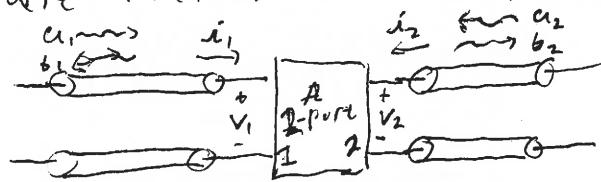


Relate bkt to S-params	Complex Power	S-param power flow	Matchng & gain Definitions	max power Xfer th <sup>2</sup> (next)
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• Talking w/o or working on S-parameters

- Measure reflections from & "through" a 2-port network



$$a_1/a_1 = \frac{V_{11}}{\sqrt{Z_0}}$$

$$\vec{b} = \vec{s} \vec{a}$$

sensitivity

- Want to relate to lumped ckt. theory
  - could just stick w/ S-params ...

- Express  $V_1$  as fn. of  $a_1$  &  $b_1$ :  $V_1 = (a_1 + b_1) \sqrt{Z_0}$

$$i_1 = (a_1 - b_1) / \sqrt{Z_0}$$

- Can flip this around to use port vars forget  $a_1$  &  $b_1$ .

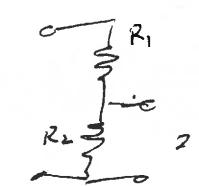
$$a_1 = \frac{V_1 + Z_0 I_1}{2\sqrt{Z_0}} \quad b_1 = \frac{V_1 - Z_0 I_1}{2\sqrt{Z_0}} \quad \text{or add & sub equations above}$$

→ Need to know so you're not confused by references ~ not match physical meaning

→ Handy as gateway between S-params & lumped models

- eg: Find S params of divider

• Can get  $S_{11}$  &  $S_{22}$  from reflection coeff. def<sup>n</sup> w/terms in place



$$S_{11} = \frac{R_1 + R_2 || Z_0 - Z_0}{R_1 + R_2 || Z_0 + Z_0} \quad \text{and} \quad S_{22} = \frac{R_2 || (R_1 + Z_0) - Z_0}{R_2 || (R_1 + Z_0) + Z_0}$$

•  $S_{12}$  &  $S_{21}$  more involved ~ so use Z-param conversion  
~ (so not  $a \rightarrow V$ , convert)

$$\vec{a} = \frac{1}{2\sqrt{Z_0}} (\vec{V} + Z_0 \vec{I})$$

$$= \frac{1}{2\sqrt{Z_0}} (Z + Z_0 \vec{I}) \vec{I} ; \vec{I} = \frac{1}{2\sqrt{Z_0}} (Z - Z_0 \vec{I}) \vec{I}$$

} combine

$$S = (Z - Z_0 \vec{I})(Z + Z_0 \vec{I})^{-1}$$

$$\text{or } Z = (I - S)^{-1}(I + S)Z_0$$

Upshot is that

$$S_{21} = \frac{2Z_0 Z_{21}}{(Z_{11} + Z_0)(Z_{22} + Z_0) - Z_{12} Z_{21}} \quad \text{call this } \Delta$$

$$S_{12} = \frac{2Z_0 Z_{12}}{\Delta}$$

- Find Z params (or Y params) & convert

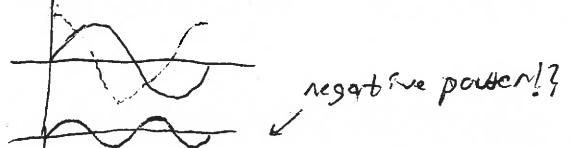
- Note ABCD params are popular

Analytic power:

- we've been using  $V(t) = V_0 e^{j\omega t}$  w/ complex  $V_0 \rightarrow$  saves us trig
- Has implications for power calculations
- eg: what is power on avg in ideal inductor?

$V_{sin wt}$    $i = \sin \omega t / j\omega L$

$$\langle P \rangle = \int_0^{2\pi} i v = \cos(2x) \cdot \frac{1}{j\omega L} \Big|_0^{2\pi} = 0$$

$$V = L \frac{di}{dt} \rightarrow i = \frac{1}{\omega L} \cos \omega t$$


- capture this with power def<sup>1</sup>

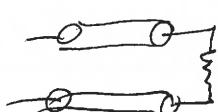
$$P = \frac{1}{2} \operatorname{Re}\{V I^*\} \quad \leftarrow \text{Need voltage & current } \underline{\text{in phase}} \text{ to dissipate power}$$

es  $\operatorname{Re}\{V_0 e^{j\omega t} \cdot V_0 e^{-j\omega t} / j\omega L\} = \operatorname{Re}\{V_0^2 / j\omega L\} = 0$

- Power in a  $\pi$  or  $\Delta$  network is  $\frac{1}{2} |q_1|^2 = \frac{1}{2} (V_{11}/\sqrt{Z_0})^2 = \frac{|V_{11}|^2}{2Z_0}$
- Power into 2 port  $\pi$   $\operatorname{Re}\{V_1 I_1^*\} = \frac{1}{2} |q_1|^2 - \frac{1}{2} |q_2|^2$

- relation of  $\Delta$  to  $\pi$  means  $b_1 = S_{11} q_1$  if terminated so  $P_{ref} = |S_{11}|^2 R_{in}$

eg 1 port



$$P_{\text{reflected}} = |\Gamma|^2 P_{in}$$

$$P_{\text{loss}} = (1 - |\Gamma|^2) P_{in}$$

Lossy 2 port

$$P_{\text{loss}} = \frac{1}{2} \alpha^* (I - S^H S) \alpha$$

positive definite