i @ P_{-1dB} &= \sqrt{\frac{4}{3} \frac{a_3}{a_1}} \cdot 0.11 \\
 V_i @ IIP3 &= \sqrt{\frac{4}{3} \cdot \frac{a_3}{a_1}}
 \end{aligned} \right\} 20 \log \frac{V_i @ P_{-1dB}}{V_i @ IIP3} = -9.6 \text{ dB}$$

- Measure P_{-1dB}
- calculate $IM3$ for
- Extrapolate to $IIP3$ P_{in} backoff from $IIP3$

Graphical rederiving of intercept points



Cascade formulas



overall $IIP3$ & $IIP2$

$$\frac{1}{IIP2} = \frac{1}{IIP2_1} + \frac{a_{11}}{IIP2_2}$$

$$\frac{1}{IIP3} = \frac{1}{IIP3_1} + \frac{a_{11}^2}{IIP3_2}$$

- Involved derivation
 - Simple exp. is the products amp'd by next stage