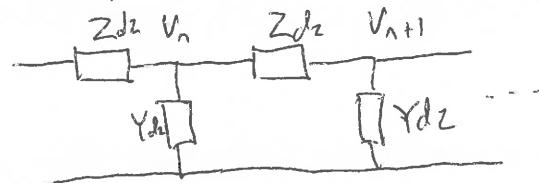


- Iterated, Telegrapher & propagation const.
- $Z_m$  &  $\gamma$  derivation w/f from divider
- $\alpha + \beta$  approx
- phase delay (velocity)
- Terminated line & reflection coeff
- VSWR & generalizes  $\gamma$  and  $Z(z)$
- $\hookrightarrow$  t example
- $\hookrightarrow$  T derivation current/field eq.
- $\hookrightarrow$  measured load 1st
- We introduced t lines as easy to design homes for waves
  - $\hookrightarrow$  introduced microstrips as an picnic example ...  $Z_0 \propto \sqrt{\mu_r \cdot h} \xrightarrow{\text{internet}} \text{FEM}$
  - $\hookrightarrow$  observed that  $E = H \rightarrow I = V$  & the house changed  $Z = \sqrt{\frac{L}{C}} \Rightarrow v = \frac{1}{\sqrt{LC}}$
  - $\hookrightarrow$  dwelled for a bit on meaning of  $Z_0$
- Trying to figure out what this looks like in a circuit
  - 
  -
- used  $Z_m = Z_0 \parallel \frac{1}{Y_{02}} \parallel Z_m \rightarrow Z_{in} = \sqrt{\frac{Z}{Y}}$
- For lossless line  $Z_m = \sqrt{\frac{L}{C}}$ , same as physics!
- For lossy line  $Z = R + j\omega L$  &  $Y = G + j\omega C$  so  $Z_m = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$
- $Z_{in}$  &  $Z_0$  are same thing b/c you see same  $Z$  measuring anywhere on the line.

- Lets compare 2 spots on the line ... do so w/ voltage div.



$$V_{n+1} = V_n \cdot \frac{\frac{1}{Y_{d2}} \parallel Z_0}{Z_{d2} + \frac{1}{Y_{d2}} \parallel Z_0}$$

$$\frac{V_{n+1}}{V_n} = \frac{Z_0 / Y_{d2}}{Z_{d2} \left( \frac{1}{Y_{d2}} + Z_0 \right) + Z_0 / Y_{d2}} = \frac{Z_0}{Z_0 Z_{d2}^2 + Z_{d2} + Z_0}$$

Take to 1st order so  $Z_{d2}^2 \rightarrow 0$  & try to form differential eq?

- $\frac{V_{n+1}}{V_n} = \frac{Z_0}{Z_{d2} + Z_0} \rightarrow \frac{V_{n+1}}{V_n} (Z_{d2} + Z_0) = \frac{Z_0}{1 + \frac{Z}{Z_0} dZ} \approx 1 - \frac{Z_0}{Z} dZ = 1 - \sqrt{ZY} dZ$

binomial

- $V_{n+1} = V_n (1 - \sqrt{ZY} dZ)$

$$\frac{V_{n+1} - V_n}{dZ} = -\sqrt{ZY} \Rightarrow \frac{dV}{dZ} = -\sqrt{ZY} \text{ in the limit}$$

- define  $\gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)} \equiv (\alpha + j\beta)$

↑ propagation constant

↑ generally complex

- solve to 1st order DE  $V(z) = V_0 e^{-\gamma z} = V_0 e^{-\alpha z} e^{-j\beta z}$

↑ attenuation

↑ phase accumulation

- Linear phase accumulation is the same as delay --- see Laplace  $e^{-\beta s}$  or Fourier  $e^{-j\omega t}$

- Generally  $T_{delay} \equiv \frac{d\phi}{dt}$  ~~see Tdelay~~

↑ group delay

$\rightarrow \alpha$  makes waves shrink (exponentially) w/  $\propto \beta$  makes them shift  
 $\hookrightarrow \beta(\omega) \Rightarrow$  fast edges move faster  $\Rightarrow$  dispersion  $\square \rightarrow \curvearrowright$

- $\alpha$  and  $\beta$  seem really important for wave evolution, but we know RLGC

- In general  $\alpha = \sqrt{0.5(\sqrt{\omega^2(LG)^2 + \omega^2(LG)^2 + (RC)^2} + (RC))^2 + \omega^2 LC)}$

↳ which is a mess, just get this from <sup>datasheet</sup> or measurements

↳ There is a similar  $\beta$  expression ↳ skin effect eg can make  $\alpha(\omega) \propto \beta(\omega)$

- can get usable design expressions in terms

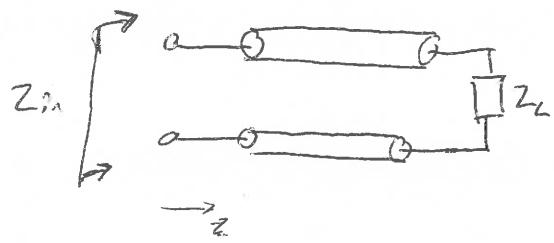
$$R=G=0 \rightarrow \alpha=0 \quad \& \quad \beta=\omega\sqrt{LC}$$

$$RG \text{ small} \rightarrow \alpha = \frac{1}{2} \left( \frac{R}{Z_0} + GZ_0 \right) \quad \& \quad \beta = \omega\sqrt{LC}$$

$$RC = GL \rightarrow \alpha = \sqrt{RG} \quad \& \quad \beta = \omega\sqrt{LC}$$

Heaviside (that one)  
discovered this & also  
using LT on ckt & Vector on  
Maxwell

- Next, lets imagine the line is finite & terminated in  $Z_L$



- If  $Z_L = Z_0$  then a propagating wave can't tell the difference between load & line

↳ in line  $V/\lambda = Z_0 \quad \} \quad$  as V wave impinges on

↳ in load  $V/\lambda = Z_0 \quad \} \quad$  load it generates exactly  
right amount of I

- Think like a wave

\* Now let  $Z_L \neq Z_0$

$\hookrightarrow$  there's going to be a reflection  $\frac{V_r}{V_i} \neq \frac{V_i}{V_{load}}$  so there's leftover current. Bounces back.

$\hookrightarrow$  system is linear, so voltages add, waves in opposite dir, so it subtracts

$$\text{At load } Z_L = \frac{V_i + V_r}{I_i - I_r} \quad - i \text{ or } r \text{ indicate incident or reflected}$$

- a boundary cond. for a wave, but it'll spec PDE

$$Z_L = \frac{V_r}{V_i} \cdot \frac{i + V_r/V_i}{1 - I_r/I_i} \quad - \text{ratio of } V_r/V_i = I_r/I_i \text{ b/c same } Z$$

- derive  $V_r/V_i \equiv \Gamma_L$  at load

$$Z_L = Z_0 \cdot \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

$$\text{can rearrange this to find values of } \Gamma_L = \frac{-Z_0 + Z_L}{Z_0 + Z_L}$$

If we look @ spot other than load

$$\text{Let } \Gamma(z) \equiv \frac{V_r(z)}{V_i(z)} = \frac{V_r \cdot e^{\gamma z}}{V_i \cdot e^{-\gamma z}} = \Gamma_L \cdot e^{2\gamma z}$$

$\Gamma$   
incident propagates fwd.  
and reflects back

if load @  $z=0$  & source @  $z=-l$

$$\left. \begin{aligned} V(z) &= V_i e^{-\gamma z} + V_r e^{\gamma z} \\ I(z) &= I_i e^{-\gamma z} + I_r e^{\gamma z} \end{aligned} \right\} \quad Z(z) = V(z)/I(z) = Z_0 \left[ \frac{1 + \Gamma_L e^{2\gamma z}}{1 - \Gamma_L e^{2\gamma z}} \right]$$

$$\frac{Z(z)}{Z_0} = \frac{2\gamma z / Z_0 - \tanh(\gamma z)}{1 - Z_0^2 / Z_0 \tanh(\gamma z)} \approx \frac{2\gamma z / Z_0 - j \tan \beta z}{1 - j Z_0 / Z_0 \tan \beta z} \quad \text{if } \alpha \text{ small}$$