

Review open reflection

- including mirror, mirror analogy

Short reflection Exercise

↳ mismatched load & short if time

Sinusoidal Reflection

↳ Get to VSWR  
↳  $t$  vs.  $z$  difference

Driving impedance of terminated line vs. length

- Transmission lines described by  $\gamma$  &  $Z_0$  w/  $V(z) = V_0 e^{-\gamma z}$   
~ Note  $\alpha$  in dB/m not native units

- Terminated lines described by  $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$  (+ maybe  $\Gamma_S$ )  
terminated in  $Z_L$

- Last time we practiced calculating  $\Gamma$

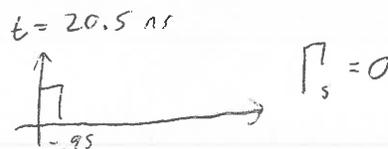
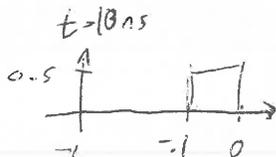
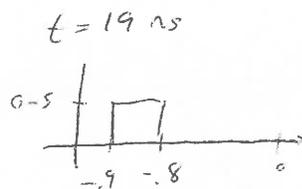
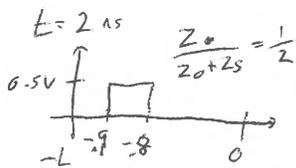
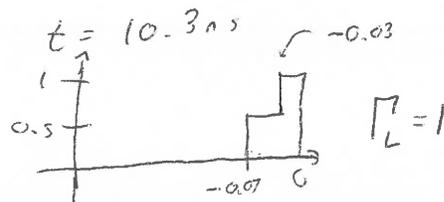
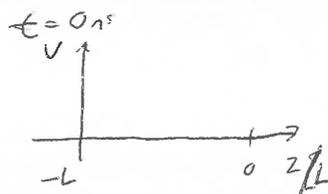
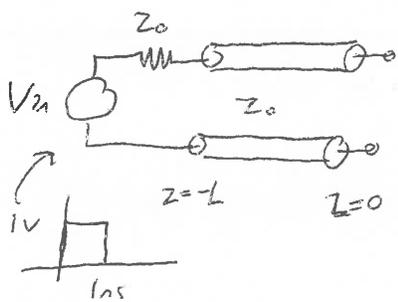
~ big surprise was that driving point impedance of w.t. line is  $Z_0$ , "see"  $Z_0$  until a reflection comes back to us.

- Ended w/ simulating square pulse propagation in lossless finite line terminated in open

~ for lossless line  $\beta = \omega \sqrt{L'C'}$ , so  $v = \frac{1}{\sqrt{L'C'}}$  for all frequencies

~ means no dispersion, so we skipped  $V(z) = V_0 e^{-\gamma z}$

& just used direct wave sol<sup>n</sup>  $V(z,t) = V_{in}(t - \frac{z}{v})$

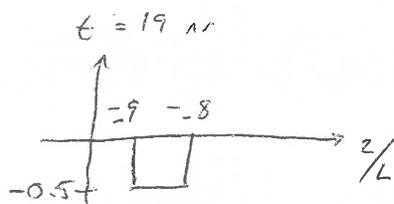
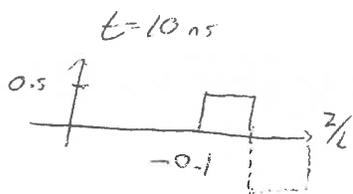
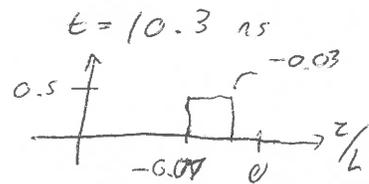
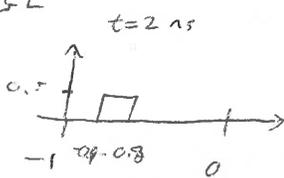
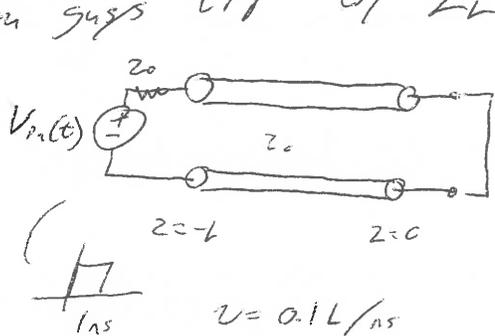


• Let  $v = 0.11$  ns

~~Young's experiment~~

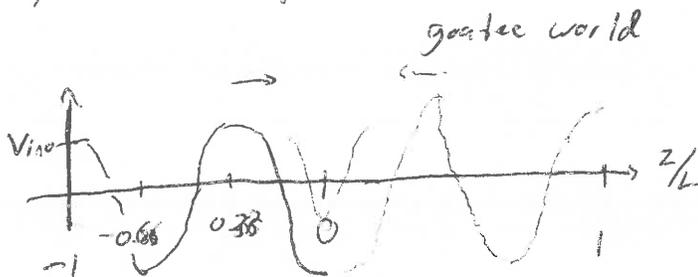
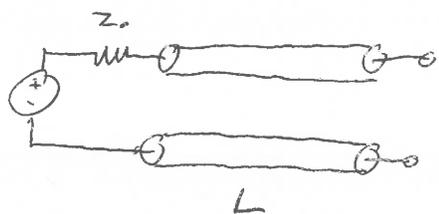
- One alternative picture I find useful is to imagine "Evil ghost twin mirror universe" t. line
- A t. line of same length has  $V_r$  launched ( $= \sum V_i$ ) & waves crash @ load.
- same thing b/c of superposition

- You guys try w/  $Z_L = 0 \Omega$



- Note that  $Z_L$  forces applied  $V = 0V$  b/c short, boundary cond not here

- What happens if we drive w/ sinusoid?



- Let  $v = 0.1 L/ns$

- Let  $V_{in} = \int \dots = \text{Re} \{ V_{in0} e^{j\omega t} \}$  ← analytic rep

- Let  $L = 1.5 \cdot v \cdot \frac{2\pi}{\omega} = 1.5 \cdot \frac{2\pi}{k}$  ← means 1.5 waves will fit on line  
⏟ period                      ⏟ wave number

- Gains to add up in funny way → let's use math instead of pictures

- Math for driving sinusoid

$$V_i(z=0, t) = V_{i0} e^{j\omega t} \quad \& \quad V_r(z=0, t) = \Gamma V_{i0} e^{j\omega t}$$

- picking  $e$  @ load makes this easy

$$V_i(z, t) = V_{i0} e^{j\omega t} e^{-\gamma z} \quad \& \quad V_r(z, t) = V_{i0} \Gamma e^{j\omega t} e^{+\gamma z}$$

← goes fwd. in bc. positive z

$$\begin{aligned} V(z, t) &= V_i(z, t) + V_r(z, t) \\ &= V_{i0} e^{j\omega t} (e^{-\gamma z} + \Gamma e^{+\gamma z}) \\ &= \underbrace{V_{i0} e^{j\omega t} e^{-\gamma z}} \left( 1 + \underbrace{\Gamma e^{2\gamma z}} \right) \end{aligned}$$

• Generalized reflection coeff  $\Gamma(z)$

• handy for mostly this case  
• ratio of  $V_r/V_i$  @ any  $z$

- pretty normal

sinusoid we're used to

- But multiplied by  $1 + \Gamma e^{2\gamma z}$  (ex freq sinusoid)

- product of sinusoids should make beats!



- Figure out maximum & minimum of beats by getting amplitude

$$\begin{aligned} |V(z, t)|^2 &= V(z, t) \cdot V^*(z, t) \\ &= V_{i0}^2 \cdot (1 + \Gamma e^{2\gamma z}) (1 + \Gamma^* e^{-2\gamma^* z}) \\ &= V_{i0}^2 (1 + |\Gamma|^2 + 2\Gamma e^{2\gamma z}) \end{aligned}$$

← same pattern w/ period  $2\gamma z$

If purely real & +ve

↳ only get ~~complex~~ real part

$$|V(z, t)|^2 = V_{i0}^2 (1 + 2\Gamma + \Gamma^2) \rightarrow |V(z, t)| = V_{i0} (1 + \Gamma)$$

If purely real & -ve  $\rightarrow |V(z, t)|^2 = V_{i0}^2 (1 - 2\Gamma + \Gamma^2) \rightarrow V(z_{min}, t) = V_{i0} (1 - \Gamma)$

- Ratio of max to min called  $VSWR = \frac{1 + \Gamma}{1 - \Gamma}$



- Unique w/  $\Gamma$ ! describes same thing!

(beat speed)  
(time evokes)  
(impedance)

- What is the spatial period of the beats?

↳ given by  $e^{2\beta z} \rightarrow$   ~~$e^{j\omega t - \beta z}$~~

↳ If lossless,  ~~$e^{j\omega t - \beta z}$~~   ~~$e^{j\omega t + \beta z}$~~

get  $e^{2\beta z}$  ←  $T_{delay} = 2\sqrt{LC}z$  on lossless line  $\sim \frac{1}{2}$

$\bullet 2\pi = 2\beta z$

$\bullet \lambda = \frac{v}{\beta} = \frac{v}{\omega/v} = \frac{v^2}{\omega} = \frac{v^2}{\omega\sqrt{LC}}$  ← half the wavelength of an input wave  
 • 1/2 sum of two opposite waves

- How does this evolve in time but moves @ 1/2 velocity

↳ unless line length exactly right beats don't stay still

↳ However, input  $\omega$  sine as wave  $\omega \rightarrow$  see canceling  $\lambda \neq v$

↳ but total accrued phase varies w/ L ... appears to have different impedance

- What is the impedance seen at ~~input~~ drive for a line of length  $z$

$$Z(z) = \frac{V_r(z) + V_i(z)}{I_r(z) + I_i(z)} = Z_0 \cdot \frac{1 + \Gamma_L e^{2\beta z}}{1 - \Gamma_L e^{2\beta z}} = \dots$$

↳ Lots of algebra making  $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

$$\dots = \frac{Z_L}{Z_0} \rightarrow \frac{Z_L \cos \beta z - j Z_0 \sin \beta z}{Z_0 \cos \beta z - j Z_L \sin \beta z}$$

if  $\alpha$  ignored

Equation is a mess (good for computers) we'll look @ graphical and.  
 Impedance is periodic in  $z$   
 ↳ 1/2  $v$  tap into diff  $\phi$  amp of VSWR pattern