

EM waves to  
IV waves

Driving point &  
Telegrapher &  
propagation constant

Microstrips, layout,  
current crowding

- steady state +  
out  
- updates

→ Labs will be done individually ↪

- Want to turn EM waves into measurable voltages & currents
- Assume sinusoidal:  $A e^{j(\omega t - kz)} = a(t, z)$  w/  $k = \frac{\omega}{v}$
- Travelling in  $\hat{z}$  →  $\vec{E}$  in  $\hat{x}$  &  $\vec{B}$  in  $\hat{y}$  (use vector math & pointing)
- Varies w/ time &  $z$  ( $\frac{\partial a}{\partial x} = 0$  &  $\frac{\partial a}{\partial y} = 0$ )

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \hat{y} \frac{\partial E_x}{\partial z} = -jk E_x(z, t) \hat{y} \quad ; \quad \nabla \times \vec{B} = \hat{x} \frac{\partial B_y}{\partial z} \\ \text{plus many zeros} \quad ; \quad = -jk B_y(z, t) \hat{x}$$

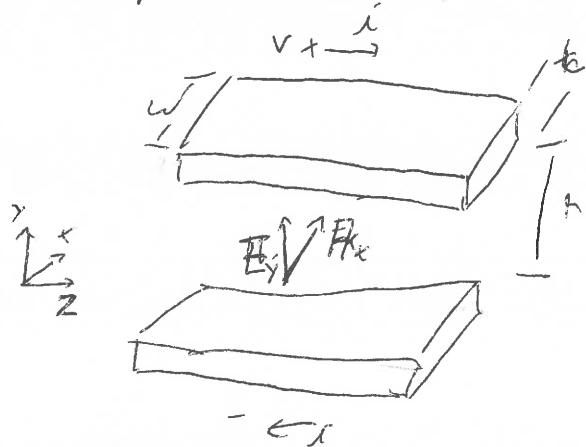
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$-jk E_x(z, t) \hat{y} = -jk \omega B_y(z, t) = -jk \mu_0 \epsilon_0 H_y(z, t)$$

$$\frac{E_x [V/m]}{H_y [A/m]} = \frac{\mu \omega}{k} = \frac{\mu \omega}{w \sqrt{\mu \epsilon}} = \sqrt{\frac{\mu}{\epsilon}} [\Omega] \quad \begin{matrix} \leftarrow \text{impedance of free space} \\ \leftarrow \text{measures ratio of } \vec{E} \text{ to } \vec{H} @ \\ \text{any point} \end{matrix}$$

can't be measured w/  
a multimeter

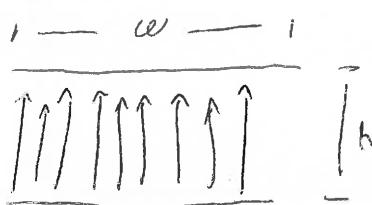
Let's put the wave in a wire\*



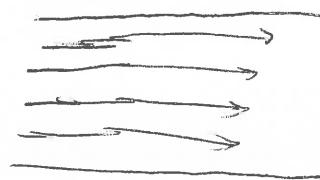
-  $H_x$  means current flows

-  $E_y$  means voltage exists

$E_y$  in front view



$H_x$  in front view  
or  $i$



- Ignore fringing

$$- V = \oint E \cdot dl = \hbar \cdot E$$

$$- \frac{C}{\text{length}} = \frac{\epsilon \omega}{h} \approx C'$$

From before:

$$\oint k E_x = \hbar \omega n H_x$$

$$\alpha \oint k E_x = \hbar \omega n H_x \cdot \frac{a}{b} \cdot b$$

$$\oint k V(z, t) = \hbar \omega L I(z, t)$$

$$\text{or } \oint k H_y = \hbar \omega \epsilon E_x$$

$$b \cdot \oint k H_y = \hbar \omega \epsilon E_x \frac{b}{a} \cdot a$$

$$\oint k I(z, t) = \hbar \omega C' V(z, t)$$

- propagating EM wave is same as propagating voltage/current wave

$$\frac{V(z, t)}{I(z, t)} = Z_0 = \frac{\omega L}{k} = \frac{\omega L}{\omega \sqrt{LC}} = \sqrt{\frac{L}{C}} \leftarrow \text{Not free space}$$

impedance b/c line affected by environment

↑  
ratio of voltage  
to current in wave  
(NOT at DC)

get  $k$  from

$$\oint k V = \hbar \omega L \left( \frac{\omega}{k} dV \right) \rightarrow k^2 L = \omega^2 L C$$

→ Summary  $Z_0 = \sqrt{\frac{L}{C}}$ ,  $k = \omega\sqrt{LC} \rightarrow v = \frac{1}{\sqrt{LC}}$

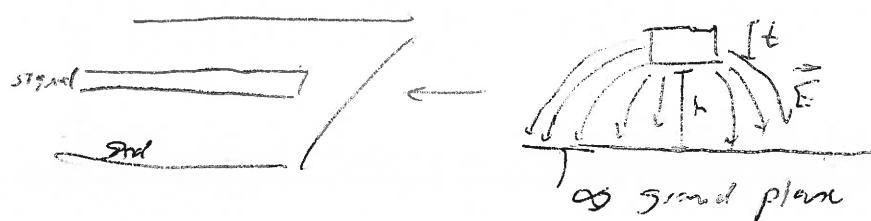
$$= \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{h}{\omega}$$

→ This environment called a transmission line

↳ structure that supports transverse electromagnetic (TEM) plane waves

→ Idealized, practical example is microstrip line

- $\sqrt{\frac{\mu}{\epsilon}} \frac{h}{\omega}$  inaccurate
- Microstrip equations & calculators abound
- Wheeler, Hammerstad



→ Issue 1: coupling risk b/c of distributed fringe (smaller the better)

→ Issue 2: corners



$L'$  and  $C'$  not uniform in this region

- ↳ issues, next lecture
- ↳ fix w/ miter

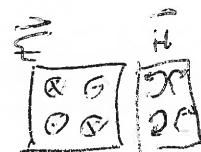
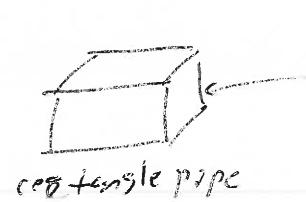
→ Aside ~ current crowding & skin effect change R vs f

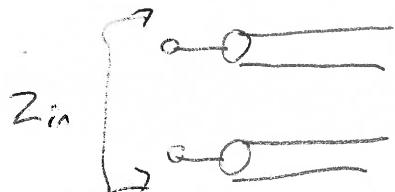


~ limits on impedance control w/ geometry, can't just make wider to lower R, change L & C

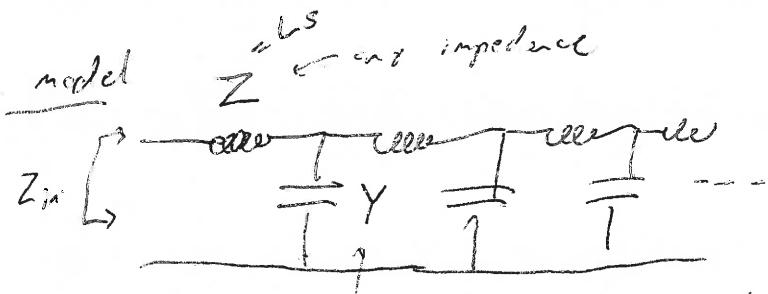
→ other t lines exist ~ coplanar waveguide, differential microstrip, BNC, etc  
~ homework

→ waveguides similar, but don't support TEM





- What  $Z_{in}$  do we see driving this line? ( $Z_0$  would be nice)
- solve for general case to add loss



Lossless  
line                          in addition  $Z_{load} = \frac{1}{Y}$

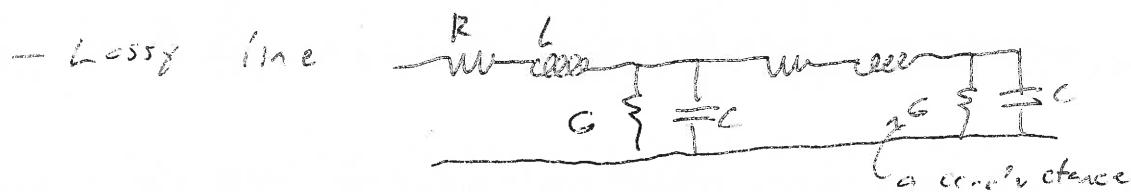
$$Y = G + jB$$

- If  $ZY$  constant called a constant  $\kappa$  line

- If  $ZY \ll 1$  then  $Z_{in} \approx \sqrt{ZY}$

- Lossless line  $\rightarrow Z_{in} = \sqrt{\frac{Z}{C}}$  theory!

Telegrapher's  
eqn



$$Z_{in} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

- characteristic impedance talks about  $V/I$  @ a point, might want  $V(x_1)/V(x_2)$
- different points from  $\gamma$  - propagation constant

voltage divider on differential

$$V_{n+1} = V_n \cdot \frac{Z_0 \parallel \frac{1}{Y_{dz}}}{Z_{dz} + Z_0 \parallel \frac{1}{Y_{dz}}} \rightarrow \text{differential}$$

$$\frac{V_{n+1}}{V_n} = \frac{Z_0}{Z_0 ZY (1/z) + Z_0 + Z_{dz}}$$

in limit, w/binomial expansion

$$\frac{dV}{dz} = V_{n+1} = V_n \left( 1 - \frac{Z_0}{Z_0} dz \right)$$

$$\frac{dV}{dz} = -\sqrt{ZY} V$$

$V(z) = V_0 e^{-\sqrt{ZY} z} = V_0 e^{-\gamma z}$

complex propagation const.

propagation const

$$\gamma = \alpha + j\beta \quad \leftarrow V(z) = V_0 e^{-\alpha z} e^{-j\beta z}$$

exp. loss  
as  $\omega$  travel  
along line

phase as  $\omega$   
travel along line

↳ delay is  $\frac{\partial \phi}{\partial \omega}$

↳ freq. dependent  $\rightarrow$  dispersion

• Finding  $\alpha + \beta$  generally a mess  $\rightarrow$  datasheet

$$\text{if } RG \text{ small} \quad \alpha \approx \frac{1}{2} \left[ \frac{R}{Z_0} + GZ_0 \right] \quad + \quad \beta = \omega \sqrt{LC}$$

$$RG \text{ zero} \quad \alpha = 0 \quad + \quad \beta = \omega \sqrt{LC}$$

$$RC = GL \quad \alpha = \sqrt{RG} \quad + \quad \beta = \omega \sqrt{LC}$$

(Heaviside)

↑  
- frequency indpt. delays  
so non-dispersive

$$- \alpha = \sqrt{LC} R$$

$$v = \sqrt{\frac{1}{LC}} \text{ m/s}$$