Department of Engineering

Lecture 21: Introduction to Noise

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Sources of Noise

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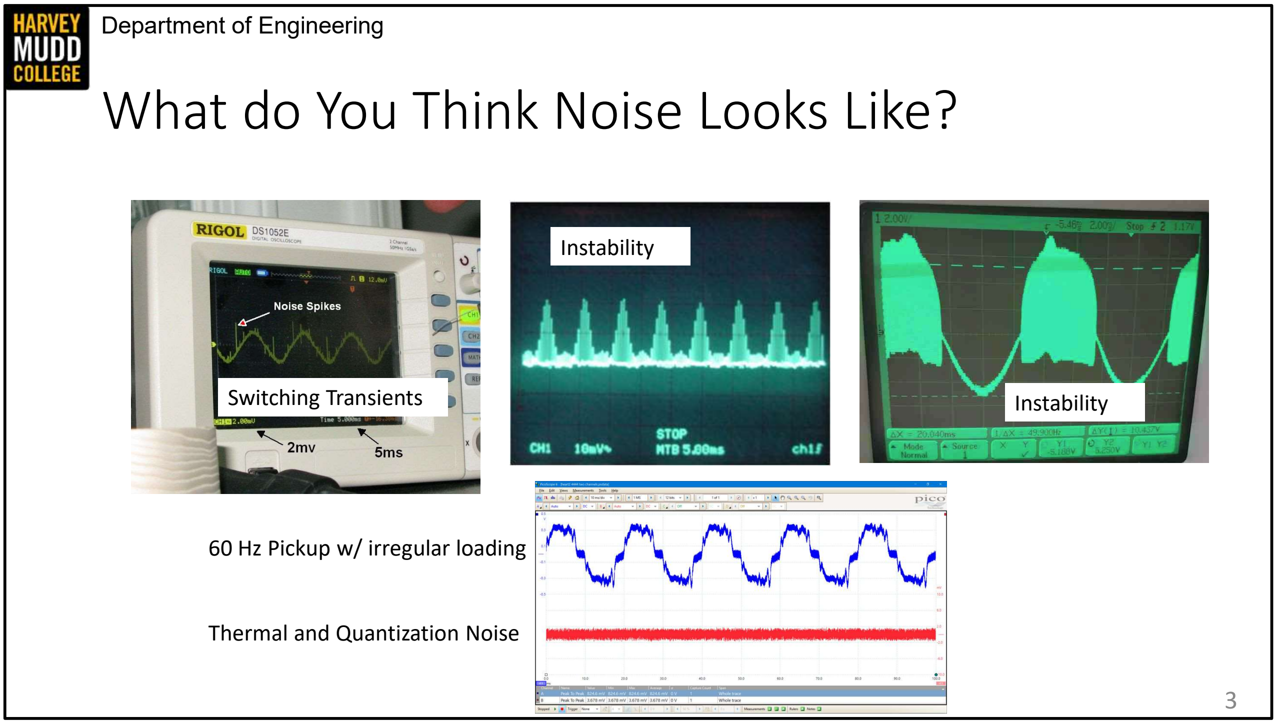
In this video we're going to talk about where noise comes from.



I've included a bunch of different traces on this slide. I'd like you to pause the video and decide which ones you would call "noisy."

OK, now that you've made your picks, I have to inform you that when we use the word "noise" in this class, we're referring to a very specific kind of purely random behavior. There are lots of signals that can make your oscilloscope traces look bad that aren't purely random. So there are very few traces on this slide that exhibit "noise" as we define it.

CLICK The large spikes in this trace are caused by a switching power supply. That power supply doesn't seem correlated with the sine wave on screen, but it's behavior isn't random. It's actually periodic, and you might be able to pick out the switching frequency from this trace.
CLICK These second two traces are showing the output of unstable amplifiers. The amplifiers create large, high-frequency sinusoids, but these are quite predictable: they only occur at one frequency.
CLICK This third trace is 60 Hz pickup in a circuit, which is also sometimes called hum in the audio amplifier community. 60Hz is the frequency of American wall power, and it's very easy to make an accidental antenna in your circuit such that you get a 60Hz signal. Pickup generally refers to signals that show up on accidental antennas.
CLICK Only this final signal qualifies as noise in this class. These random fluctuations, which

are often small, are caused by two sources of noise called thermal and quantization noise.

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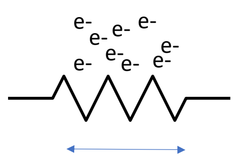
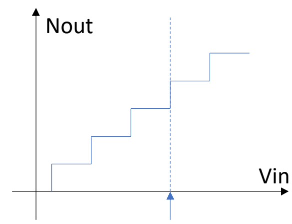
Get Random Noise from Resistors, Rounding

Thermal Noise comes from e- Thermal Motion
(AKA Johnson noise)



Current varies w/ # of e- at end

Quantization Noise comes from resolution



Input gets rounded up or down

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I mentioned two noise sources on the last slide: thermal noise and quantization noise. These are the only two noise sources we'll be considering in this class. Thermal noise comes from random motion of electrons in a resistor: if they all happen to move in the same direction then we get a current, and that current interacts with resistance by creating a small noise voltage. This is sometimes also called Johnson noise after one of the early derivations of its existence. Quantization noise come from rounding errors when we turn analog signals into digital ones. For example, the voltage we are applying to the analog-to-digital transfer function on the right could get round up or could get rounded down, either way, the number that we pick to represent the voltage will be pretty far from the actual value. We'll be diving into greater detail on these noise sources in the next few videos.

Summary

- Strictly defined, noise is purely random
- Pickup, instability, switching transients and hum are not noise
- Thermal noise comes from random fluctuations of e- in resistors
- Quantization noise comes from rounding in analog-to-digital converters

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Thermal Noise Units and Noise Temperature

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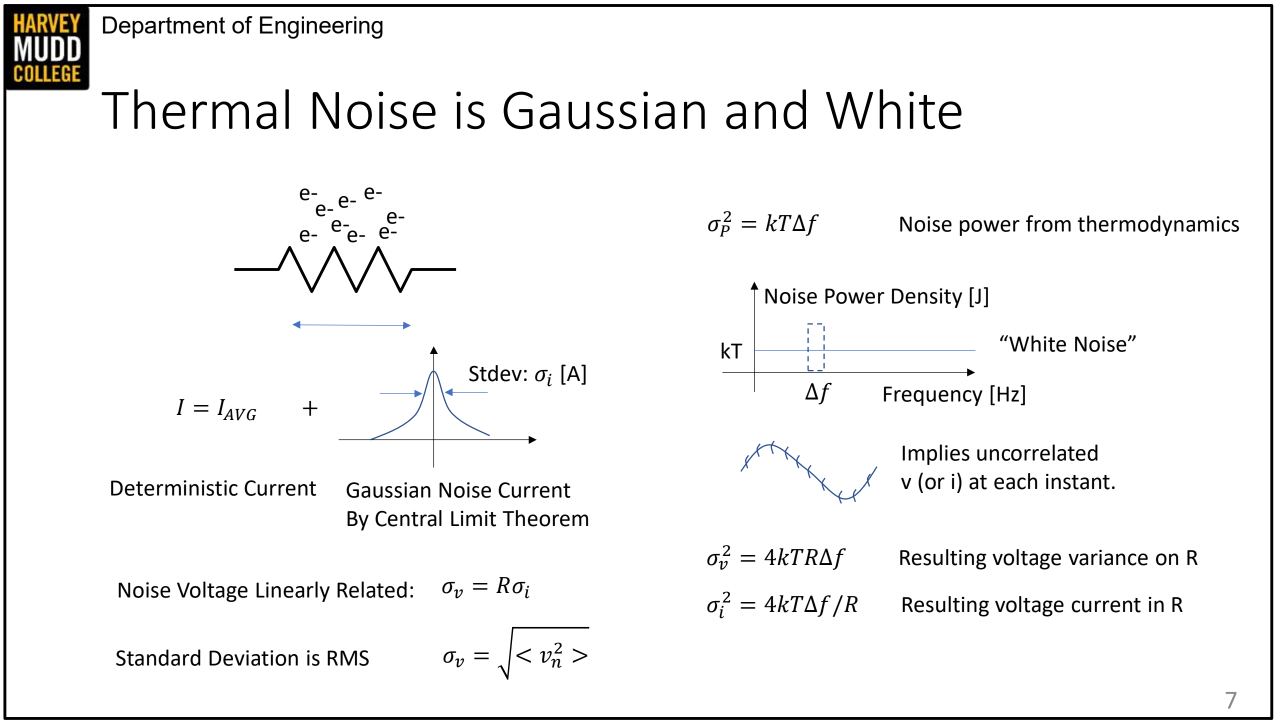
In this video we're going to examine what thermal noise looks like and how we describe it.



Thermal noise consists of random vibrations of electrons in a resistor.
CLICK That means that the total current in a resistor is going to be given by some amount of deterministic current flowing in the resistor plus a random amount of current that is normally distributed. We can say that the noise is Gaussian because it's the sum of many independent random processes – each electron "deciding" whether to run left or right in a time differential – which converges to a Gaussian by the central limit theorem. This Gaussian has some standard deviation, which we call sigma_i, and that standard deviation is a measure of how "strong" the noise is. We could measure this Gaussian by looking at a resistor for a long time and making a histogram of the currents we observe in it, which would be called an ensemble distribution. A property called ergodicity, which we're not digging into, means that the distribution at each unit of time matches the ensemble distribution.
CLICK This device is still a resistor, so the current and voltage are linearly related. That means the noise voltage is also a Gaussian, but the standard deviation is R times bigger than the standard deviation of the current. As a quick reminder, the standard deviation of a distribution is the same thing as the root, mean square value of the distribution. That fact is good to keep in mind because there are lots of notations for noise, some of which represent RMS voltages instead of standard deviations.
CLICK We are going to pull a few results from statistical mechanics with little justification for the rest of the slide. Statistical mechanics says that the variance of noise power is equal

to Boltzmann's constant times the temperature in each differential bin of a frequency spectrum. I've represented that with an equation at the top of this column and with a picture in the middle. This statement says that there's some noise energy at every frequency. This is called white noise because white light has power at many frequencies. That convention gets extended sometimes: noise that has its energy clustered at low frequencies is called pink.

CLICK In the time domain, this means that every spot on a signal has some Gaussian added on top of it, which I've tried to indicate with the centipede cartoon in the middle of the slide, and that those Gaussians are uncorrelated moment to moment. Knowing noise now doesn't give you any information about noise in the future.

CLICK Finally, a different bit of statistical mechanics tells us how this noise power turns into noise voltage and noise current variances. This is the amplitude of voltage or current fluctuations on a noisy resistor that would explain the total noise power it emits.

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There are Lots of Units for Thermal Noise

Quantity	Symbol	What is?	In a resistor	Units	
Noise current variance	i_n^2, σ_i^2	Variance	$4kT\Delta f/R$	[A^2]	
Noise current	i_n, σ_i	Stdev	$\sqrt{4kT\Delta f/R}$	[A]	
Noise voltage variance	v_n^2, σ_v^2	Variance	$4kTR\Delta f$	[V^2]	
Noise voltage	v_n, σ_v	Stdev	$\sqrt{4kTR\Delta f}$	[V]	
Noise voltage variance density	$\overline{v_n^2}$	Variance density	$4kTR$	[V^2/Hz]	Called "spot noise"
Noise voltage density	$\overline{v_n}$	Stdev density	$\sqrt{4kTR}$	[V/rt. Hz]	A crazy unit!
Noise power	P_n, σ_P^2	Variance	$\sigma_P^2 = \sigma_v^2/4R$	[W]	
Noise power density	$\overline{P_n}, \overline{\sigma_P^2}$	Variance density	$\overline{\sigma_P^2} = \sigma_v^2/4R\Delta f$	[J]	
Noise Temperature	T_n	Variance density	$\sigma_v^2/4kR\Delta f$	[K]	

$$P_n = kT_n\Delta f \rightarrow T_n = \frac{P_n}{k\Delta f}$$

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We've introduced a lot of different measurements of noise and it's worth comparing them. The first column here has the name of quantities, the second has a symbol we're going to use for them, the third explains whether they are standard deviations or variances, and also whether they are total values or densities. A density in this situation refers to a quantity that appears at every value of frequency, and we can chase it through our equations by looking for whether a quantity is dependent on some differential bandwidth Delta f. We'll be exploring how to find the value of Delta f in these equations two videos from now. However, it's important to call densities out in this table to differentiate them from standard deviations and variances. Standard deviations and variances describe the noise processes that show up on oscilloscope traces, while densities are mathematical constructs that talk about how much noise power lives at different frequencies. The fourth column gives an expression for each of these quantities in a resistor, which are all just simple algebraic manipulations of the equations from the previous page. Finally, the last column gives the units of each of these noise quantities.

There's a lot to call out here, and working through the algebraic manipulations in the fourth column is good practice and a way to explore this material for yourself. However, I'm going to confine myself to a few notations. First, noise voltage variance density, or sometimes the noise power density depending the calculation your'e doing, is called spot noise. Spot noise refers to to the white noise level in a system. Second, noise voltage density has units of volts per root Hertz, which is a really crazy unit! We'll see soon that we mostly care about powers and variances in noise calculations, so this square root is just an artifact of trying to do a calculation with standard deviations that makes more sense with variances. Third, I've added one new quantity to this slide called the noise temperature. Noise temperature is just a different way of specifying a noise power density or a noise voltage density. If you know a noise power density level, noise temperature is the temperature that would produce it. Since we've only seen resistors in these videos, noise temperature is trivially equal to the actual physical temperature right now, but as we add more complicated elements to our noise analysis, noise temperature will be a useful shorthand for indicating the noise in our system.

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Summary

- Thermal noise is white and Gaussian (in voltage and current).

$$\sigma_v^2 = 4kTR\Delta f$$

$$\sigma_i^2 = 4kT\Delta f/R$$

- Thermal noise implies a noise power density (Gaussian squared)

$$P_n = kT\Delta f$$

- Noise temperature is the temperature that would explain an observed noise power level.

$$T_n = \frac{P_n}{k\Delta f}$$

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Adding and Referring Noise

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In this video we're going to examine how noise in one part of a circuit causes noise in another part. That process is called referring noise because the original noise doesn't go away, but it will appear larger or smaller when you examine its effect at other locations.

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Voltage (and Current) Noise Variances Add

$v_{n1}^2 = 4kTR_1\Delta f$

$v_{n2}^2 = 4kTR_2\Delta f$

$$v_{ntot}^2 = 4kT(R_1 + R_2)\Delta f = v_{n1}^2 + v_{n2}^2$$

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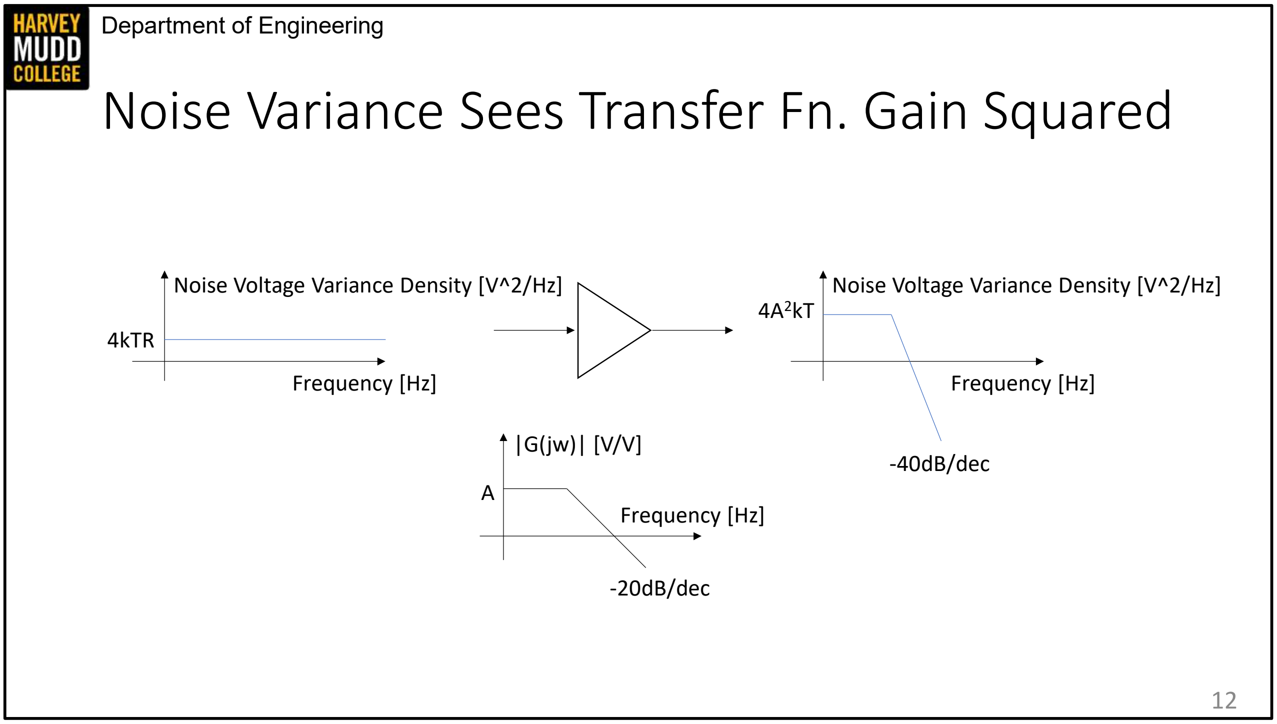
The rule of moving noise around is that noise variances add, which means that noise currents or voltages add in quadrature. Like last video, we're not going to be diving deep into the derivations of where these noise rules come from, but a quick demonstration of this principle is to imaging two resistors in series. Each resistor crease some voltage noise variance, we can see that on the left. On the right, I've evaluated the voltage noise variance of their series combination, which is equal to the sum of each of their individual voltage noise variances. If the resistors were in parallel, you'd see that the same behavior holds for current noise variances.

The big takeaway here is that variances add, so don't directly add noise voltages together.



The next important rule for moving noise around is that noise variance is scaled by the square magnitude of transfer functions. We're talking about noise variance here because we know noise variances add, so it's common to think about all of the noise in a system as variances. One weak justification for that is that the units of noise variance are volts squared while the units of a transfer function are volts per volt. We need to square the transfer function to match the input units. Real justifications involve autocorrelations and power spectral densities, which you can learn about in a random processes class down the road.

You can see this rule in action on this slide. A voltage noise variance is fed into an amplifier that has a DC gain of A and a first order rolloff at high frequencies, which is common for amplifiers. We can see that the noise spectrum at the output of the amplifier is scaled by A squared at DC, and it rolls off at 40dB per decade, indicating that the noise spectrum at the input has been scaled and shaped by the transfer function squared.

Summary

- Noise variances add, so don't just add noise voltages or currents.
- Noise variance is referred from one part of a circuit to another by the transfer function squared.

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Noise Bandwidth

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In this video we're going to talk about the relationship between noise density and observed noise levels.

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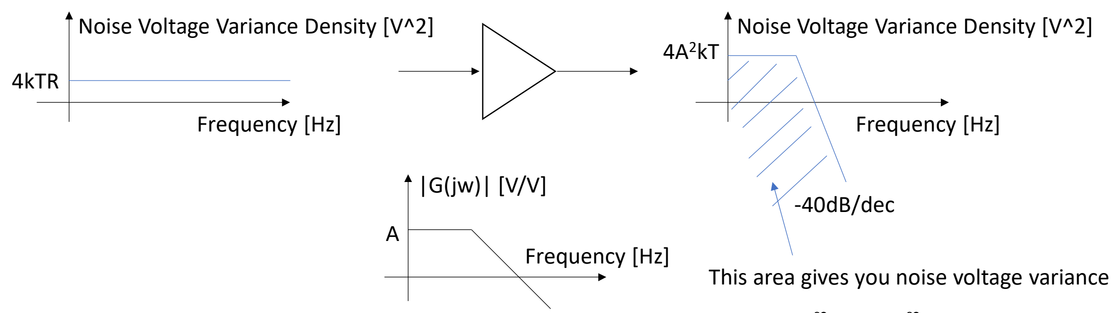
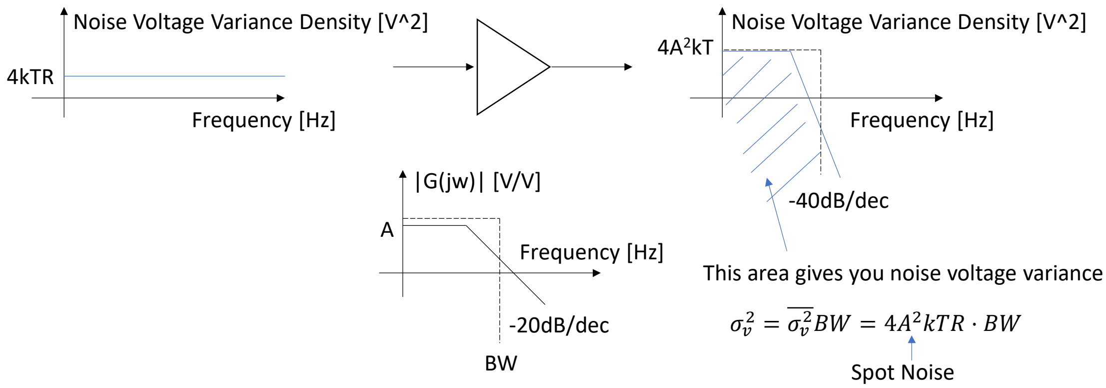
Observed Noise Set by "Captured" Density



$$\sigma_v^2 = \int_0^\infty \overline{\sigma_v^2} = \int_0^\infty 4kT|G(j\omega)|^2 \Delta f$$

The observed noise variance in a system is set by how much total noise power the system captures from a white noise density spectrum. In other words, systems with wide bandwidths will let in lots of noise variance density, while systems with narrow bandwidths will only let in a little noise variance density. You can see this in the shaded area of the spectrum on the right: the noise variance at the output of this amplifier is going to be given by the area under the noise spectrum. We find that area by integrating the transfer function of our circuit.

We Approximate with Brick Wall Bandwidths



$$\sigma_v^2 = \overline{\sigma_v^2} BW = 4A^2 kTR \cdot BW$$

Spot Noise

$$\frac{\sigma_v^2}{4R} = P_n = kT_n BW \rightarrow T_n = A^2 T$$

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We often simplify this calculation by pretending that the bandwidth of our systems is represented by a sharp cut-off, which is sometimes called a “brick wall filter”. I’ve indicated this made-up filter on the transfer function and our noise transfer function. Though the approximation is optimistic if you set your bandwidth wrong, you can get a decent estimate of noise density using this technique. This means the integral in the lower left turns into a simple multiplication of our spot noise by the bandwidth.

CLICK Alternatively, we can rearrange that equation in terms of noise power, which we know to be k times our noise temperature times our bandwidth. This tells us the noise temperature is A squared times the physical temperature. This matches expectations because the noise temperature is just a linearly scaled shorthand for noise power density, a variance, so we’d expect it to be referred through a system using the transfer function squared as well.

Summary

- Observed noise variance depends on how much noise density makes it through your system's transfer function.
- We often approximate system transfer functions with brick wall cutoffs to simplify the math

$$\sigma_v^2 = \overline{\sigma_v^2} BW = kT_n BW$$

- Noise temperature is referred by the transfer function squared, just like other noise variance densities.