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Lecture 20: Linearity and Distortion 2

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2^{nd} Order Intermodulation

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In this video we're going to study another form of distortion that arises when you drive nonlinear elements with two sinusoids.

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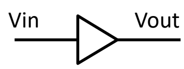
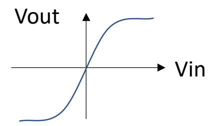
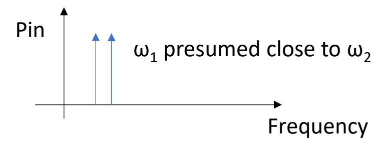
We Get New Distortion Behavior with 2 Tones







$$V_{out}(t) = a_1 V_{in}(t) + a_2 V_{in}^2(t) + a_3 V_{in}^3(t) + \cdots \qquad \text{Let } V_{in}(t) = V_1 \cos(\omega_1 t) + V_2 \cos(\omega_2 t)$$

$$V_{out}(t) = a_1(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t) + a_2(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t)^2 + a_3(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t)^3$$

$$a_2(V_1^2 \cos^2\omega_1 t + 2V_1V_2 \cos\omega_1 t \cos\omega_2 t + V_2^2 \cos^2\omega_2 t)$$

Second order harmonic distortion for V1

A new thing! Second order intermodulation!

Second order harmonic distortion for V2

$$2V_1V_2 \cdot \frac{1}{2}[\cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t]$$

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We're starting with our picture of a nonlinear amplifier represented by a Taylor Series, and we're going to assume that our input is two sinunsoids of similar frequency, which is different than the single sinusoid we've been testing with so far. This input signal is called a two-tone test.

CLICK We can substitute this input into our non-linear amplifier model
CLICK and if we focus on the second order term of the model, we can expand the term into a polynomial. We find that two of the terms in the polynomial are familiar, they look like the second order harmonic distortion that we'd expect to see from each of the V1 and V2 tones. However, we also get a third term that is new. This term is referred to as second order intermodulation because it comes from the interaction of the two input tones.
CLICK We can use the angle addition formula, which I'm not going to go through the trouble of deriving because I actually remember this one reliably, to expand the second order intermodulation term to individual tones. Per the angle addition formula, these tones show up at omega1 plus omega2 and omega1 minus omega2.

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Second Order Intermodulation is Out of Band

$$V_{out}(t) = a_1(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t) + \cdots$$

$$a_3(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t)^3 + \cdots$$

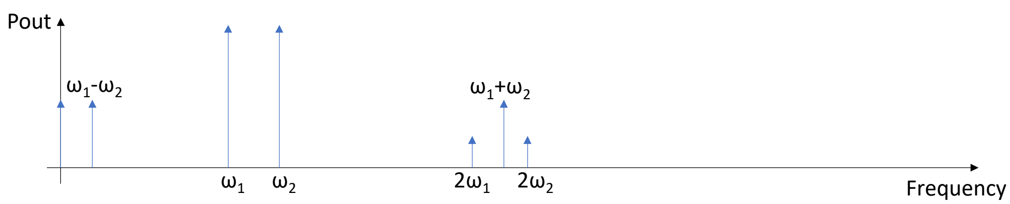
A lurking monster

$$\frac{a_2}{2}\Big[\underbrace{V_1^2(1+\cos 2\omega_1 t) + V_2^2(1+\cos 2\omega_2 t)}_{\text{Offset and 2}^{\text{nd}}\text{ harmonics like before}} + 2V_1V_2\cos(\omega_1+\omega_2)t + 2V_1V_2\cos(\omega_1-\omega_2)t\Big]$$

Close to 2nd harmonics

Close to DC

$$IM3 \stackrel{\text{def}}{=} \frac{\text{amplitude of 2nd intermodulation with } V_1 = V_2 = V_{zp}}{\text{amplitude of one tone test fundamental}} = \frac{a_2 V_{zp}^2}{a_1 V_{zp}} = \frac{a_2}{a_1} V_{zp} = 2 \cdot HD2$$



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Plugging our harmonic distortion and intermodulation distortion expansions into our model results in this monstrous equation. As noted, we're still sleeping on what the third order term will look like once we expand it out, but we can learn a few interesting things from the second order term.

CLICK The second order term contains second order harmonic distortion, so it should be no surprise that those terms turn into DC offset and second harmonics of each of the input tones. However, the intermodulation creates new frequency content. There is a term at omega1 plus omega2, which is close the second harmonics, and there's a term at omega1 minus omega2, which is close to DC. These intermodulation terms are each bigger than the harmonic distortion term by a factor of two. However, these tones aren't any worse than normal second order harmonic distortion: they're about a factor of two away from the fundamental in frequency, so a sharp filter can reduce their effect. One exception is that some receivers, called direct downconversion receivers, are very sensitive to signals near DC, and second order intermodulation creates a signal near DC.
CLICK We can define an IM2 measure of intermodulation. This measure is defined as the amplitude of the 2nd intermodulation product with equally sized tones divided by the size of one fundamental output tone. It turns out to be twice as big as the HD2 distortion measure.
CLICK The tones generated by the two-tone test are summarized in this graph. We can see

the DC offsets, the sum and difference intermodulation terms, and the harmonic distortion terms.

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Summary

- Two tones in a non-linear system result in intermodulation: additional distortion products beyond harmonic distortion
- 2nd order intermodulation produces tones at $\omega_1 + \omega_2$ and $\omega_1 - \omega_2$
- 2nd order intermodulation can be described using the IM2 product

$$IM2 \stackrel{\text{def}}{=} \frac{\text{amplitude of 2nd intermodulation with } V_1 = V_2 = V_{zp}}{\text{amplitude of one tone test fundamental}} = \frac{a_2}{a_1} V_{zp} = 2 \cdot HD2$$

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3^{rd} Order Intermodulation

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In this video we’re going to continue analyzing intermodulation terms in our amplifier model.

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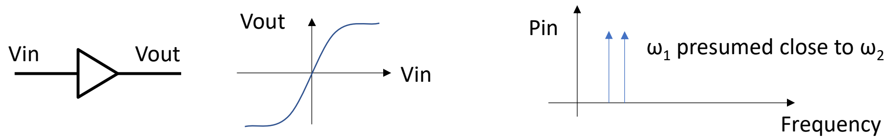
We Get New Distortion Behavior with 2 Tones



$V_{out}(t) = a_1 V_{in}(t) + a_2 V_{in}^2(t) + a_3 V_{in}^3(t) + \cdots$

Let $V_{in}(t) = V_1 \cos(\omega_1 t) + V_2 \cos(\omega_2 t)$

$$V_{out}(t) = a_1(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t) + a_2(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t)^2 + \underbrace{a_3(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t)^3}$$

$$a_3(V_1^3 \cos^3\omega_1 t + 3V_1^2 V_2 \cos^2\omega_1 t \cos\omega_2 t + 3V_1 V_2^2 \cos\omega_1 t \cos^2\omega_2 t + V_2^3 \cos^3\omega_2 t)$$

Third order harmonic distortion for V1

New things! Third order intermodulation!

Third order harmonic distortion for V2

$$3\frac{V_1^2 V_2}{4}[2\cos\omega_2 t + \cos(2\omega_1 + \omega_2)t + \cos(2\omega_1 - \omega_2)t] + 3\frac{V_1 V_2^2}{4}[2\cos\omega_1 t + \cos(2\omega_2 + \omega_1)t + \cos(2\omega_2 - \omega_1)t]$$

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The remaining term of interest is the third order term under the effects of a two-tone test.

CLICK This term expands into a polynomial that contains third order harmonic distortion terms for each tone and two third order intermodulation products.
CLICK Those intermodulation products expand into this expression, which you can prove to yourself at home using our previous expression for cosine squared and the angle addition formula. I encourage you to pause the video and do this short derivation for yourself.

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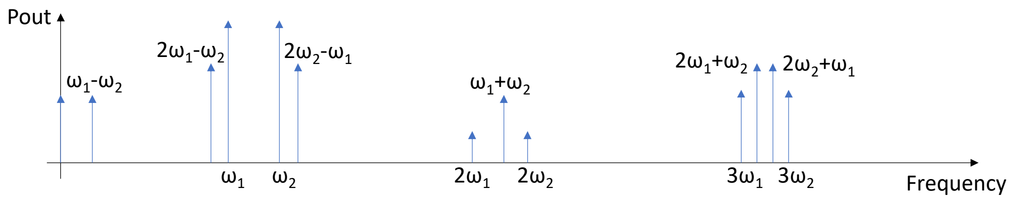
Third Order Intermodulation is In Band

$$V_{out}(t) = a_1(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t) + \cdots$$
$$\frac{a_2}{2}[V_1^2(1+\cos 2\omega_1 t) + V_2^2(1+\cos 2\omega_2 t) + 2V_1V_2\cos(\omega_1+\omega_2)t + 2V_1V_2\cos(\omega_1-\omega_2)t] + \cdots$$
$$\frac{a_3}{4}[V_1^3(\cos 3\omega_1 t + 3\cos\omega_1 t) + V_2^3(\cos 3\omega_1 t + 3\cos\omega_1 t) + \cdots$$
$$3V_1^2V_2\underline{(2\cos\omega_2 t + \cos(2\omega_1+\omega_2)t + \cos(2\omega_1-\omega_2)t)} + 3V_1V_2^2\underline{(2\cos\omega_1 t + \cos(2\omega_2+\omega_1)t + \cos(2\omega_2-\omega_1)t)}]$$

$$IM3 \stackrel{\text{def}}{=} \frac{\text{amplitude of in band 3rd intermodulation with } V_1 = V_2 = V_{zp}}{\text{amplitude of one tone test fundamental}} = \frac{3a_3V_{zp}^3/4}{a_1V_{zp}} = \frac{3}{4}\frac{a_3}{a_1}V_{zp}^2 = 3\cdot HD3$$



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I've included the whole output expression for a two-tone test on this slide.
CLICK The new developments, which I've just underlined, are that third order intermodulation produces more gain compression on each input tone and a handful of new tones at 2omega1 plus/minus omega2 and 2omega2 plus/minus omega1. The gain compression is no different than harmonic distortion, and the sum terms – 2omega1 + omega2 and 2omega2 plus omega1 -- are close to third harmonics, which makes them easy enough to filter out. However, the difference terms – the frequency components at 2omega1 minus omega2 and 2omega2 minus omega1 – fall very close to the fundamental frequencies, which makes them difficult to filter from your signal. These difference terms are often said to be "in band", referring to the range of frequencies your input signal occupies.

CLICK We can define a metric for third order intermodulation by comparing the amplitude of the in-band intermodulation products with one fundamental tone. This results in a metric that is three times larger than HD3.

CLICK The frequencies from the full amplifier output expression above appear here. You can see that there's a pattern of intermodulation making tones near to harmonic distortion terms such that you are left with little pyramids in your spectrum. These pyramids extend further if you apply input powers sufficiently high to create fourth and fifth order harmonic distortion. Again, the most problematic parts of this spectrum are the “in-band” terms at 2omega1 minus omega2 and 2omega2 minus omega1.

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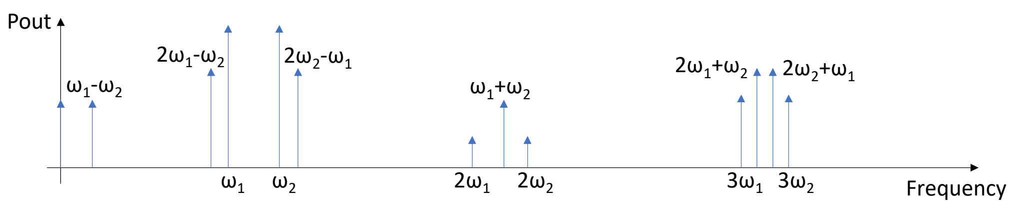
Summary

- 3^{rd} order intermodulation products at $2\omega_1 \pm \omega_2, 2\omega_2 \pm \omega_1, \omega_1, \omega_2$

- 3^{rd} order intermodulation described by IM3 product

$$IM3 \stackrel{\text{def}}{=} \frac{\text{amplitude of in band 3rd intermodulation with } V_1 = V_2 = V_{zp}}{\text{amplitude of one tone test fundamental}} = \frac{3}{4}\frac{a_3}{a_1}V_{zp}^2 = 3 \cdot HD3$$



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Creates a rich spectrum that consists of these little pyramids around each harmonic of your test tones

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Third Order Intercept Point (IP3, IIP3 and OIP3)

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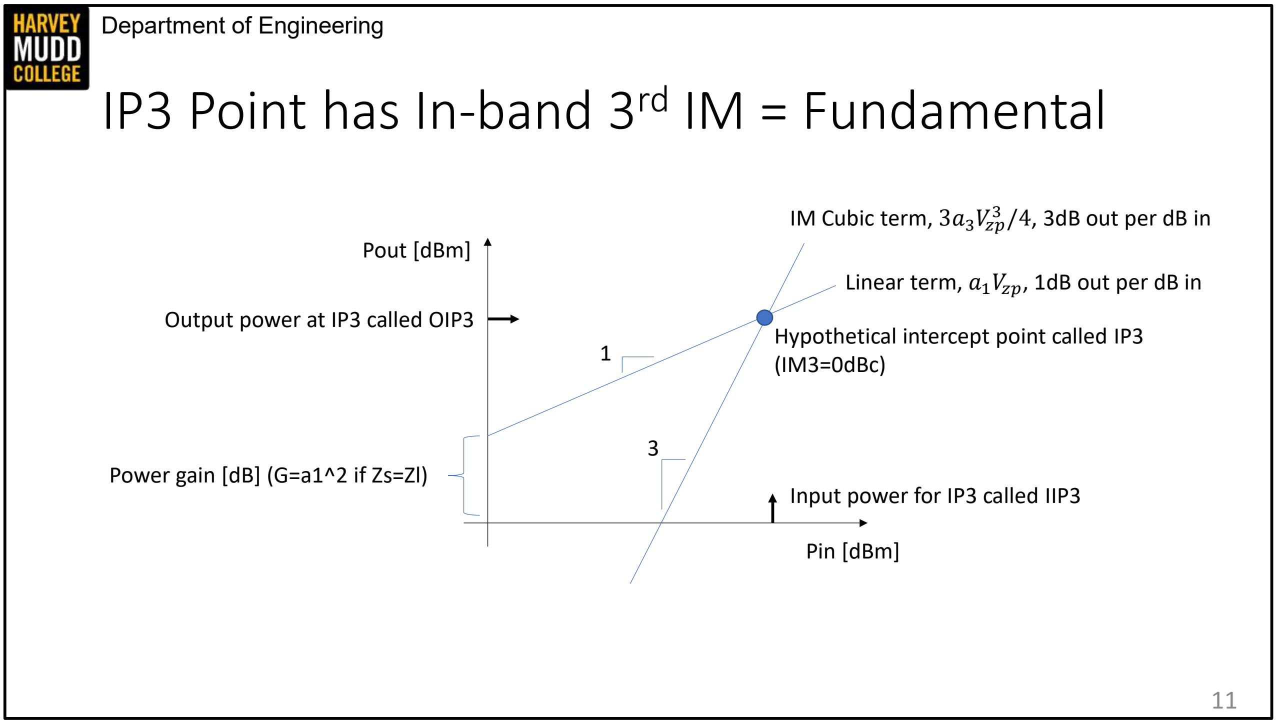
In this video we're going to discuss one more important measure of distortion, called the third order intercept point. We're going to see that this intercept point allows us to quickly calculate the power of distortion products, which is handy because IIP3 is often included on the datasheets of amplifiers.



We're going to approach this new measure by plotting the first order and third order components of an amplifier's output on logarithmic axes. I've noted that the axes are logarithmic by indicating that the units are dBm.

CLICK The first order component of amplifier power shows up as a straight line with an offset. The offset represents the power gain of the amplifier, which will be a1^2 if both the input and the output of the amplifier are matched to the characteristic impedance of their transmission lines. The line is straight and has a slope of 1 because a linear term in a normal plot will still be a linear term in a log-log plot – plotting y=x is the same as plotting log y = log x. Even though the definition of this line comes from a relation between output voltage and zero-to-peak input voltage, we can still plot the line on power axes because you can write a definition of decibels in terms of voltages.

CLICK The third order component of amplifier power shows up as a straight line as well, but it has a slope of three because the voltage curve that causes it is cubed. Extending our earlier analogy, plotting y=x^3 becomes log y = 3 log x on a logarithmic scale.

CLICK These two lines intersect at some place on the plot because the third order line has a steeper slope than the first order line, and that point is called the third order intercept point or IP3. Because the third order harmonic component is equal to the first order component at IP3, we can also say the IM3 is equal to 0dBc at IP3. The input power that results in IP3 is called the input-referred third-order intercept point, or IIP3, and the output power that results from an input of IIP3 is called the output-referred third order intercept point, or OIP3. OIP3 is related to IIP3 by the power gain of the system because the points are on the a1*Vzp line. IP3 is impossible to measure directly because amplifiers would blow up if you applied IIP3 to their input; creating a third order term the same size as the fundamental is way outside the practical capabilities of most amplifiers.

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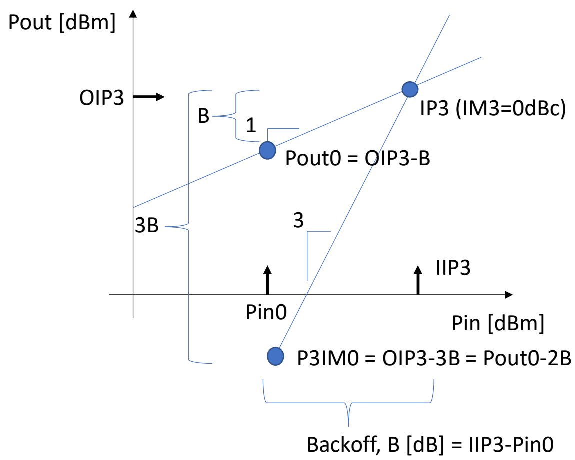
If You Know IP3, You Can Find P3IM from Pin



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So why keep track of IP3 if we can't measure it? We're most interested in IP3 because it's useful for calculating the power of intermodulation products.

CLICK We do that calculation by defining some input power Pin0 and observing that Pin0 is some distance from IIP3. We're going to call that distance our backoff, or B.
CLICK Because we know the slope of the first order and third order lines, we can find the power of both of those lines in terms of IP3. We know that the first order line is going to be B below OIP3 because the slope is one. And we also know that the third order line is going to be 3B below OIP3 because the slope is 3.
CLICK That means we can write expressions for the first order output power and the third order intermodulation power in terms of OIP3 and the backoff, which is the same thing as saying we can use the vertical version of backoff to find the powers we care about. We can also observe that the distance between the first order output power and the third order intermodulation power is given by 2B, so we know how big the 3^{rd} order intermodulation is compared to the fundamental.

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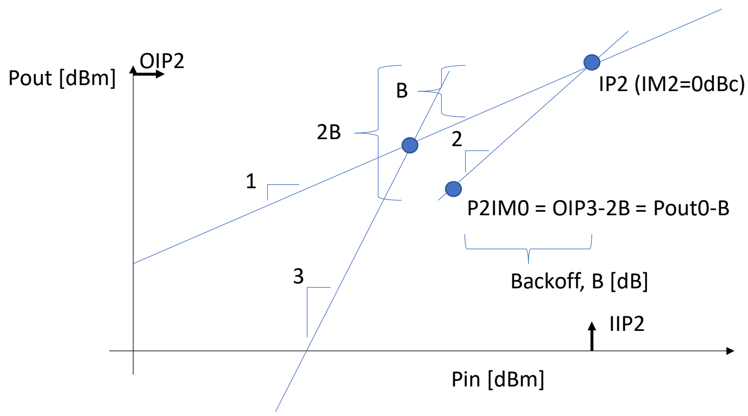
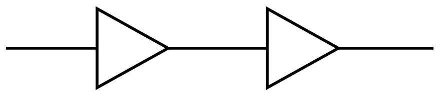
IP2 Also Exists, Only Matters in Corner Cases



Occasionally find IIP2 or OIP2 on datasheet, important for direct downconversion

It's worth noting that IP2 also exists, but it's often at a higher input power than IP3 because a2 is a small coefficient. When you're calculating your second order intermodulation products, you need to know that the second order term has a slope of 2 on this log-scale graph, which means your second order intermodulation product will be 2B below OIP3 and B below the first order line.

IIP2 and IIP3 Get Worse In Cascaded Amplifier



Stage 1	Stage 2
$a_{1,1}$	$a_{2,1}$
$IIP2_1$	$IIP2_1$
$IIP3_1$	$IIP3_1$

$$a_{total} = a_{1,1}a_{1,2}$$

$$\frac{1}{IIP2_{total}} = \frac{1}{IIP2_1} + \frac{a_{1,1}}{IIP2_2}$$

$$\frac{1}{IIP3_{total}} = \frac{1}{IIP3_1} + \frac{a_{1,1}^2}{IIP3_2}$$

Linearity matters most in early stages because distortion products amplified

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It’s possible to calculate the IIP2 and IIP3 for cascades of amplifers using the formulas I’ve shown here, which are referred to as cascade formulas. Note that these are linear equations, they aren’t in units of decibels.

I don’t use these often because I prefer to make a spreadsheet that tracks the power of each frequency component at every spot in the amplifier, even in between these two stages. However, these formulas convey one very useful physical insight, which is that the IIP2 or IIP3 of the first amplifier contributes more to the total IIP3 than the second amplifier. We can see that because the IIP2_2 and IIP3_2 contributions to the total equations are discounted by the gain of the first stage. That’s because the distortions product of the first stage are amplified by the rest of the amplifier chain, so even small distortion terms can be amplified to high levels. If you have more stages than two, each subsequent stage is discounted by all the gain that came before it.

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Summary

- Intercept points are hypothetical input powers where harmonics have the same power as the fundamental.
- You can calculate intermodulation power levels by backing off from intercept points.
- IIP2 and IIP3 have cascade formulas

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Relationships Between Distortion Quantities

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In this video we're going to summarize what we've learned about distortion quantities by comparing all of our distortion metrics at once.

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Quantity	Expression	Relation (linear)	Relation(dB)	What Causes?
HD2	$\frac{1}{2}\frac{a_2}{a_1}V_{zp}$			HD2
HD3	$\frac{1}{4}\frac{a_3}{a_1}V_{zp}^2$			HD3
IM2	$\frac{a_2}{a_1}V_{zp}$	$2 \cdot HD2$	$HD2 + 6\text{dB}$	IM2
IM3	$\frac{3}{4}\frac{a_3}{a_1}V_{zp}^2$	$3 \cdot HD3$	$HD3 + 9.54\text{dB}$	IM3
aeff	$a_1 + \frac{3}{4}a_3 V_{zp}^2$			HD3
P-1dB	$20\log\left(\frac{a_{eff}}{a_1}\right) = -1$		$IIP3 - 9.6\text{dB}$	HD3
Vzp@P-1dB	$V_{zp} = \sqrt{10^{-\frac{1}{20}}}\sqrt{\frac{4}{3}\left\|\frac{a_1}{a_3}\right\|}$	$\sqrt{0.11} \cdot V_{zp}@IIP3$	$V_{zp}@IIP3 - 9.6\text{dB}$	HD3
IIP3	$20\log(IM3) = 0$		$P_{-1dB} + 9.6\text{dB}$	IM3
Vzp@IIP3	$V_{zp} = \sqrt{\frac{4}{3}\left\|\frac{a_1}{a_3}\right\|}$		$V_{zp}@P_{-1dB} + 9.6\text{dB}$	IM3

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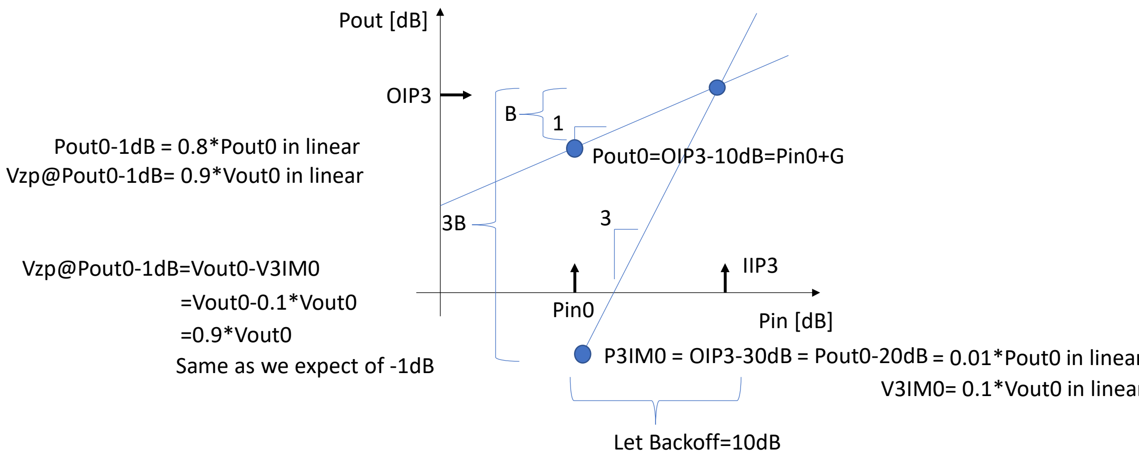
So here's a roundup of all of our distortion quantities in one place. There are two interesting portions of the table. The first is the top half that notes that the harmonic distortion products are pretty tightly related to intermodulation products. Conveniently, you can remember that IM2 and IM3 are each bigger than their linked harmonic distortion by their order.

The bottom half of the table has the most important relationship to remember from this video, which is that IIP3 is ~10dB above P-1dB. We can find that most easily by comparing the Vzp @ IIP3 to the Vzp @ P-1dB. They're related by a factor of sqrt(10^(-1/20)), which turns out to be 9.6dB. Because decibels are always ratios of powers, we can also say that IIP3 and P-1dB are related by 9.6dB.

This relationship is really useful because we can measure P-1dB while we can't measure IIP3. By measuring P-1dB for an amplifier, you can calculate IIP3 and in turn find your third order intermodulation. You can also find your harmonic distortion power based on the relation between IM3 and HD3. That makes this relationship one of the keys of interpreting amplifier datahseets.

IIP3 is ~10dB above P-1dB, Graphical Version



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Because the relation between P-1dB and IIP3 is so important, we're going to show another way to find it. We can also use our backoff calculation to show that P-1dB is 10dB below IIP3. We do that by letting our backoff be 10dB, and then showing that the third order intermodulation power would reduce the first order power by 1dB.

CLICK We're going to need to subtract our third order voltage amplitude from our first order voltage amplitude to do this because powers don't add directly and dB definitely don't, so we convert our third order modulation power back into a linear scale and then back into a voltage. We're neglecting the system impedance in this calculation because all of these powers are created by voltages driving the same impedance, and every voltage is expressed relative to another voltage, that means the impedance would fall out of these equations.

CLICK We follow that up by figuring out how big 1dB is. Reducing Pout0 by 1dB is the same as multiplying it by 0.8. If we convert that into the corresponding 0 to peak voltages, we see it's the same as reducing the zero-to-peak input to 90% of it's initial value.

CLICK Finally, we can bring those two calculations together to show that the 3rd order intermodulation, which is 0.1 Vout0, reduces the voltage output of the amplifier to 0.9Vout0. That matches the voltage we expect from our P-1dB calculation, so backing off by 10dB from IIP3 puts us at P-1dB.

Summary

- IIP3 is ~10dB above P-1dB
- Intermodulation is about order times worse than harmonic distortion