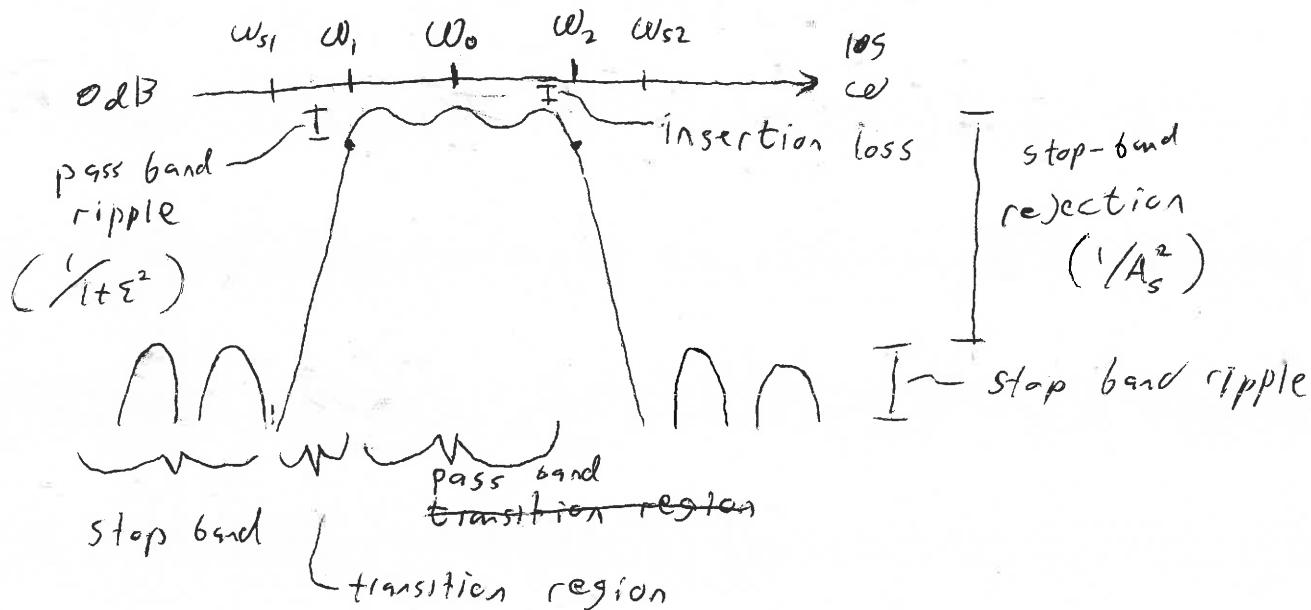


22.1

22.8

- Filter description
 - Filter Synthesis
 - ↳ transforms
 - ↳ transforms
 - ↳ Relation to S-params
 - ↳ Laplace Picture
 - ↳ Z-match U/VNA
- We're talking about S-parameters ~ generalization of Π to 2-port systems. Going to talk more
- Need to go over filters for design project ... ok S-param example
- Filters are linear systems that are frequency selective

Lots of common specifications



- ω_0 called the center frequency

- ω_1 & ω_2 define edge of pass band

in general
end edge
 $f = -3\text{dB}$
out $\frac{1}{1+\epsilon^2}$

{ - magnitude @ ω_1 & ω_2 often $-3\text{dB} \sim \frac{1}{2}$ power BW

- Bandwidth $BW = \omega_2 - \omega_1$

- Fractional BW means ω_2 & ω_1 determined from ω_0

↳ e.g.: $F\text{BW} = \frac{BW}{\omega_0}$... good invariant measure of "difficulty"

- $\omega_0 = \sqrt{\omega_1 \omega_2}$ b/c x-axis is $\log \omega$

- Max pass band deviation of $\frac{1}{1+\epsilon^2} \rightarrow \min$ stop band $\frac{1}{A_s^2}$

- Steepness of rolloff in transition region $\propto \omega^n$ where n is called order
 - \hookrightarrow related to # of poles used to make filter

$$\text{Filter prototype } |H(s)|^2 = \frac{1}{1 + \epsilon^2 F(s)}$$

- \hookrightarrow function $F(s)$ is polynomial or ratio of polynomials w/ order n
- \hookrightarrow specific function determines shape of filter
- \hookrightarrow Order \rightarrow poles \rightarrow # energy storage \Leftarrow # stored states in memory

Butterworth

Chebyshev I

Chebyshev II

elliptic



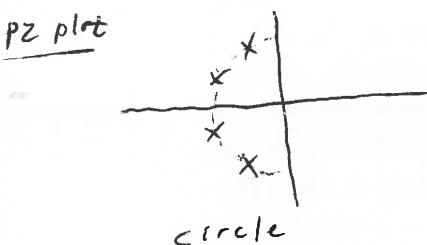
$$F(s) = \left(\frac{\omega}{\omega_p}\right)^n$$

passband edge

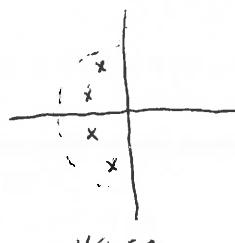
Chebyshev
polynomial
 $C_n(\omega/\omega_p)$

inverse chebyshev
polynomial
 $1 - C_n(\omega_p/\omega)$

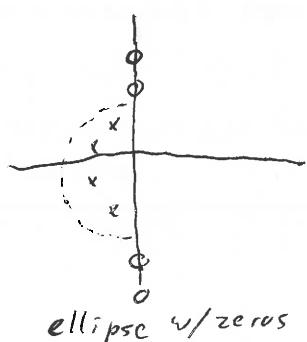
Chebyshev rational
polynomial $N(s)/D(s)$



circle



ellipse



ellipse w/zeros

— same

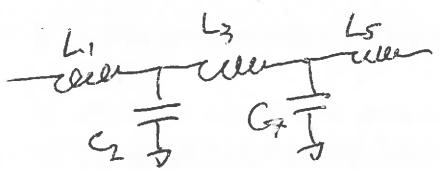
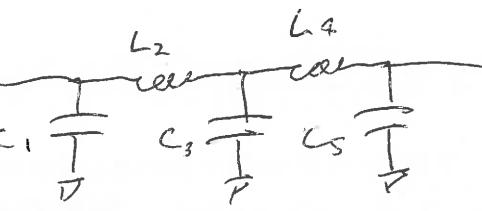
→ phase for these usually nuts

→ Bessel-Thomson filters approximate linear phase (pretty well)

② expense of ripple $\Leftarrow F(s)$ are Bessel functions

→ Butterworth often called maximally flat \sim type II chebyshev technically flatter

- How do we make filters?
- By tradition, start w/ low pass & learn about high pass transformation

- 1st pick $A_s, \varepsilon \geq n$ (for allowable stopband monotonicity)
 - $\varepsilon = \sqrt{10^{0.18} - 1}$
 - \hookrightarrow ~~some filter has~~ formula relating ε to passband deviation
 - $\hookrightarrow A_s$ vs. n plots or equations: Butterworth $n = \frac{\ln A_s/\varepsilon}{\ln \omega_p/\omega_0}$
 - \hookrightarrow Look up or simulate
(Matlab pretty good)
- Now need to turn into circuit
 - need n energy storage elements
 - $n=3$: 
 - or
 - 

- get component values from a filter table

\hookrightarrow tables normalized to $Z_{in} = 1 \Omega$ & $\omega_p = 1 \text{ rad/s}$

\hookrightarrow de-normalize Z by multiplying Z_n by Z_0 (so ω)

$$\hookrightarrow L = L \cdot Z_0 \quad \& \quad C = C / Z_0$$

\hookrightarrow un-normalize ω by dividing all elements by ω_p

$$(\text{recall } Z_{in} \approx \sqrt{L/C} \quad \& \quad \omega_0 \approx \sqrt{L/C})$$

Example Butterworth:

→ Go from low pass to high pass, band pass, band stop

$$\frac{L}{\omega_0} \left| \begin{array}{c} 1 \\ \text{---} \\ C \end{array} \right| \rightarrow \frac{1}{\omega_0 L} \left| \begin{array}{c} 1 \\ \text{---} \\ T \end{array} \right| \quad \frac{Y_{BW}}{\omega_0} \left| \begin{array}{c} 1 \\ \text{---} \\ T \end{array} \right| \frac{BW}{\omega_0^2 L} \quad \frac{L-BW}{\omega_0^2} \left| \begin{array}{c} 1 \\ \text{---} \\ T \end{array} \right| \frac{1}{BW \cdot L}$$

$$\frac{C}{\omega_0} \left| \begin{array}{c} 1 \\ \text{---} \\ T \end{array} \right| \rightarrow \frac{1}{\omega_0 C} \left| \begin{array}{c} 1 \\ \text{---} \\ C \end{array} \right| \quad \frac{BW}{\omega_0^2 C} \left| \begin{array}{c} 1 \\ \text{---} \\ T \end{array} \right| \frac{C}{BW}$$

high pass

band pass

$$\left| \begin{array}{c} 1 \\ \text{---} \\ T \end{array} \right| \frac{1}{BW \cdot C}$$

band stop

$$\frac{C \cdot BW}{\omega_0^2}$$

↳ band elements \sim parallel tanks rans open \Rightarrow series ring short
 $(Y=0)$ $(Z=0)$

S-parameters

$$\Rightarrow |H(s)|^2 = |S_{21}|^2$$

→ S_{11} goes low when S_{21} is high \sim no reflection in pass band

→ reflect otherwise b/c lossless filters

↳ can make lossy RC or RL ladders